TM 11-681

DEPARTMENT OF THE ARMY TECHNICAL MANUAL

ELECTRICAL FUNDAMENTALS

(ALTERNATING CURRENT)

DEPARTMENT OF THE ARMY • *DECEMBER 1951*

AGO 1736A—Nov

TECHNICAL MANUAL

ELECTRICAL FUNDAMENTALS (ALTERNATING CURRENT)

TM 11-681
CHANGES No. 1

DEPARTMENT OF THE ARMY
WASHINGTON 25, D. C., *3 October 1955*

TM 11-681, 27 December 1952, is changed as follows:

4. Direction of an Induced Emf

a. The direction of * * * following simple rules:

(1) (Superseded) If the thumb, forefinger, and middle finger of the right hand are held at right angles to one another, the thumb pointing in the direction of the motion of the conductor and the forefinger pointing in the direction of the magnetic field, then the middle finger points in the direction of the induced emf; that is, toward the positive terminal. This is known as *Fleming's Right-Hand Rule*. If the directions given above are applied to the left hand rather than the right, the middle finger points in the direction of the current instead of the induced emf. It is then known as the left-hand generator rule. Refer to TM 11-661, Electrical Fundamentals (Direct Current), for a description of this rule.

* * * *

31. Capacitance

* * * * *

d. An examination of * * * neutral uncharged condition. **The arrows between the plates of the capacitor represent *electrostatic lines of force*. These lines of force make up an *electrostatic field* between the plates. The electrostatic field is caused by, and its proportional to, the difference of potential across the plates. Refer to TM 11-661 for a detailed description of electrostatic fields.**

* * * * *

33. The Parallel Plate Capacitor

* * * * *

b. (Superseded) The formulas above hold for a parallel-plate capacitor in a vacuum, that is, with nothing between the plates. The introduction of some insulating substance between the plates intensifies the electrostatic field and, hence, a greater charge is held on the plates for the same difference in potential. In an uncharged capacitor, the atoms that make up the insulating material are in a normal or neutral state. That is, the electrons of each atom revolve around its nucleus in normal paths (orbits) as shown in A, figure 44.1. The figure shows only two atoms (greatly enlarged) of the many millions that make up any piece of insulating material. When the capacitor is charged, an electrostatic field is produced between the plates. Since the insulating material is within the electrostatic field, its electrons are affected by the force of the field. The electrostatic field tends to put a strain on the electrons and, although they do not break away from the atoms, their orbits are distorted (B, fig. 44.1). The distortion of the electron orbits, known as *orbital strain*, sets up an electrostatic field within the insulating material. This field opposes the original electrostatic field between the plates and decreases its strength. The strength of the original electrostatic field must be proportional to the voltage across the capacitor. Since the voltage across the capacitor has not changed, the electrostatic field will tend to build up to its original strength. To do this, more electrons are attracted from the positive plate through the battery to the negative plate; in other words, the charge on the capacitor increases. Since the voltage across the capacitor remains the same while the charge increases, the capacity is increased. This can be shown by the formula for capacity:

$$C = \frac{Q}{E}$$

The degree of orbital strain and, hence, the amount the capacitance is increased, varies with different types of insulating materials. Thus, by proper selection of this insulating material, known as the *dielectric*, capacitance may be increased, since the ratio Q to E is increased. Various materials have

been tested experimentally to determine the ratio of this increase compared to a vacuum, which is taken as 1. This ratio is called the *dielectric constant, K*, of the material and is a factor independent of area or thickness, since in any single case it is a comparison with a vacuum of the same dimensions. The formulas above then become:

$$C = 8.84 \times 10^{-8} \frac{KA}{d}$$

and:

$$C = 22.45 \times 10^{-8} \frac{KA}{d}$$

where K is the dielectric constant. This formula reveals that the greater the dielectric constant K, the greater will be the capacitance for a given set of dimensions.

* * * * *

35. Capacitance in Ac Circuits

* * * * *

b. This effect of * * * in the other. For example, the **dc** resistance of a 2-microfarad wax paper capacitor is very high (millions of ohms), yet the same capacitor placed across a 400-cycle ac source shows an appreciable current flow. Figure 47 illustrates * * * such a current.

c. From the observations * * * is 1 ampere. Therefore, by Ohm's Law, the **opposition** offered by the capacitor is only 200 ohms. This opposition is * * * to current flow.

70. Impedance and Reactance in the Ideal Parallel Tank Circuit

a. In the series * * * Then: $\frac{E}{Z} = I_L - I_C$.

And:

$$Z = \frac{E}{I_L - I_C} = \frac{E}{\frac{E}{X_L} - \frac{E}{X_C}} = \frac{E}{\frac{E(X_C - X_L)}{X_L X_C}}$$

$$Z = \frac{X_L X_C}{X_C - X_L}.$$

* * * * *

71. Q of a Parallel Resonant Circuit

* * * * *

b. In the *series* * * * that is, by R. Therefore:

$$Q = \frac{\frac{X_L X_C}{R}}{X_C} = \frac{X_L}{R}.$$

Thus, the Q * * * then: $I_C = QI$.

* * * * *

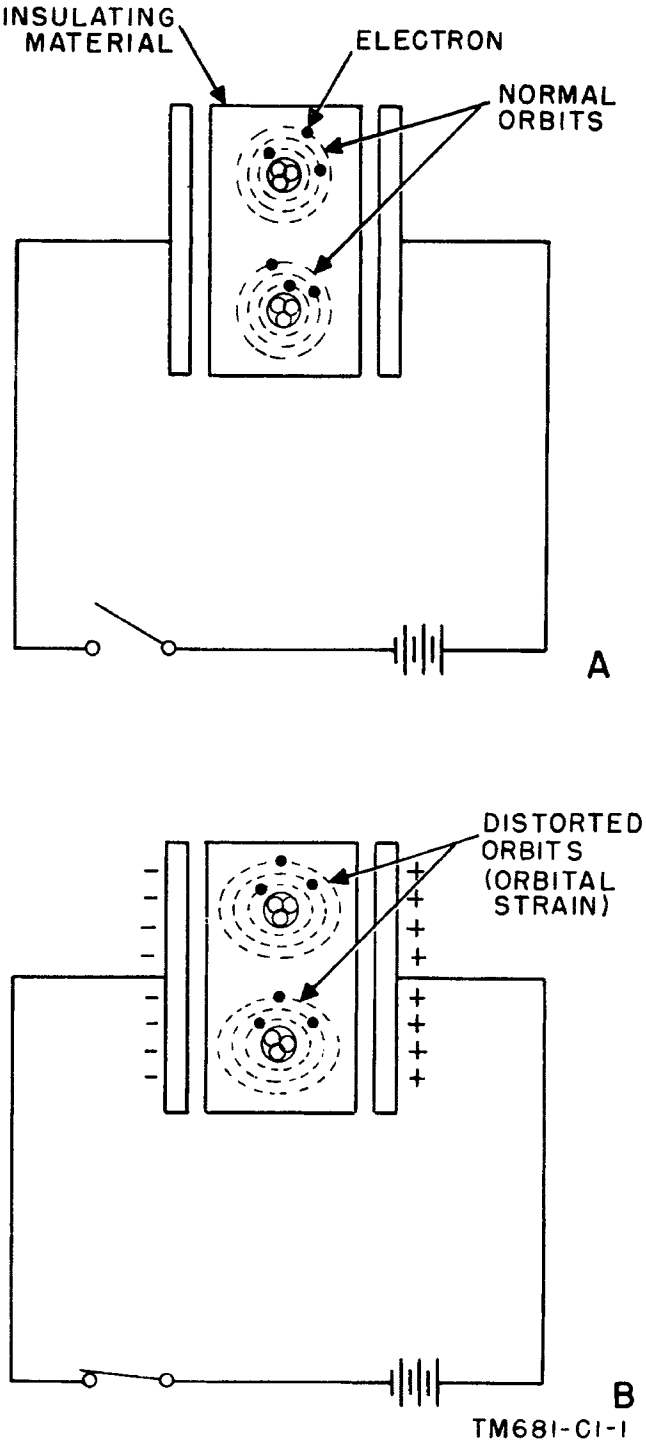

Figure 44.1 (Added) *Orbital strain in dielectric of capacitor.*

Figure 112. The direction of the arrows showing the flux linkage, ϕ_m, and the primary flux leakage is reversed.

78. Theory of Transformer Operation

* * * * *

b. The importance of * * * in chapter 2. At any given instant of time, the **primary** current is in the direction shown. The direction of the flux lines is determined by the **left-hand rule** *for*

coils and solenoids: if the fingers of the left hand point in the direction of electron flow in the coil winding, the thumb points in the direction of the magnetic field (**TM 11–661**). At this same * * * of figure 114.

* * * * *

f. The total action * * * it was designed. An ideal transformer would show the current and voltage vectors superimposed (θ_s and θ_p equal to 0°)—that is, voltage and current in each side in phase, and the secondary voltage and current 180° out of phase **with the primary voltage and current.** Such a transformer * * * current shifted 180°.

88. Radio Frequency Transformers

* * * * *

b. (Superseded) The typical radio frequency transformer consists of an untuned primary, which is connected to the output circuit of a vacuum tube, and a tuned secondary connected to the input circuit of the following vacuum tube (A, fig. 142). The two coils are wound next to each other, or one over the other, for close coupling, and with the number of turns in each dependent on the function of its particular circuit. Thus, the primary winding has many turns because its principal function is to offer high impedance to the first vacuum tube; the secondary

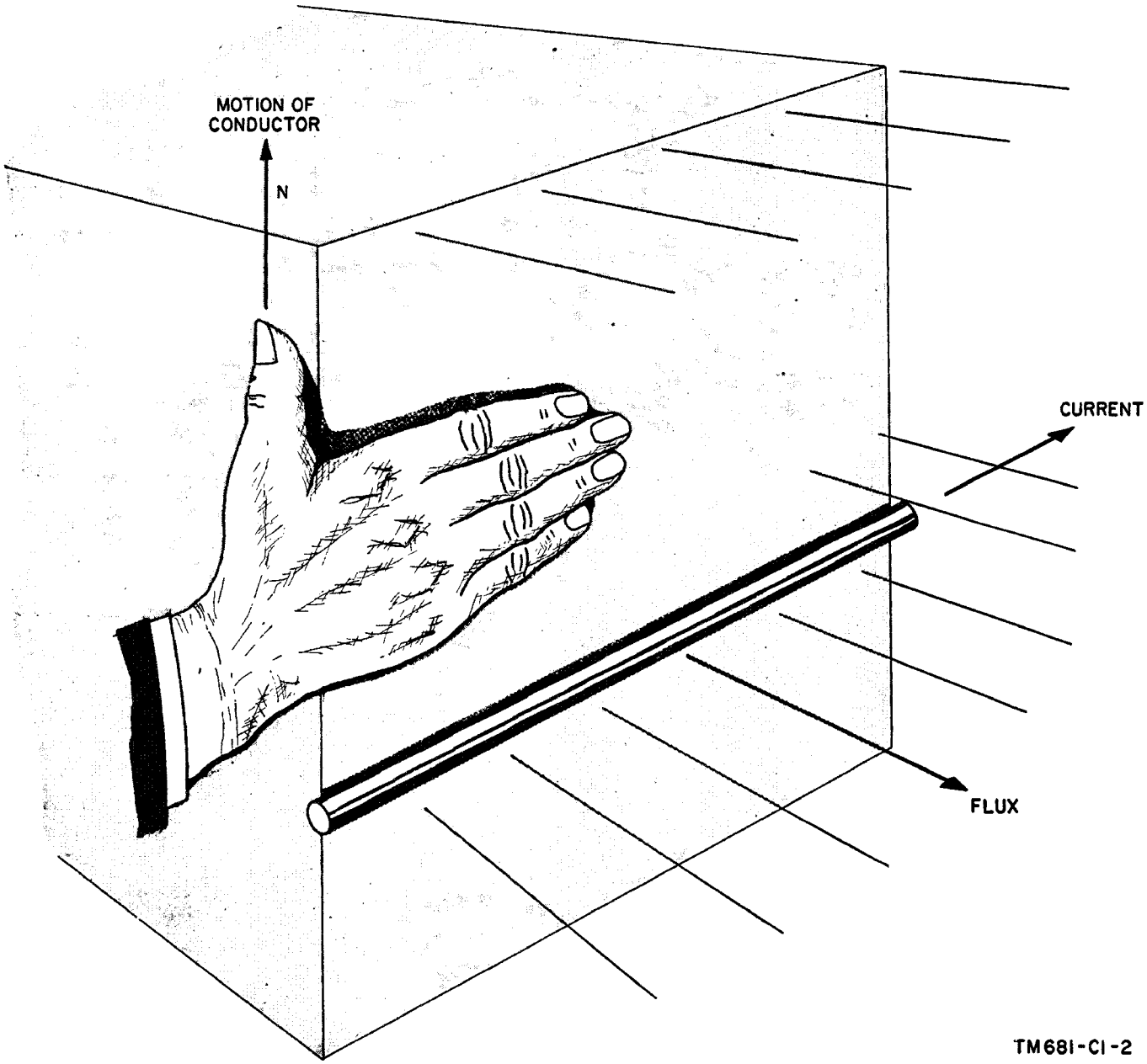

Figure 171. (Superseded) *Right-hand rule for motors.*

winding is determined by the amount of inductance needed for the band of frequencies to be tuned. When the resistance of the primary circuit is much greater than the inductive reactance of the primary, the resonance curve of current in the secondary circuit is as shown by the solid line curve of B, figure 142. When the reactance of the primary is greater than the resistance of the primary circuit, the Q of the secondary circuit falls and its resonant frequency is shifted to a higher frequency, as shown by the dotted line curve in B, figure 142. This is because of the mutual coupling between the two circuits. Because of this coupling, conditions in the primary circuit affect the secondary. When the primary circuit becomes inductively reactive, it effectively couples an impedance consisting of capacitance and resistance in series with the secondary circuit. The effect of series added capacitance increases the resonant frequency while the added resistance lowers the Q.

* * * * *

Figure 152. The direction of the arrows showing current in the loops of B and D, figure 152, is reversed.

94. Direct-Current Generator

By replacing the * * * called an *armature.*

* * * * *

b. A graph of 1 cycle of operation is shown in figure 155. The generation of the emf for positions A, B, and C is the same as for the basic ac generator, but, at position ***D,*** commutator action reverses the current in the external circuit, and the second half-cycle has the same waveshape as the first half-cycle. For this reason, * * * one direction only.

* * * * *

Figure 155. The direction of the meter needle in D, figure 155, is changed to indicate the same direction as the one shown in B of the same figure.

103. Right Hand Motor Rule

There is a * * * flow are known.

BY ORDER OF THE SECRETARY OF THE ARMY:

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For explanation of abbreviations used, see SR 320-50-1.

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TECHNICAL MANUAL

ELECTRICAL FUNDAMENTALS (ALTERNATING CURRENT)

TM 11-681
CHANGES No. 2

HEADQUARTERS,
DEPARTMENT OF THE ARMY
WASHINGTON 25, D. C., *5 June 1958*

TM 11-681, 27 December 1951, is changed as follows;

Note. The parenthetical reference to previous changes (example: "page 2, C 1") indicates that pertinent material was published in those changes.

Page 1, paragraph 1. Add the following note after subparagraph *a*:

Note. The above description applies to the type of waveshapes shown in figure 1. Other types of ac waveshapes are covered in TM 11-669, Transients and Waveforms.

Page 7, paragraph 4*c*.

Line 8. Delete "into" and substitute: **out of.**

Line 11. Delete "out of" and substitute: **into.**

Page 9, paragraph 6*a*, sixth sentence. Delete "revolution" and substitute: **10 revolutions.**

Page 29, paragraph 24*d*, second equation. Delete "$Z=\frac{R\times X_L}{R\times X_L}$ and substitute: "$Z=\frac{R\times X_L}{R+X_L}$.

Page 39, paragraph 31*c*. Delete the third sentence and substitute: **The flow of current charges the capacitance of the circuit (fig. 43) and this charge will be stored indefinitely until it leaks away or is used at some later date.**

Page 47, paragraph 38*c*, last line. Change "inductive" to: **Capacitive.**

Page 48, paragraph 39*a*, line 8. Change "since" to: **sine.**

Page 90, paragraph 60*a*, line 10. Delete "$X_L=6.28\times2\times10^{-2}$" and substitute: $X_L=6.28\times2\times10^{2}$.

Page 91, paragraph 61*a*. Delete the last equation and substitute:

$$\text{Then: } f_r=\frac{1}{2k\sqrt{LC}}=\frac{1}{6.28\times\sqrt{2\times10^{-3}\times8\times10^{-11}}}$$

$$=\frac{1}{6.28\sqrt{16\times10^{-14}}}=\frac{1}{6.28\sqrt{\frac{16}{10^{14}}}}$$

$$=\frac{1}{\frac{25.12}{10^7}}=\frac{10^7}{25.12}$$

$$=398{,}000 \text{ cps.}$$

Page 93, paragraph 63*a*, line 5. Change "derectly" to: **directly.**

Page 94, paragraph 65*a*, third sentence, equation. Change "=15,000 cps" to: =**159,000 cps.**

Page 95, paragraph 65*c*, Third equation, Change

"$L=\frac{X_L}{2\pi f}=\frac{1{,}365\times10^{-3}}{6.28\times455\times10^{3}}$," to:

"$L=\frac{X_L}{2\pi f}=\frac{1{,}365}{6.28\times455\times10^{3}}$

Last equation. Change

"$f_r=\frac{107}{22}$=455,000 cps" to:

$f_r=\frac{10^7}{22}$=455,000 cps.

Page 98, paragraph 68*b*. Delete the third sentence and substitute: **Thus, these two currents are 180° out of phase with each other and each**

current is 90° out of phase with the applied voltage. Partial or complete cancellation will take place in the line, depending on the amount of each of the branch currents.

Page 103 (page 2, C 1), paragraph 71*b*, sixth sentence. Change

"Therefore: $Q=\frac{X_L X_C}{\frac{R}{X_C}}=\frac{X_L}{R}$." to:

Therefore: $Q=\frac{\frac{X_L X_C}{R}}{X_C}=\frac{X_L}{R}$.

Page 105, paragraph 72*c*, sixth sentence. Change

"Band pass$=\frac{f_r}{Q}=\frac{100{,}000}{50}=5{,}000$ cycles." to:

Band pass$=\frac{f_r}{Q}=\frac{100{,}000}{50}=2{,}000$ cycles.

Page 127, paragraph 85*a*, line 9. Change "E_o" to: $\mathbf{R_o}$.

Page 133, paragraph 87*b*, sixth sentence. Delete the equations and substitute:

$$\frac{N_p}{N_s}=\sqrt{\frac{Z_p}{Z_s}}=\sqrt{\frac{100}{200{,}000}}$$

$$\frac{N_p}{N_s}=\sqrt{\frac{1}{2{,}000}}=\frac{1}{44.6}.$$

Page 156, paragraph 104, first sentence. Change "d-c generator" to: **dc motor.**

Page 168, paragraph 114*b*, line 8. Change "I_2" to $\boldsymbol{L_2}$.

Page 172, paragraph 115*f*, line 6. Change "208-volt" to: **120-volt.**

Page 173, paragraph 115*f*, last equation. Change "$\frac{E_y}{\sqrt{3}}=20{,}000\ 1.73=11{,}600$ volts (approx)" to:

$$\frac{E_y}{\sqrt{3}}=\frac{20{,}000}{1.73}=11{,}600 \text{ volts (approx)}.$$

Page 183, Appendix I, paragraph 2*d*. In all of the formulas change "b" to: **B.**

Figure 202. Change the caption to:

Graph of $3x+4y+12=0$.

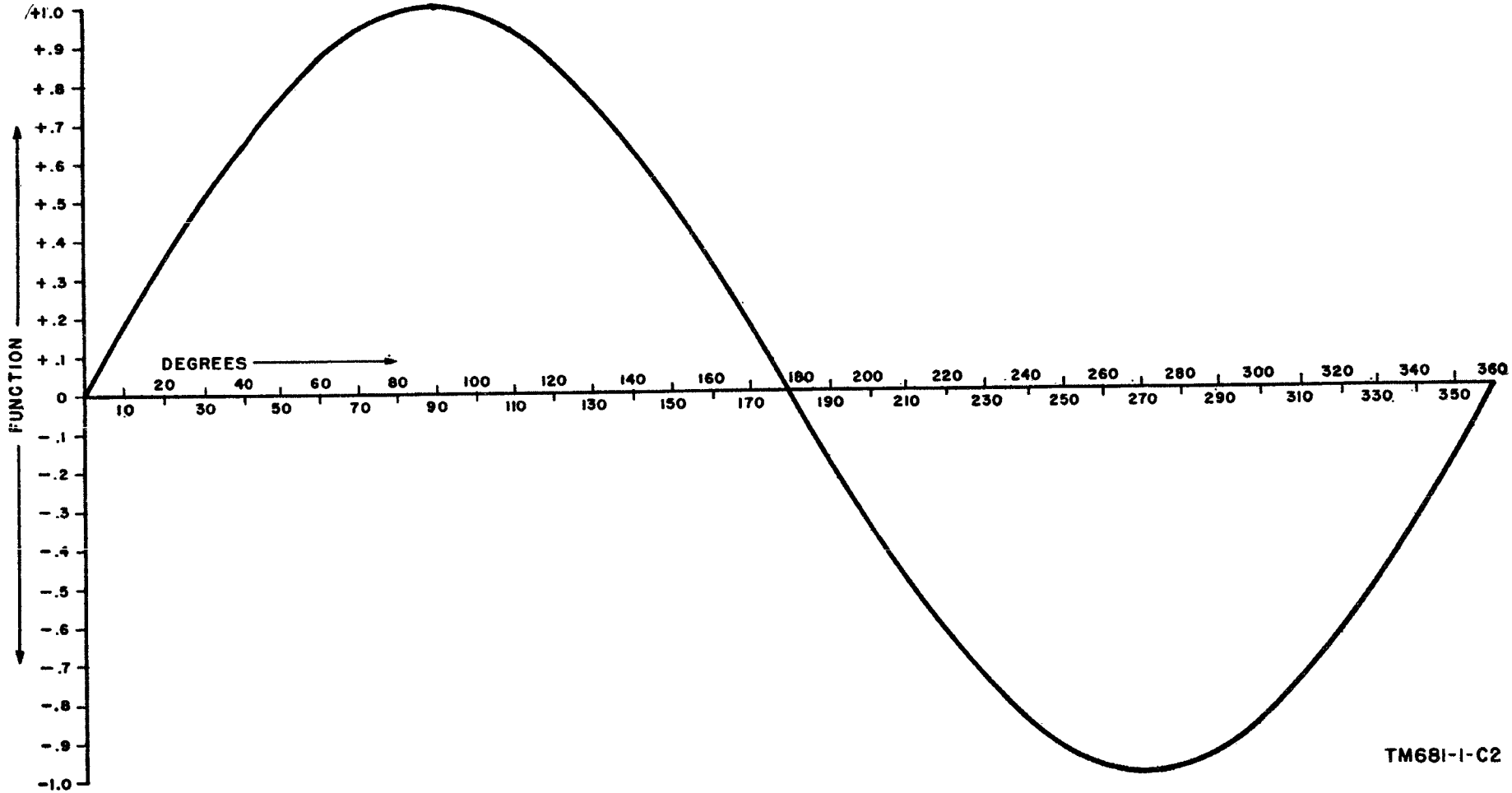

Figure 246. (Superseded) *Graph of sine curve.*

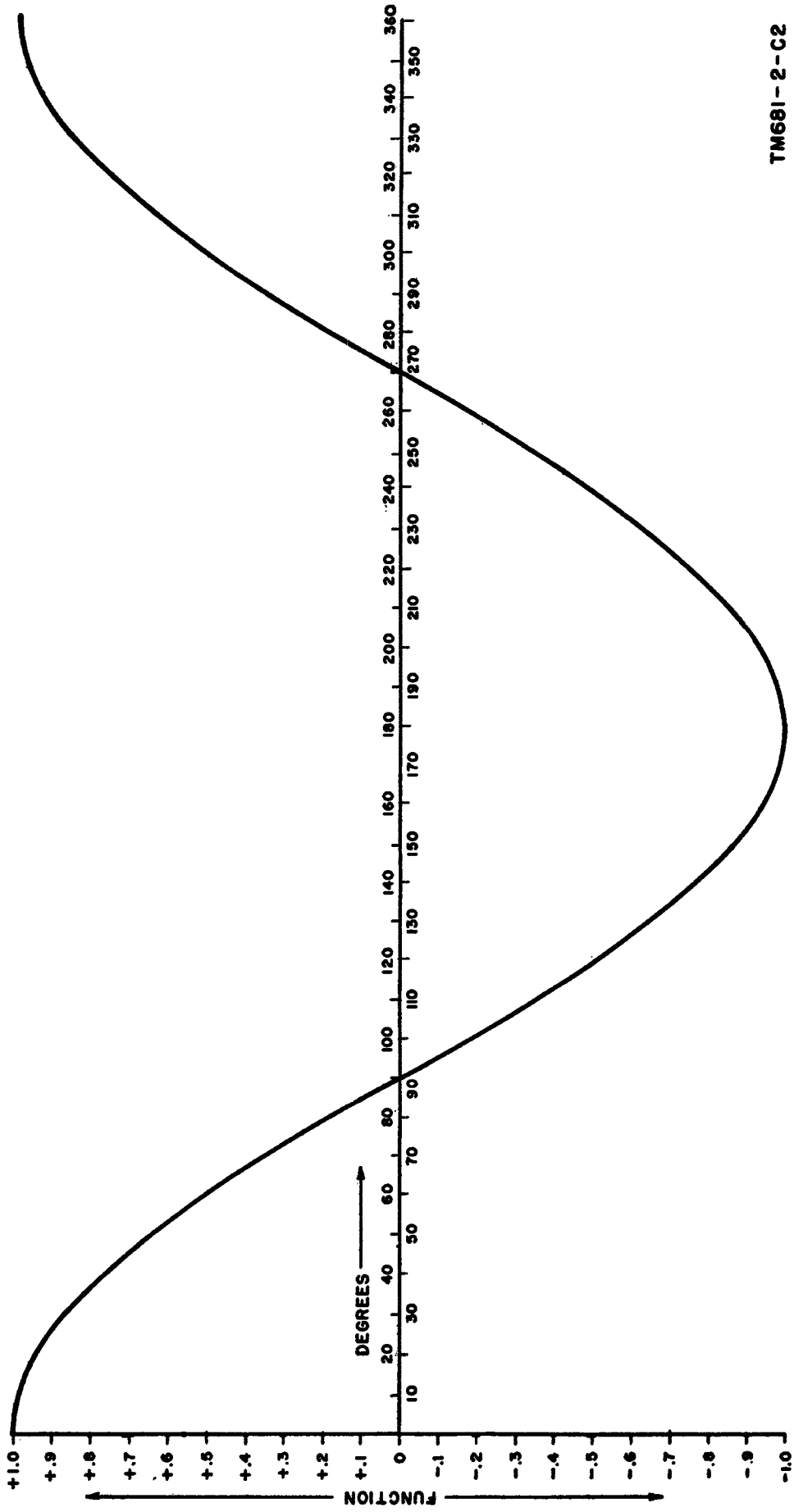

Figure 247. (Superseded) *Graph of Cosine Curve.*

[AG 412.42 (23 May 58)]

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TM 11-681

This manual supersedes so much of TM 11-490, 10 August 1944, as pertains to alternating-current electricity.

ELECTRICAL FUNDAMENTALS

(ALTERNATING CURRENT)

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DEPARTMENT OF THE ARMY

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TM 11–681 is published for the information and guidance of all concerned.

[AG 412.42 (1 Nov 51)]

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For explanation of distribution formula, see SR 310–90–1.

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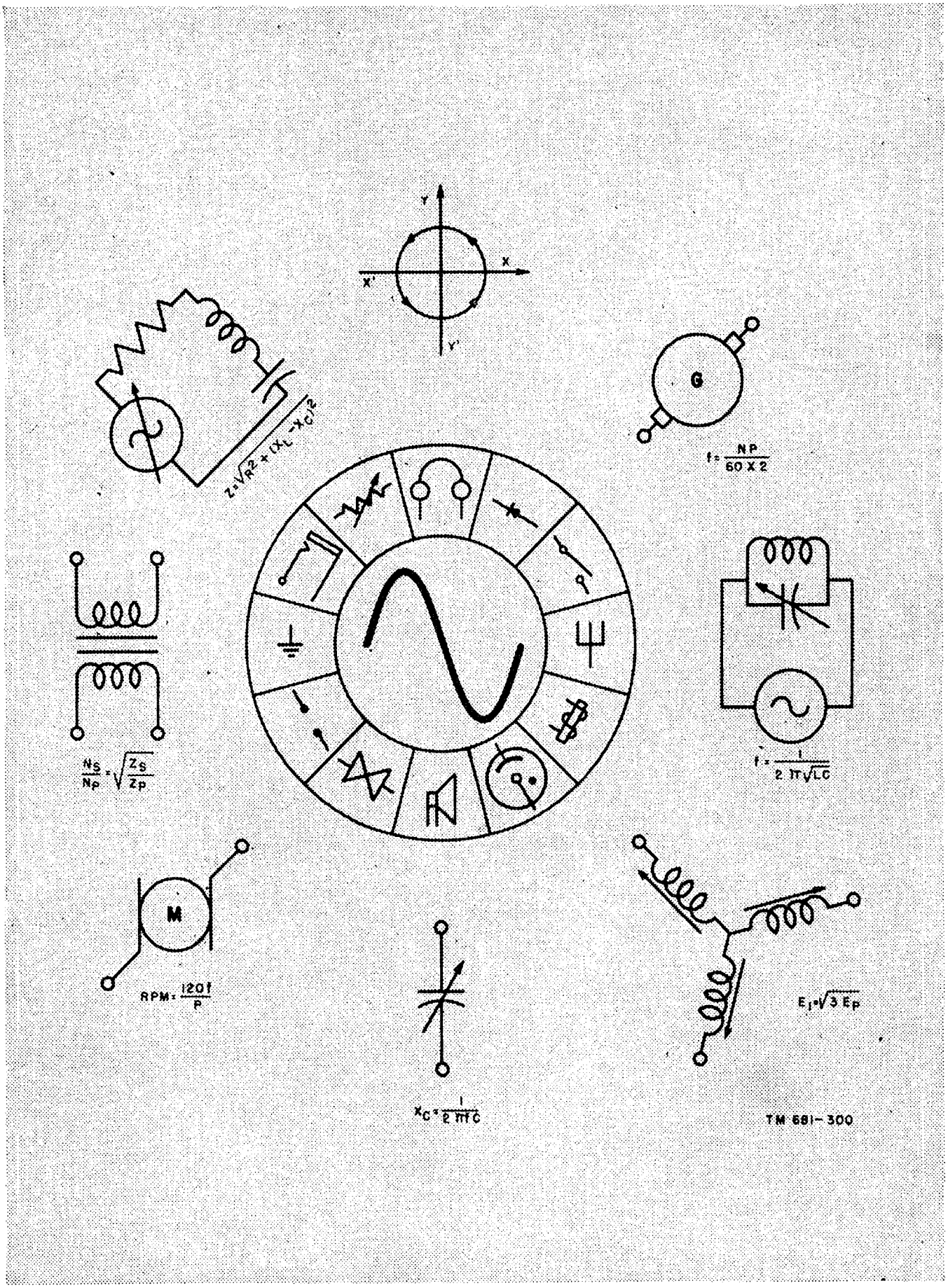


Frontispiece.

This manual supersedes so much of TM 11-490, 10 August 1944, as pertains to alternating-current electricity.

CHAPTER 1

ALTERNATING CURRENT

1. Alternating Current and Direct Current

a. Alternating current is an electric current which moves first in one direction for a fixed period of time and then in the opposite direction for the same period of time. Unlike direct current (A of fig. 1), which reaches some value, or magnitude, in a short time and then remains at that magnitude as long as the circuit is completed, alternating current is constantly changing in magnitude. From a zero value, alternating current builds up to a maximum in a positive direction, and then falls off to zero value again (the condition of no current flow) before building up to a maximum in the opposite or negative direction and then finally returning again to zero (B of fig. 1). For this reason, alternating current is defined as a current which is constantly changing in magnitude (either building up or falling off) and periodically (at set intervals of time) changing direction.

b. Although direct current was the first type of current to be widely used and the first whose characteristics were understood, it is limited in the number and kind of its applications to power machinery and to electronic circuits, and is used today only in special circumstances. The characteristics of alternating current, on the other hand, were not clearly understood until toward the end of the last century, and therefore, its efficient use as a source of power is a more recent development. In fact, there are today older sections of American cities, such as Greenwich Village in New York City, still supplied with direct current because these sections were originally electrified before the use of alternating current machinery had become widespread. Direct current must continue to be supplied to these areas until all their electrical apparatus and machines have been changed or converted to the use of alternating current.

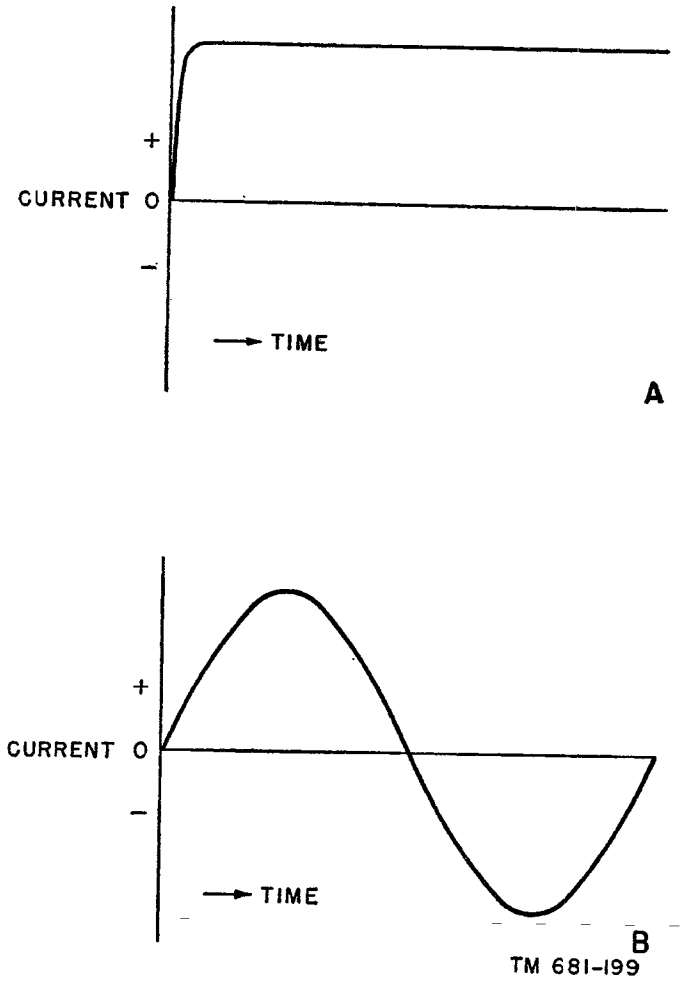


Figure 1. Graph of a direct current, A, and an alternating current, B.

c. As indicated above, the first electric power machinery was designed for use on direct current, which at that time seemed simpler to handle than alternating current. It was soon evident, however, that the use of direct current,

although simpler, had certain disadvantages which were not characteristic of the use of alternating current. Following is a listing of the most important of these disadvantages:

(1) *Direct current cannot be transmitted over long distances without a considerable loss of power.* The total resistance of the wires from the point of origin to the point of use would consume most of the power as heat before it arrived at its destination. Alternating-current power, on the other hand, may be sent across country without appreciable loss as low current at high voltage on the now familiar high-tension wires; these voltages are transformed at the place of use into suitable current and voltage for consumption in factory or home.

(2) *Direct-current power cannot be radiated from an antenna.* The whole art of radio communication as we know it today depends in large measure on the capability of alternating current to radiate its energy into space from an antenna and to project itself for long distances across the surface of the earth.

d. Accordingly, alternating current gradually became recognized as a more suitable and versatile source of power than direct current, particularly since alternating current has the overall advantage of being converted with relative ease to direct current. On the other hand, the conversion of dc (direct current) to ac (alternating current), though possible, is often neither convenient nor easy. This does not mean that dc is outmoded or useless; there are many situations in which direct current is the proper if not the only source of power, particularly in mobile installations such as the automobile and the airplane in which the initial source of power is the d-c storage battery.

2. Electromagnetic Induction

a. The first type of electric current generally used was direct current from wet cells and batteries. In 1819, Hans Christian Oersted, a Danish physicist, was experimenting with dc and accidentally discovered that a wire carrying an electric current affected a compass needle and was, therefore, itself a kind of magnet. This was called an electromagnet to distinguish it from a natural or artificial magnet. In either case, the magnetic lines of force, or the magnetic field about a wire or coil carrying a current is the same as that about a natural magnet. Oersted's discovery meant that electricity and magnetism were closely related, since one could be used to produce the other. However, it was not until 1831, 12 years later, that Michael Faraday in England and Joseph Henry in America were able to prove the converse—namely, that a magnet could be made to produce an electric current.

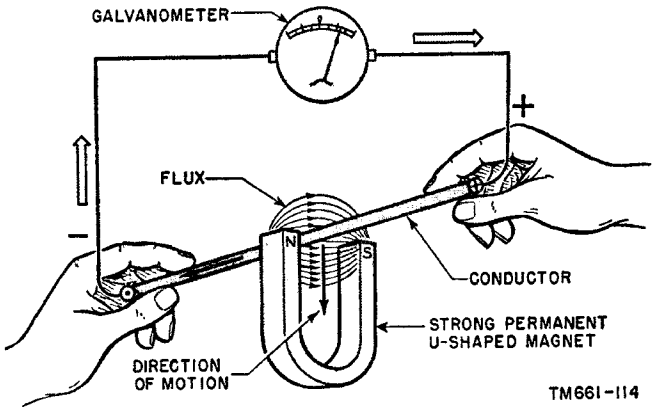


Figure 2. Inducing an emf.

b. Faraday, in his now classic experiment, connected a sensitive galvanometer across a coil and found that when a magnet was thrust into the coil a current flowed in the coil, and that when the magnet was withdrawn a current flowed in the opposite direction. Current flow, however, resulted only during the time the magnet was *moving*—that is, when the lines of force about the magnet cut the wires of the coil. The opposite condition was also found to be true, that if the magnet was held stationary and the coil moved, current flowed during the time of moving. Thus, an alternating current had been produced. Figure 2 illustrates this principle in terms of a single conductor and a horseshoe magnet.

3. Magnitude of Induced Emf

a. MOVEMENT. It has been found that the faster the movement of a magnet toward a wire, or vice versa, the greater the emf (electromotive force) or voltage produced, and also, the

stronger the magnetic field the greater the voltage. From these observations it can be concluded that the measure of the emf produced depends directly on the number of magnetic lines cut by the wire of the coil in a given unit of time. Thus, to increase emf it is necessary either to move the magnet faster or to increase the number of lines present by means of a stronger magnet. This may be stated generally: The emf induced in a moving conductor is proportional to the number of magnetic lines of flux cut by the conductor per second. To express the electromotive force in volts, a practical unit, rather than in units of the cgs (centimeter-gram-second) electromagnetic system, it is necessary to multiply the number of flux lines by the factor 10^{-8}, resulting in the following formula:

Induced emf (in volts) = flux lines cut per second $\times$ 10^{-8}.

is equal to the area in cm^2 times the number of lines per square cm—

$$1\ cm^2 \times \frac{10^9\ lines}{cm^2} = 10^9\ lines.$$

Thus, 10^9 lines are cut by the conductor in 1 second.

(2) The voltage e induced in the conductor is therefore:

$$e = \frac{10^9\ lines}{sec} \times 10^{-8} = 10\ volts.$$

c. ROTATION. It is mechanically awkward to produce an alternating emf by moving a magnet first into a coil and then out again, or by moving the coil back and forth about a stationary magnet. But a simple generator of alternating current may be set up by rotating a single turn coil within a stationary magnetic field. Thus, continuous uniform movement is possible, and the direction of the induced emf reverses as the coil

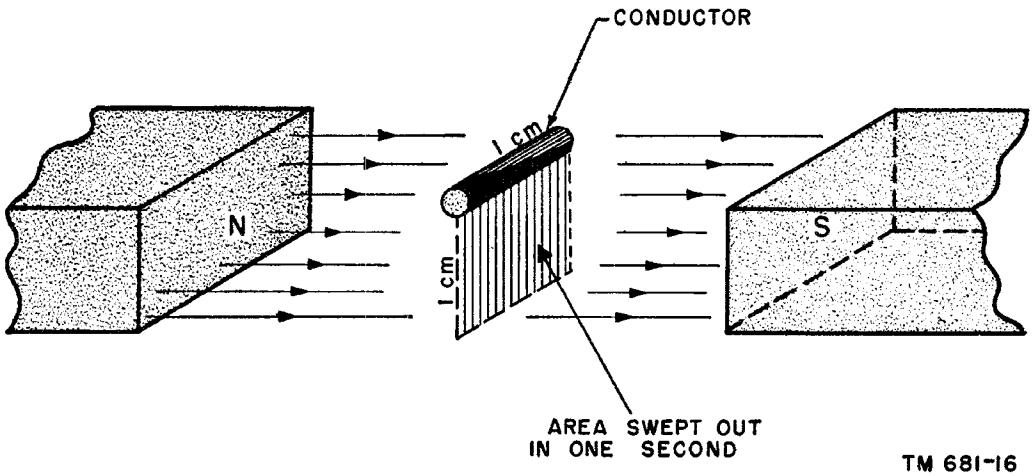


Figure 3. Conductor moving vertically down through a magnetic field.

b. EXAMPLE NO. 1: Assume that the poles of the magnet shown in figure 3 produce a flux density of 10^9 lines per square cm (centimeter) or 10^9 lines/cm^2. Also, assume that the conductor is 1 cm in length and is moving vertically downward at a velocity of 1 cm/second. The voltage induced in the conductor may be found as follows:

(1) The area swept out at right angles to the flux lines by the conductor in 1 second is equal to the length of the conductor in cm times the distance of travel in cm, or 1 cm^2. This area is shown shaded in figure 3. The number of flux lines passing through this area

turns, since as the conductor moves down through the field a voltage is induced in one direction, and induced in the other direction as it moves up through the field. However, before considering the method for determining the *direction* of this induced emf, the effect on the magnitude of the induced emf because of the rotation of the coil must be considered. The following example will illustrate that the moving conductor will cut fewer flux lines when moving at an angle to the flux lines than when the conductor moves directly downward or upward across the field of the magnet.

d. EXAMPLE NO. 2. The same conductor used in example No. 1 is moved with the same veloc-

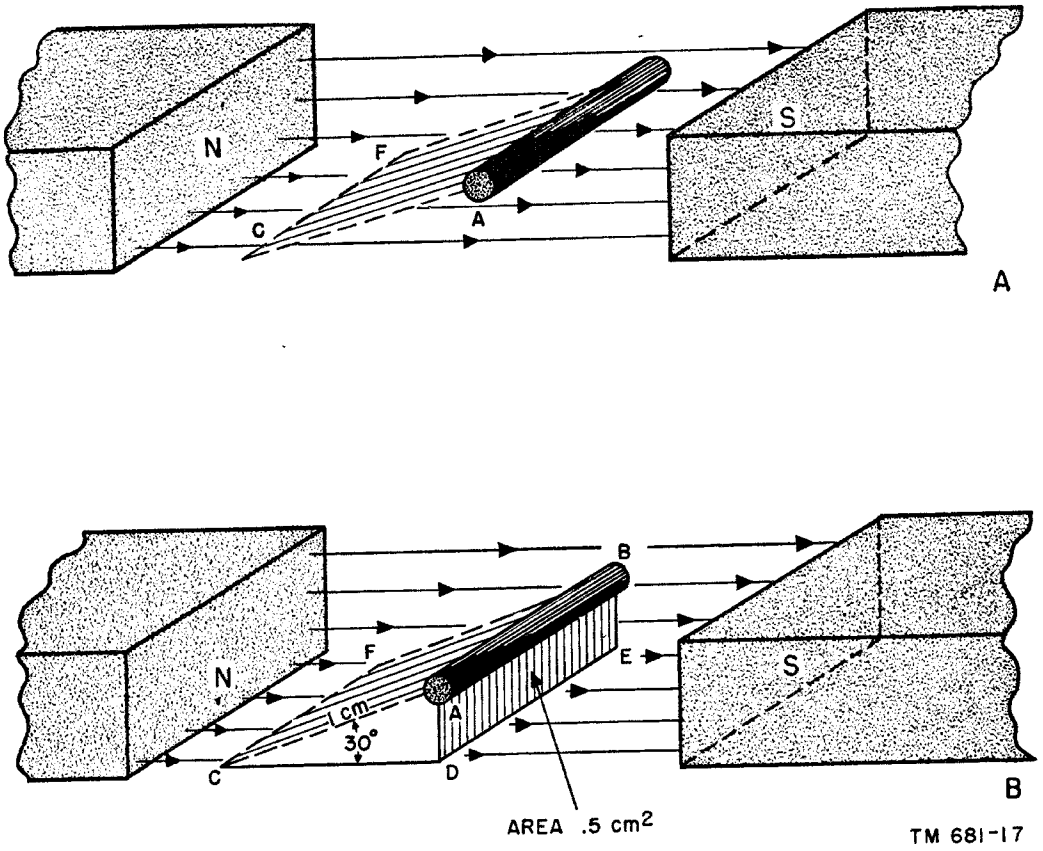


Figure 4. Conductor moving through a magnetic field at an angle to the flux lines.

ity of 1 cm per second but in a direction making an angle of 30° with the flux lines (A of fig. 4). The emf induced in the conductor may be found in the following manner: The area swept out by the conductor in 1 second is shown by the shaded lines in the figure. This area is equal to the length of the conductor times the distance of travel, or 1 square cm. The flux lines passing through this area are the lines cut by the conductor in 1 second. It is apparent from A of figure 4 that the number of flux lines cut by the conductor are fewer than the number of flux lines cut by the same conductor moving the same distance vertically downward (fig. 3). The maximum number of lines are cut when the conductor moves vertically downward. Now, to find the voltage induced in the conductor moving at a 30-degree angle from the flux lines, it is necessary to find the number of flux lines cut by the conductor in 1 second. This can be done in the following manner:

(1) Construct the right triangle ACD as shown in B of figure 4. The hypotenuse of this triangle, *AC*, is known to be 1 cm, representing the distance traveled by the conductor. Also, the angle of 30° is known. Thus, by trigonometry, we can find side *AD:*

$$\text{Sin } 30° = \frac{\text{AD}}{AC}.$$

From the table of trigonometric functions or from the curve in appendix II, it is found that:

$$\text{Sin } 30° = .5$$

Therefore: $$.5 = \frac{\text{AD}}{AC}.$$

Substituting the known value of 1 cm for the side, AC:

$$.5 = \frac{\text{AD}}{1\ cm}.$$

Cross-multiplying:

$$.5 \times 1 \text{ cm} = AD.$$

Transposing:

$$AD = .5 \times 1 \text{ cm}.$$

Therefore: $AD = .5$ cm.

(2) Construct rectangle ABED by making *BE* parallel to, and equal in length to, *AD*, and DE parallel to, and opposite, *AB* (B of fig. 4). We then have a rectangle 1 cm long and .5 cm wide. The area of this rectangle is 1 cm times .5 cm or .5 cm^2.

(3) Examination of B of figure 4 reveals that every flux line that enters face ABFC must leave face ABED. Therefore the number of flux lines cut by the conductor is exactly the same as the number of flux lines passing through area ABED (.5 square cm).

(4) The number of flux lines passing through area ABED is equal to the area in cm^2 times the number of lines per square cm, or:

$$\frac{.5\ cm^2 \times 10^9\ lines}{cm^2}$$
$$= .5 \times 10^9\ lines$$
$$= 5 \times 10^8\ lines.$$

(5) The voltage induced in the conductor is:

$$5 \times 10^8\ lines \times 10^{-8} = 5\ volts$$

where: 10^{-8} is the multiplying factor used to convert the induced emf to volts.

e. VARIATION. From the preceding, it is seen that the voltage induced in a conductor moving vertically downward across the flux lines is greater than the voltage induced in the same conductor moving at an angle of 30° to the flux lines. Further, the emf induced in a conductor moved *parallel* to the magnetic field is zero. From figure 5 it is evident that if the conductor moves parallel to the magnetic field, the area swept out perpendicular to the field is zero (the total area swept out is parallel to the field). This means that the conductor cuts *no* lines of force and, therefore, the induced emf is zero.

f. FORMULA. A general formula, which reduces the effects described in the paragraphs above to a readily usable form, may now be stated. This formula gives the magnitude of the emf induced in any conductor of length l, moving with a velocity v, in such a direction as to make an angle θ with the flux lines. The flux density of the magnetic field through which the conductor moves has the value B, which is the number of lines of force per square centimeter. Figure 6 illustrates these quantities.

$$e = B \times l \times v \times \sin\theta \times 10^{-8}$$

where: e = induced emf in volts
B = flux density in lines per cm^2
l = length of the conductor in cm
v = velocity of the conductor in $\frac{cm}{sec}$
θ = angle of movement of the conductor with relation to the lines. (The value of sin θ for any angle may be obtained from the table of trigonometric functions or from the graph in figure 246, app. II.)

g. SUMMARY. The following conclusions may be drawn concerning the magnitude of the emf induced in a conductor rotating in a magnetic field:

(1) The emf induced in a conductor is at a maximum when the conductor cuts the magnetic lines of flux at right angles ($\sin 90° = 1$).

(2) The induced emf is zero when the conductor moves parallel to the magnetic field ($\sin 0° = 0$).

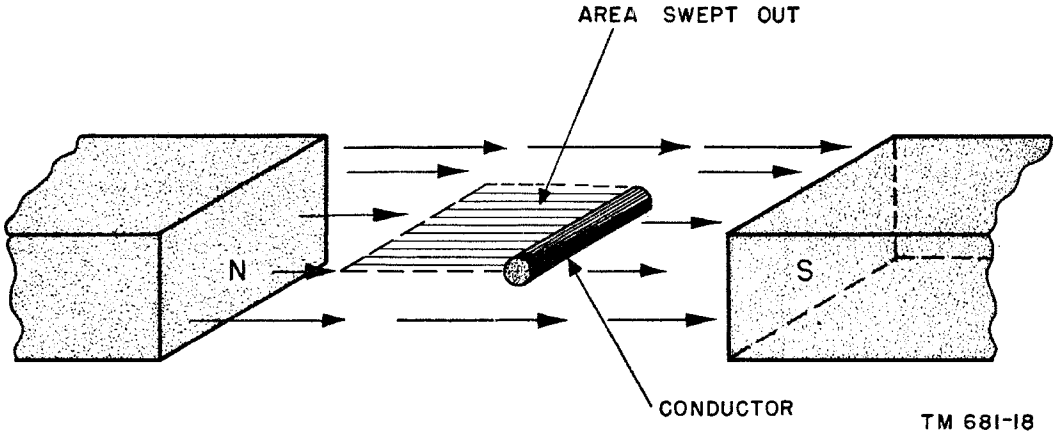


Figure 5. Conductor moving through, and parallel to, a magnetic field.

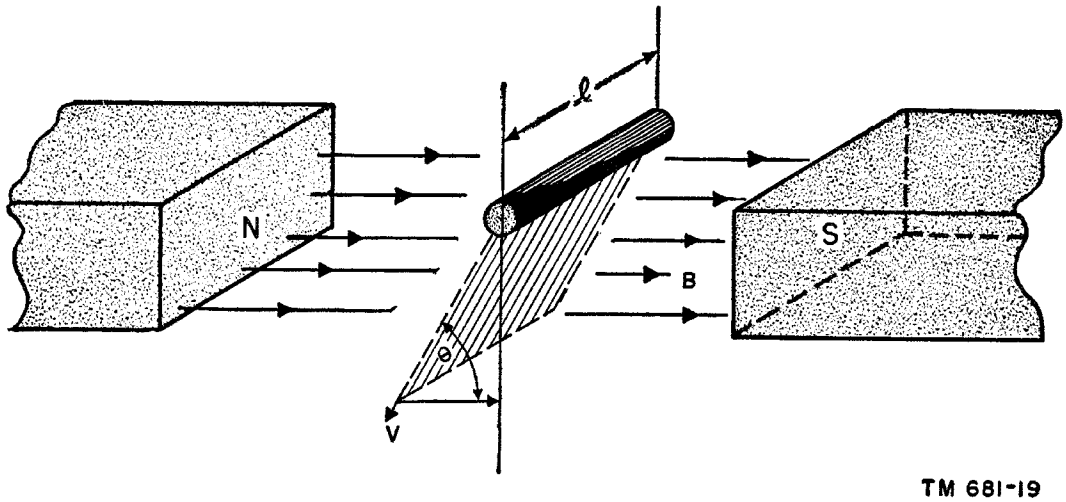


Figure 6. Conductor moving through a magnetic field.

(3) If the conductor is moving in a direction neither at right angles nor parallel to the field, the emf induced in it will depend on the sine of the angle that the line of movement (called the velocity vector) of the conductor makes with the magnetic lines of flux. Figure 8 shows this angle for eight positions during a single rotation.

(4) Increasing either the magnetic flux density, the length of the conductor, or the velocity of the conductor relative to the field will result in an increase in the induced emf.

4. Direction of an Induced Emf

a. The direction of the emf induced in a conductor may be determined by either of the following simple rules:

(1) If the thumb, forefinger, and middle finger of the right hand are held at right angles to one another, the thumb pointing in the direction of the motion of the conductor and the forefinger pointing in the direction of the magnetic field, then the middle finger points in the direction of the induced emf. This is known as *Fleming's Right-Hand Rule.*

(2) At the conductor, draw a vector in the direction of the velocity (a line tangent to the circle of rotation) and a vector in the direction of the flux lines. Now, turn the velocity vector through the *smallest angle* toward the vector representing the direction of the flux lines, called the B-vector or flux density vector. Then, the direction in which the axis of a *right-hand screw* would move is the direction of the induced emf. This is known as the *right-hand screw rule.*

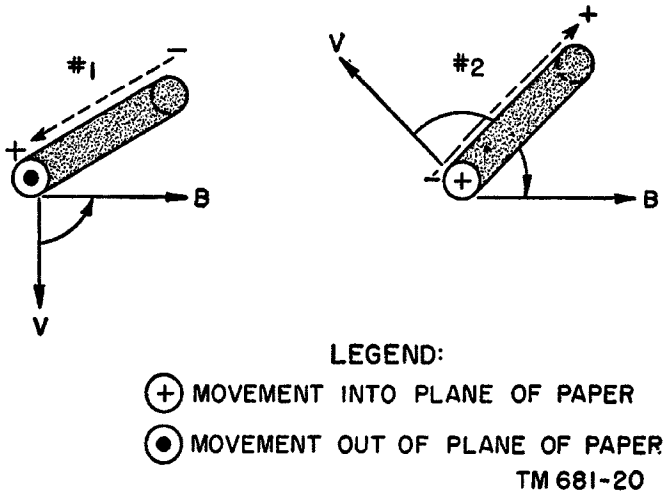


Figure 7. Direction of induced emf in a conductor.

b. The direction of the induced emf in conductor No. 1 and conductor No. 2 (fig. 7) may be taken as an example of the right-hand screw rule. A dot • indicates movement out of the paper and a cross + indicates movement into the paper. The curved arrows show the V-vector being turned into the B-vector through the *smallest possible angle.* For conductor No.

1 the right-hand screw would be turned counterclockwise and therefore move out of the paper. The same reasoning may be applied to conductor No. 2: The right-hand screw would be turned clockwise and therefore move into the paper. The dotted arrows show the polarity of the induced emf in each conductor.

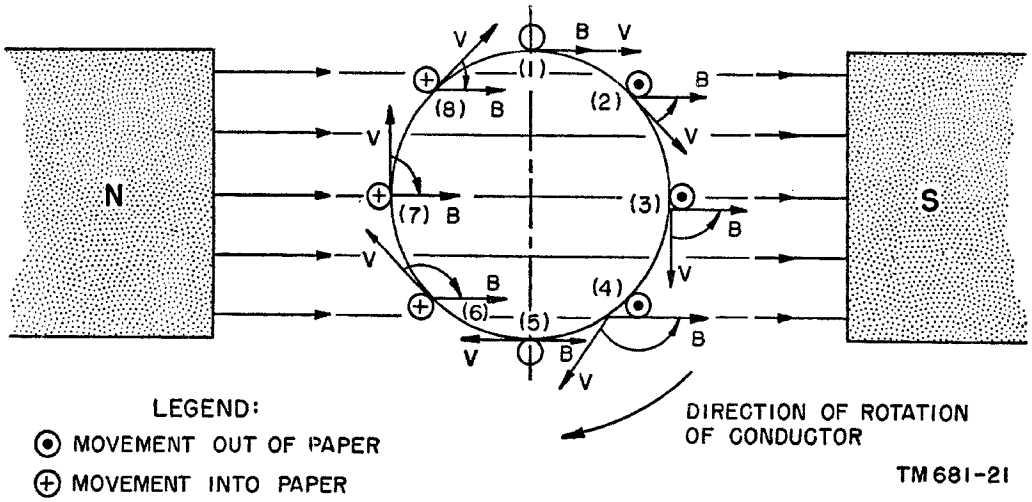


Figure 8. Conductor rotating in a magnetic field.

c. The direction of the induced emf in the cross section of the conductor in figure 8 may be verified for each of the indicated positions of the conductor. The velocity and flux density vectors are drawn at each position. The curved arrows show the V-vector turned into the B-vector. Notice that on the right-hand side (under a south pole) the induced emf's are all into the plane of the paper; on the left-hand side (under a north pole) the induced emf's are all out of the plane of the paper. In positions 1 and 5, the conductor is travelling parallel to the field and the generated emf is zero; therefore, no direction is indicated. These observations will be used in the next paragraph in explaining the operation of a simple a-c generator.

5. Operation of a Simple Two-pole A-c Generator

a. CHARACTERISTICS. The fundamental generation of an induced voltage and current in a conductor, with its direction and magnitude included, has been explained above. A simple a-c generator making use of these principles is shown in figure 9. However, certain features of this generator are basic to the design of all a-c generators, and accordingly are listed below:

(1) Some means of providing a magnetic field is necessary. In the present case, the north and south poles of a permanent magnet are used.

(2) The coil must be rotated through the magnetic field.

(3) A coil is composed of two *coil sides* in which emf's are induced.

(4) A coil of more than one turn presents a coil side of more than one conductor. The total emf induced in a coil side is equal to the emf induced in one conductor multiplied by the number of conductors.

(5) At any instant, the emf induced in one coil side is equal and opposite in direction to that induced in the other coil side. The two emf's appear in series between the collector rings because of the *back connection.*

(6) Each coil side is connected to a metallic collector ring.

(7) A brush is continually in contact with each collector ring. The brushes serve to conduct the induced current in the coil to the load.

b. THE CYCLE. The voltage produced by this generator is an alternating voltage. One complete revolution of the coil will produce 1 cycle of voltage—that is, a voltage which builds up from zero to a maximum, falls off to zero, builds up again in the opposite direction to a maximum, and falls off to zero to complete the cycle. Such a cycle of alternating current or voltage usually is represented as a *sine* curve

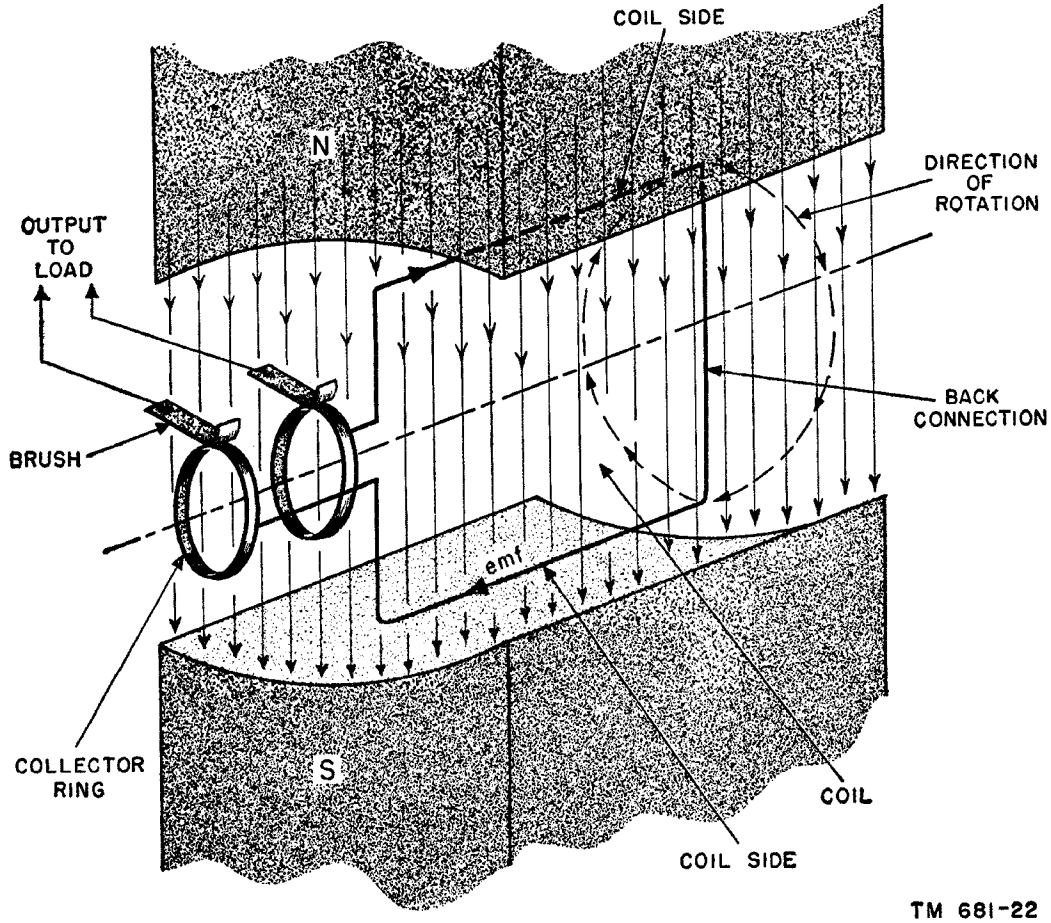


Figure 9. Simple a-c generator.

(see section III, appendix I). Figure 10 shows the method by which the rise and fall of the voltage in the simple two-pole generator is plotted against time and generates the sine curve. Since the emf generated in a coil is twice the emf induced in one coil side, it is only necessary to investigate how the emf of one coil side changes with time.

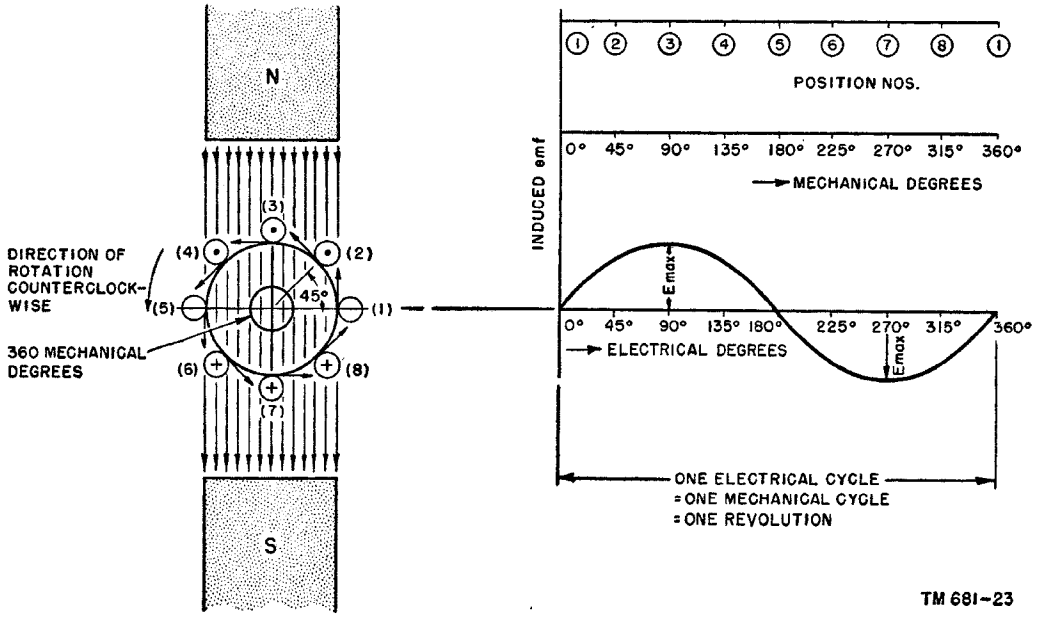


Figure 10. Generation of a sine-wave emf.

c. SINE-CURVE GENERATION. Figure 10 shows a coil side in eight positions, spaced 45° around the axis of rotation. The rotation of the coil is counterclockwise, and the heavy arrows show the velocity of the conductor at each position. In each position the angle of rotation will *determine* the number of lines of force cut by the conductor, since at 0°, 180°, and 360° no lines will be cut, and at 90° and 270°, a maximum number will be cut. It should be noted that the number of lines cut is *not* proportional to the angle itself but to the *sine* of the angle (par. 3); and since the emf induced in the coil is proportional to the number of lines of force cut by the conductor, the emf induced at each position will, therefore, be proportional to the sine of the angle of rotation of the conductor. At positions 1 and 5, the conductor is moving parallel to the lines of force and the induced emf is zero. This corresponds to 0° and 180° on the time axis of the sine curve. At positions 3 and 7, the conductor cuts the flux lines at right angles and the emf is a maximum. This corresponds to 90° and 270° on the sine curve. The horizontal or X-axis of the sine curve represents time, and 1 complete cycle represents the period of rotation of the coil. Therefore, the time axis may be divided directly into mechanical degrees of rotation. A quarter turn of the conductor is one-quarter of the time of 1 cycle and therefore is marked as 90° rather than actually in fractions of seconds of time. The vertical or Y-axis is a measure of the sine of the angle of rotation of the conductor, and since this sine is proportional to the emf, it may be divided directly into units of voltage or current.

d. THE HALF-CYCLE. The part of the sine curve above the horizontal axis is called the *positive* half-cycle or alternation, and the part below the line is called the *negative* half-cycle or alternation. This use of the terms positive and negative refers to the direction of the induced emf in reference to a fixed position, the negative voltage being in the opposite direction from the positive and occurring in time 1 half-cycle later. It will be remembered that the direction of the induced emf at each position of the coil side was obtained by application of the right-hand screw rule. Thus, whether the first half-cycle of the sine curve of figure 10 is positive or negative depends on whether the generator is rotated counterclockwise or clockwise.

e. SUMMARY. The following conclusions may be drawn concerning the operation of a simple two-pole a-c generator:

(1) The emf in a coil as it rotates is an alternating emf that varies as the sine of the angle of rotation. This emf is called a sine wave of voltage.

(2) The emf is a maximum when each coil side is under the center of a pole.

(3) The emf is zero when each coil side is in the space between poles (interpolar spaces 1 and 5 of fig. 10).

(4) The polarity of the induced emf in the coil side under the north pole is opposite to that induced in the coil side under the south pole. These emf's add together to make the emf of the generator *twice* that of either coil side.

(5) When the coil has completed 1 mechanical revolution, the induced emf has completed 1 electrical cycle.

6. Operation of a Four-pole Generator

a. As noted above, 1 mechanical revolution of the coil of a simple two-pole generator equals 1 electrical cycle. If the coil rotates once each second, the *frequency* of the alternating emf is 1 cycle per second. Ten revolutions of the coil each second would produce a frequency of 10 cycles per second. *Frequency then is defined as the number of cycles per second.* In 10 revolutions, the coil goes through 10 times 360 or 3,600 mechanical degrees each second (1 revolution is equal to 360 mechanical degrees). And for every revolution of the coil, the emf goes through 10 cycles (10 times 360°) or 3,600 electrical degrees. It may be seen then that for a *two-pole machine,* 1 mechanical degree is always equal to 1 electrical degree.

b. This last statement must be modified when applied to a four-pole a-c generator. Since the induced emf is a maximum under the center of a pole and is zero in the interpolar spaces, we can readily find the emf generated in a conductor rotating in a magnetic field produced by four poles. The distribution of the flux lines is clearly shown in figure 11. In position 1 the

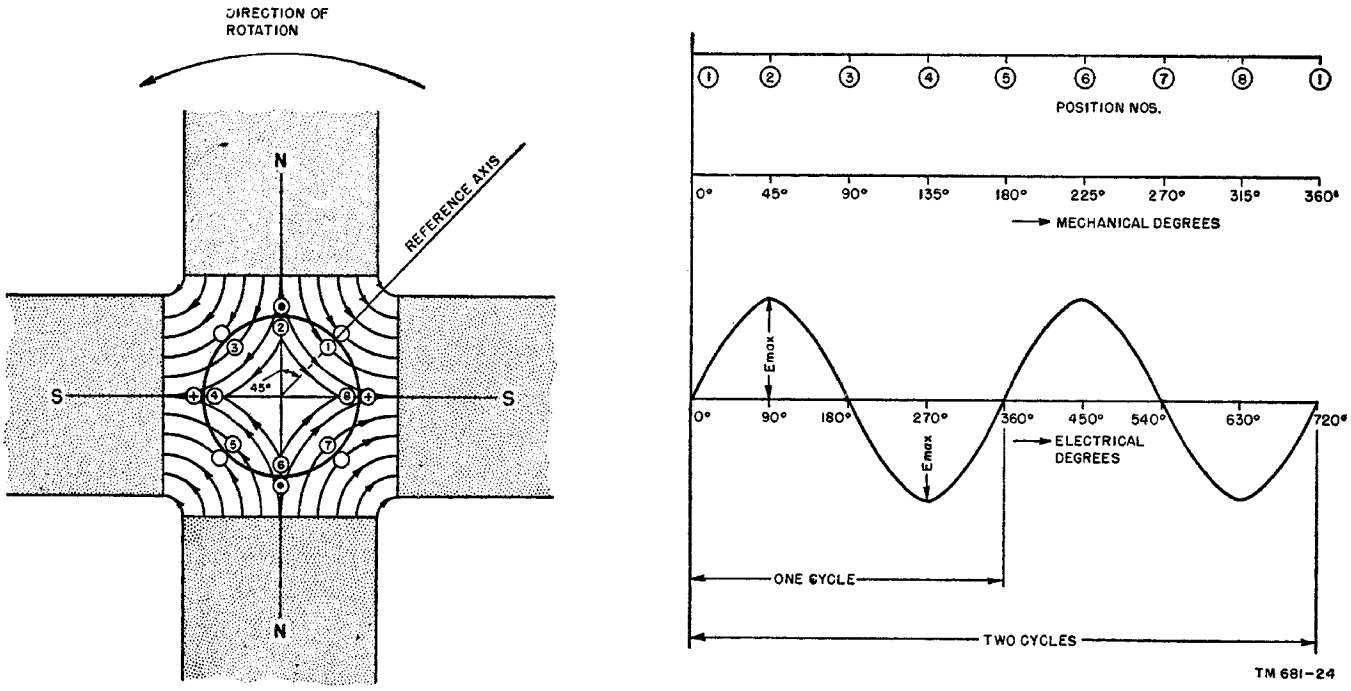


Figure 11. Four-pole a-c generator.

conductor is in the interpolar space and its emf is zero. In position 2, 45 mechanical degrees later, the conductor is under the center of the north pole and its emf is a maximum. The induced emf has gone through 90 electrical degrees (from zero to a maximum) when the conductor has rotated only 45 mechanical degrees. In other words, 90 electrical degrees equals 45 mechanical degrees, or 1 mechanical degree is equal to 2 electrical degrees. In 1 mechanical revolution, the induced emf has gone through 2 electrical cycles. For example, assume that we have a four-pole machine and the number of revolutions per second is 10. The frequency of the generated emf may be found as follows. For every revolution, 2 electrical cycles are performed. Every second, the machine makes 10 revolutions and therefore every second the emf goes through 10 times 2 or 20 cycles. The frequency, then, is 20 cycles per second.

7. Frequency of Multipolar A-c Generators

a. By an analysis similar to that used in determining the frequency of a four-pole machine, it may be shown that for a six-pole generator 1 mechanical degree is equal to 3 electrical degrees, and that for an eight-pole generator, 1 mechanical degree is equal to 4 electrical degrees. Thus, to convert mechanical degrees to electrical degrees, mechanical degrees must be multiplied by one-half the number of poles. Stated as a formula:

$$1 \text{ mechanical degree} = \frac{P}{2} \text{ electrical degrees.}$$

Since 1 mechanical revolution equals 360 mechanical degrees, it is equal to P/2 times 360 electrical degrees. And since 360 electrical degrees equals 1 electrical cycle, the induced emf will go through P/2 cycles in one revolution.

b. The number of revolutions of a generator is usually given as rpm (revolutions per minute). If a coil makes N revolutions per minute, it makes N/60 revolutions per second. Therefore, the number of cycles per second of an induced emf equals N/60 times P/2. Since the number of cycles per second is, by definition, the frequency of the generator, then—

$$f = \frac{P \times N}{2 \times 60} = \frac{P \times N}{120} \text{ cycles per second}$$

where: f = frequency of generated voltage
P = number of poles
N = revolutions per minute

8. Instantaneous and Peak Value of Voltage

The *instantaneous* value of a sine wave of voltage is the value of the emf generated at any instant of time. Note that all instantaneous values of either alternating current or voltage

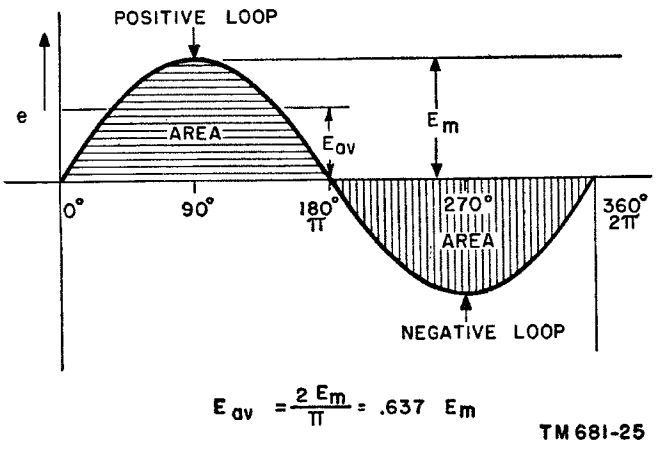


Figure 12. Average value of sine-wave voltage.

are generally indicated by small letters, whereas all average, effective, and maximum values are indicated by capital letters. Figure 12 illustrates this notation. A 0°, the instantaneous value e of the emf is 0. Between 0° and 90° the value of e is increasing from 0 to the maximum value. At 90° the value of e is a maximum and is equal to the *peak* voltage—that is, to E (maximum), written E_m. Between 90° and 180° the value of e is decreasing from peak voltage to 0. In the next half-cycle, e increases and decreases in the same manner but in the opposite direction. Thus, the instantaneous voltage varies constantly during 1 complete cycle of 360 electrical degrees. *Peak voltage* E_m may be defined, therefore, as the point of maximum instantaneous voltage. Peak voltage may be either the negative peak voltage or the positive peak voltage. For a *pure* sine wave of voltage, the positive peak is equal to the negative peak.

9. Average Value of Voltage or Current

The average value of a *complete cycle* of sine wave voltage or current is zero, since the negative loop is equal and opposite to the positive loop (fig. 12). However, the term *average value* when applied to an alternating current or voltage is restricted to the average value of one loop, either the positive or negative. The positive loop of the sine wave is identical to the negative loop, and therefore, the area under each is the same. The average value of a sine function is defined as the area under one loop divided by the base of the loop. The base of one loop is 180° or π radians in length. But in order to find the area of such an irregular surface as a sine loop, the figure must be broken down into a series of small rectangles whose arcs may be easily determined. The sum of all these small areas will then be the area of the loop (see illustration in negative loop of figure 12). In this manner, it is found that the area of one loop of a sine curve is $2\ E_m$, where E_m is the maximum value of the voltage; or the area is equal to $2\ I_m$, where I_m is the maximum value of the current. The average value may then be defined by the following formula:

Average value of voltage =

$$\frac{2\,E_m\ (\text{area of loop})}{\pi\ (\text{length of base})} = \frac{E_m}{\frac{\pi}{2}} = .637 \times E_m.$$

Average value of current =

$$\frac{2\,I_m\ (\text{area of loop})}{\pi\ (\text{length of base})} = \frac{E_m}{\frac{\pi}{2}} = .637 \times I_m.$$

10. Effective or Rms Value of Voltage or Current

a. INSTANT POWER. In the study of direct current it was shown that the power dissipated as heat in a resistance R is equal to the square of the current multiplied by the resistance, or to I^2R. But when an alternating current flows through a resistance, the power dissipated does not remain constant over a full cycle, since the instantaneous current changes constantly with relation to time. However, the power absorbed by the resistor at any instant is equal to the square of the instantaneous current i multiplied by the resistance, or to i^2R. At this point it should be noted that i^2 is always positive, even though i might be a negative value, since the square of a negative number is always positive and greater than zero.

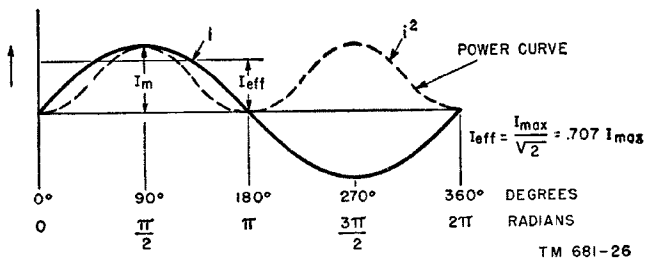


Figure 13. Effective or rms value of sine-wave current.

b. EQUIVALENT POWER. Therefore, over a given period of time, such as 1 half-cycle, a cer-

tain amount of energy is delivered to the resistance in the form of heat. But it may be determined that a certain value of *direct current* flowing through the same resistance for the same amount of time produces the same heat dissipation as the alternating current. The value of this direct current is called the equivalent heating value or the *effective value* of the alternating current. The effective current is defined as the area of one loop of the i^2 graph divided by π, or as the square root of the average value of i^2 over one loop, known as the *root mean square* or rms. For this reason, the effective value is often called the rms value. Figure 13 shows the curve of i^2; as mentioned above, all the loops are positive. To find this effective or rms value of current, it is necessary to proceed in a manner similar to that used in determining the average value. The effective current then may be shown to be equal to the maximum current I_m divided by the square root of 2, and in like manner the effective voltage to be equal to the peak voltage E_m divided by the square root of 2. The effective value may then be expressed in the following formulas:

$$\text{Rms value of current} = \frac{I_m}{\sqrt{2}} = \frac{I_m}{1.414} = .707\ I_m.$$

$$\text{Rms value of voltage} = \frac{E_m}{\sqrt{2}} = \frac{E_m}{1.414} = .707\ E_m.$$

Note. The value of the current or voltage indicated by the usual a-c ammeter or voltmeter is the effective value, that is, the root-mean-square value.

c. EXAMPLE NO. 1. The voltage measured at a household electric outlet is 117 volts. This is the effective or rms value of the voltage. What is the peak voltage, the average voltage?

$E_{rms} = 117$ volts
$E_{rms} = E_m \times .707$
$117 = E_m \times .707$
$E_m = \frac{117}{.707}$
$E_m = 165.5$ volts approx = peak voltage.
Also: $E_{av} = E_m \times .637$
$E_{av} = 165.5 \times .637$
$E_{av} = 105.44$ volts = average voltage.

d. EXAMPLE NO. 2. An electric toaster uses an alternating current of 4 amperes. This is the rms current. What is the maximum current, the average current?

$I_{rms} = 4$ amperes
$I_{rms} = I_m \times .707$
$4 = I_m \times .707$
$I_m = \frac{4}{.707}$
$I_m = 5.65$ amperes = maximum current.
Also: $I_{av} = I_m \times .637$
$I_{av} = 5.65 \times .637$
$I_{av} = 3.59$ amperes = average current.

e. CONVERSION FORMULAS. In the examples given above, it will have been noticed that the peak voltage and the maximum current in an electric circuit are considerably in excess of what is generally the voltage or current given by the rms value. For this reason, circuits must be designed to withstand these values, even though they are instantaneous values and present only twice each cycle. The following list of formulas, therefore, summarizes the findings in the paragraphs above and provides a useful means of converting one value to the other. The relationships among these values are also shown graphically in figure 14.

$E_{rms} = \frac{E_m}{1.414}$
$E_{rms} = E_m \times .707$
$E_{av} = \frac{E_m}{\frac{\pi}{2}}$
$E_{av} = E_m \times .637$
$E_m = E_{eff} \times 1.414$
$E_m = E_{av} \times \frac{\pi}{2}$
Also: $E_{rms} = E_{av} \times 1.11$
$E_{av} = E_{rms} \times .9$

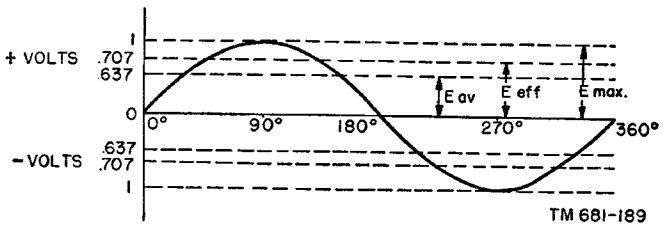


Figure 14. Relationship of maximum, rms, and average voltages.

11. Phase Difference: Angle of Lead or Lag

a. TIME. As explained in the discussion of the generation of a sine wave, the time of a cycle of alternating current or voltage is usually ex-

pressed in electrical degrees. Thus 90° is said to be one-fourth of a cycle and represents an amount of time dependent on the frequency of the voltage, or the number of cycles per second. If the voltage considered is the usual 60-cycle ac used in the home, 1 cycle is completed in one-sixtieth of a second. Then 90° or one-fourth cycle represents actually one-fourth of one-sixtieth of a second—that is, one-fourth times one-sixtieth or one-two hundred and fortieth of a second. The *phase* of this voltage is also 90°, or one-two hundred and fortieth of a second. Phase then is defined as the difference in time between any point on a cycle and the beginning of that cycle. The beginning of a cycle generally is taken to be the point at which the cycle passes through zero moving in a positive direction. Such a consideration of phase, although seldom used in reference to a single voltage, is of immediate practical importance when two ac voltages or currents are present in the same circuit. It is necessary then to determine the position of one with relation to the other at any given instant in time.

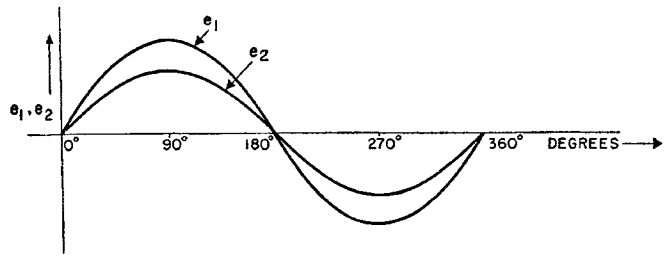


Figure 15. Two sine-wave voltages in phase.

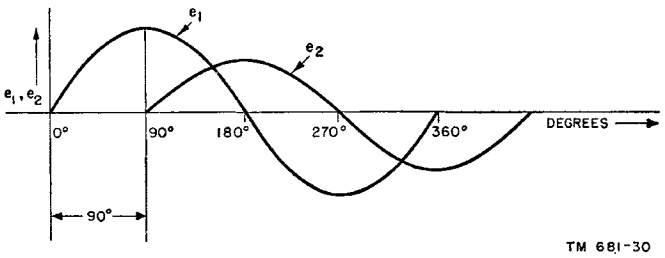


Figure 16. Two sine-wave voltages 90° out of phase.

b. LEAD AND LAG. Thus, if two 60-cycle a-c generators are put in operation at the same time and connected to the same circuit, the two varying voltages rise and fall and reverse direction at the same time. These voltages are said to be *in phase* (fig. 15). If one generator is started one-two hundred and fortieth of a second after the other, the two voltages do not rise and fall together but are separated by a definite period of time which may be expressed in degrees as a fraction of a cycle. The voltages from the two generators are then said to be *out of phase;* and the first is said to *lead* the second, or the second is said to *lag* the first, by the number of degrees expressing this time difference, that is, by 90°. In figure 16, this condition is illustrated as voltage e_2 lags voltage e_1, since the starting point of e_2 is 90° to the right of e_1. Over the time interval 180° to 270°, e_2 is positive and e_1 is negative. Note that the X-axis, or time axis, moves from left to right, and therefore, any point to the right of any other point is later in time or lags the other. Figure 17 shows two voltages 180° out of phase. Both voltages go through their zero points and maximum points at the same time, but e_1 is in the opposite direction from e_2—that is, they are always of opposite sign: When e_1 is negative, e_2 is positive. If these two voltages are of equal value, or amplitude, and present in the same circuit, the resultant voltage is zero, since the voltages cancel completely. However, if the two voltages are in phase, as discussed above and shown in figure 15, the resultant voltage is the sum of the two voltages. For out-of-phase relationships less than 180°, the resultant voltage is the *vector* sum of the two voltages (see appendix I).

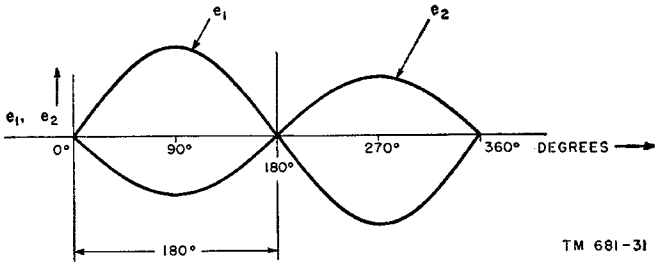


Figure 17. Two sine-wave voltages, 180° out of phase.

c. PHASE DIFFERENCE. It should also be carefully noted that when the time difference in starting the two previously mentioned 60-cycle generators is one-sixtieth of a second, or 1 cycle, the voltages remain *in phase.* But a time difference of one and a quarter cycles, for instance, is expressed as the fractional part of 1 cycle, or 90-degree phase difference. From this it follows that if the time difference is any whole number

multiple of 1 cycle, the voltages are in phase, and that in any other case, the time difference is expressed simply as the fractional part of a single cycle. In addition, phase difference is usually expressed in degrees from 0° to 180° because any angle greater than 180°—for instance 210° *lead* by the first generator—may be expressed in terms of the second generator as a 150-degree *lag*.

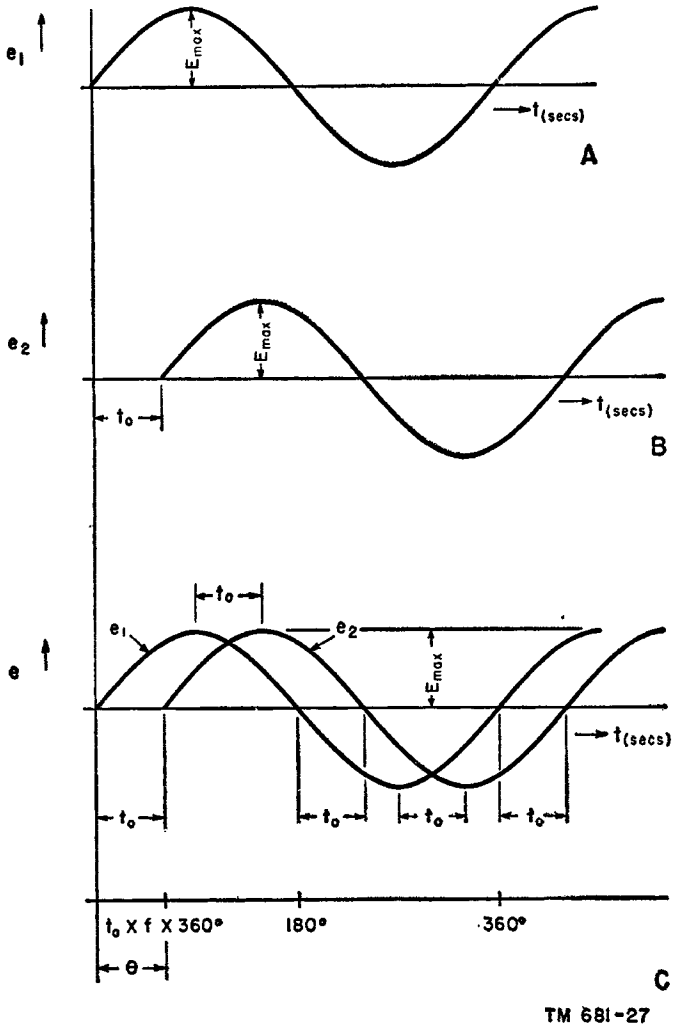


Figure 18. Phasing of a-c generators.

d. EXAMPLE NO. 1. These points may now be expressed generally and illustrated by figure 18. Two a-c generators are put in operation at different times. Generator No. 2 is started t seconds later than generator No. 1. Both machines deliver sine-wave voltages at the same frequency. In figure 18, A is a graph of the voltage of generator No. 1, B is a graph of the voltage of generator No. 2, and C is a graph of the voltages of generators No. 1 and No. 2 plotted on the same time base. Note that voltage e_2 does not begin until a definite time interval (t_0 seconds) has elapsed since the beginning of voltage e_1. Thus, if t_0 equals 1 millisecond, and voltage e_1 has the value of 1 volt after 2 milliseconds, then voltage e_2 will attain the same value after 2 + 1 or 3 milliseconds. Figure 18, then, illustrates the following important points:

(1) The zero points of e_1 and e_2 are separated by t_0 seconds and the maximum points are separated by t_0 seconds.

(2) Voltage e_1 goes through its zero value t_0 seconds before e_2.

(3) The graph of e_2 can be reproduced by shifting that of e_1 to the right t_0 seconds.

e. PHASE ANGLE. The time lag of e_2 (t_0 seconds) may be converted to a lag in electrical degrees by the following equation:

Electrical degrees = t_0 (seconds) × frequency × 360°.

For example, if two 60-cycle a-c generators are started 1 millisecond apart, the phase shift in seconds is equal to 10^{-3}. Then the phase shift in electrical degrees is equal to 10^{-3} times 60 times 360° or 21.6°. The second generator to start is then said to lag the first by 21.6° (fig. 19).

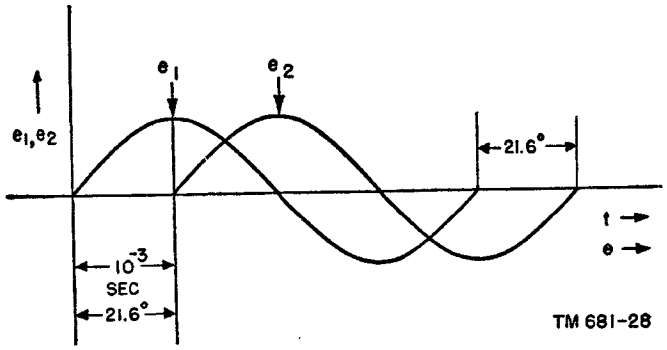


Figure 19. Two sine-wave voltages, 1-millisecond phase difference.

f. EXAMPLE NO. 2. If three generators are started at different times, each generator leads or lags the other two generators by a specified time. Figure 20 illustrates this condition. Generator No. 1 is started first, generator No. 2 next, and generator No. 3 last. Then voltage e_2 lags e_1; e_3 lags e_1 and also lags e_2. From the graph—

$$t_{3,2} = t_3 - t_2.$$

For example, three 60-cycle a-c generators are put into operation. Generator No. 2 is started 2 milliseconds after generator No. 1, and genera-

tor No. 3 is started 1 millisecond after generator No. 2. Using the formula described in the preceding paragraph, these time differences may be converted to equivalent electrical degrees of phase difference.

$t_2 = 2$ milliseconds $= 2 \times 10^{-3}$ seconds

$t_{3,2} = 1$ millisecond $= 1 \times 10^{-3}$ seconds

$t_3 = t_2 + t_{3,2} = 2 + 1 = 3 \times 10^{-3}$ seconds.

Generator 2 lags generator 1 by $2 \times 10^{-3} \times 60 \times 360° = 43.2°$.

Generator 3 lags generator 2 by $1 \times 10^{-3} \times 60 \times 360° = 21.6°$.

Generator 3 lags generator 1 by $3 \times 10^{-3} \times 60 \times 360° = 64.8°$.

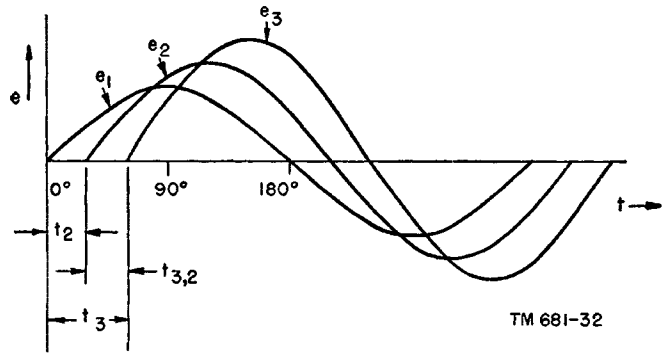


Figure 20. Three sine-wave voltages, phase difference.

12. Summary

a. Direct current was the first type of current widely used, but it is limited in its applications. The principal disadvantages of dc are:

(1) Dc cannot be transmitted over long distances without considerable loss of power.

(2) D-c power cannot be radiated from an antenna.

b. In 1819, H. C. Oersted discovered by accident that a conductor carrying a current possessed the properties of a magnet. This type of magnet is known as an *electromagnet.*

c. In 1831, Michael Faraday and Joseph Henry working independently discovered that a magnet thrust into a coil caused a current to flow in the coil, and that removing the magnet caused the current to flow in the opposite direction.

d. The *magnitude* of the electromotive force induced in a conductor by a magnet is proportional to the number of magnetic flux lines cut per second.

Emf (volts) = flux lines cut per second $\times 10^{-8}$.

e. A conductor moving parallel to the flux lines cuts no flux lines per second. A conductor moving at right angles to the flux lines cuts a maximum number of lines per second. A conductor moving at any angle to the flux lines cuts a number of flux lines proportional to the *sine* of the angle. Expressed formally: Any conductor of length l, velocity v, moving at an angle θ to the flux density B has an emf in volts induced in it as indicated:

$$e \text{ (volts)} = B \times l \times v \times \sin \theta \times 10^{-8}.$$

f. The *direction* of the emf induced in a conductor is the direction in which the axis of a right-hand screw would move when turned with the velocity vector as it moves through the smallest angle toward the flux density vector. This is known as the right-hand screw rule.

g. Alternating current is a current which in a certain fixed period of time first moves in one direction, then reverses and moves in the opposite direction. Therefore, alternating current is defined as a current which is *constantly* changing in magnitude and *periodically* changing direction.

h. The simple two-pole generator:

(1) The total emf induced in a coil side is equal to the emf induced in one conductor multiplied by the number of conductors.

(2) The emf induced in one coil side is equal and opposite in direction to that induced in the other coil side.

(3) These emf's of the two coil sides add together in series to make the emf of the generator twice that of either coil side.

(4) The emf induced in the coil as it rotates is an alternating emf that varies as the sine of the angle of rotation. This emf is called a sine wave of voltage.

(5) The emf is at a maximum when each coil side is under the center of a pole.

(6) The emf is zero when each coil side is in the interpolar spaces.

(7) When the coil has completed 1 mechanical revolution, the induced emf has completed 1 electrical cycle.

i. The *frequency* of an alternating current is defined as the number of electrical cycles per

second. The frequency of a simple two-pole generator is the number of mechanical revolutions per second. One mechanical degree is equal to P/2 electrical degrees (where P is equal to the number of poles). Therefore, the frequency of a multipolar generator is equal to P/2 times N/60 (where N is the number of revolutions per minute). Expressed formally:

$$f = \frac{P \times N}{120}.$$

j. The *instantaneous* value of a sine wave of voltage is the value of the emf generated at any instant in time. *Peak* voltage, E_m, is the point of maximum instantaneous voltage. The *average* value of a sine wave of voltage is the average of the instantaneous values over one loop, and is equal to the area of the loop divided by the length of the base, or to $2E_m/\pi$. The *effective or rms* value of a sine current is the value of the direct current producing the same amount of heat through the same resistance during the same period of time. It is equal to $I_m/\sqrt{2}$.

k. Conversion formulas for maximum, effective, and average values of ac follow:

(1) $E_m = E_{rms} \times 1.414$
(2) $E_m = E_{av} \times \frac{\pi}{2}$
(3) $E_{rms} = \frac{E_m}{1.414}$
(4) $E_{rms} = E_m \times .707$
(5) $E_{rms} = E_{av} \times 1.11$
(6) $E_{av} = \frac{E_m}{\frac{\pi}{2}}$
(7) $E_{av} = E_m \times .637$
(8) $E_{av} = E_{rms} \times .9$

l. Phase is the difference in time between any point on a cycle and the beginning of that cycle. *Phase difference* is the difference in time between any two cycles—that is, between the time of the passing of one cycle through a given point as compared with the time of the passing of the second cycle through the same point. Phase difference is usually expressed in electrical degrees.

m. If the phase difference between two cycles is zero, they are said to be *in phase.* If the phase difference is one quarter-cycle, the two cycles are said to be *out of phase* by 90° . The first cycle may be said to *lead* the second, or the second cycle may be said to *lag* the first by 90°. Two cycles which pass through their zero points and maximum points at the same time but in different directions are 180° out of phase. Phase difference is expressed in degrees from 0° to 180°; an angle of lead, for instance, greater than 180° may be expressed as an angle of lag less than 180°.

13. Review Questions

a. What is alternating current? How does it differ from direct current?

b. What are the principal disadvantages of the use of direct current?

c. What major discovery in electricity is credited to Oersted? To Faraday and Henry?

d. What factors determine the magnitude of an induced emf?

e. What is the formula for calculating the magnitude in volts of an emf induced in a conductor moving at an angle to the flux lines?

f. What is the effect on the magnitude of the emf when the angle of rotation is 90°? 0°? 45°?

g. How is the direction of an induced emf determined?

h. List the *physical* characteristics of a simple two-pole generator.

i. List the *electrical* characteristics of a simple two-pole generator.

j. Why is the sine curve the characteristic curve of an alternating current?

k. How is the frequency of an a-c generator determined?

l. Define the instantaneous, peak, maximum, effective, and average values of an alternating current or voltage.

m. Give the conversion formulas for each of the foregoing.

n. Why is a household circuit designed to withstand higher voltages than the 117 volts listed at the outlet?

o. What is the relationship between time and phase?

p. What is meant by the term *lead*? By *lag*?

q. What is meant by the terms: in phase? 90° out of phase? 180° out of phase?

r. What is the phase angle?

s. How is a phase angle greater than 180° expressed?

t. How is a phase angle determined in degrees?

CHAPTER 2

INDUCTANCE

14. Inductance

a. Inductance is that property of an electrical circuit which tends to prevent a change of current.

b. In the analysis of dc and d-c circuits previously considered, any opposition to the flow of current was termed *resistance* and indicated by the symbol *R*. The current in such a typical d-c circuit may be compared to an object in motion, such as an automobile, which is retarded only by the friction or resistance of the surface on which it moves. If, however, a d-c circuit is broken suddenly, by opening a switch for instance, a considerable spark jumps across the contacts of the switch as it opens. It may be said then that opening a d-c circuit is like suddenly stopping an object in motion. But from Newton's *first* law of motion, it is known that an object in motion tends to remain in motion, and that a considerable force must be exerted to bring it to a stop. In the case of a speeding automobile stopped suddenly by a stone wall, this inertia or momentum which tends to keep the car moving will smash the car and dissipate itself as heat. In the case of a d-c circuit suddenly opened, particularly one carrying heavy current, the inertia or momentum of the current meeting the very high resistance of the open circuit produces a high voltage and dissipates itself as heat in a fat blue spark (fig. 21).

c. Furthermore, Newton's first law of motion states that an object at rest tends to remain at rest unless acted on by an external force. For example, an automobile must exert considerable power in order to start; after reaching speed, the only power necessary to keep it moving is the power used to overcome friction. In like manner, an electric current cannot be started instantaneously; there is a delay in time between the application of the voltage and the rise of the current to its maximum amount. This effect of inertia in a d-c circuit generally is not noticed, since the starting delay is slight, and except in circuits carrying large amounts of current the spark resulting from stopping the current, or opening the circuit, is not dangerous. It should be noted, however, that in d-c circuits carrying heavy power, provision must be made to open the circuit by gradually increasing resistance until current reaches a safe value.

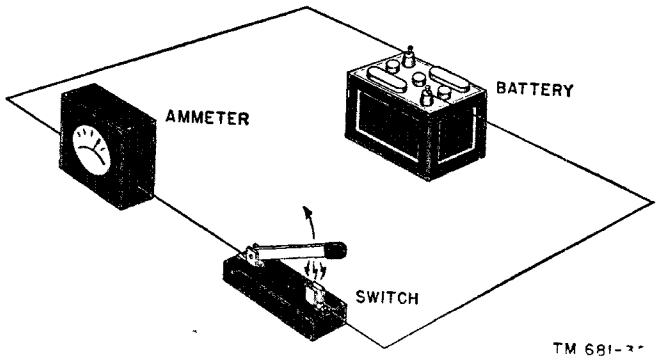


Figure 21. Effect of electrical inertia.

d. In an a-c circuit, however, this effect of electrical inertia, or inductance, is ever present, since by definition an alternating current is one constantly changing in magnitude and periodically changing direction. Thus, if the current in a circuit is increasing, the inductance of the circuit is defined as that property of the circuit which tends to prevent the increase; if the current is decreasing, the inductance of the circuit tends to prevent the decrease. The greater the inductance of the circuit, the greater the opposition to a *change* in current.

15. Counter or Back Electromotive Force

a. In a-c circuits there is ever present an opposition to the flow of current other than the d-c resistance of the circuit. This additional opposition is caused by a *counter* or *back elec-*

tromotive force, so named to distinguish it from the applied voltage, which is the original force tending to set up a current flow. Back emf or counter emf is analogous to the force which opposes a change in motion of a mass. Stated generally, it may be said that the velocity of a mass cannot be changed instantaneously, whether that mass be an automobile, an electron, or an electrostatic charge.

b. In an electric circuit, the cemf (counter electromotive force) is an *induced* emf or voltage. This voltage is induced in conductors of the circuit not by means of an external magnetic field, as in the case of the simple two-pole generator, but by means of the magnetic field already surrounding any conductor carrying a current. Any change in current changes the intensity of this magnetic field, and the resultant emf induced, the counter emf, is a *self induced* voltage. Thus, the property of a circuit which produces such an emf is called *self-inductance.* Actually, all elements in a circuit, including connecting wires, show some self inductance, but for all practical purposes only those elements designed to make use of this property to advantage are known as *inductances* or *inductors.* Moreover, it may be said that counter emf is present in any a-c circuit, but its effect is negligible in a circuit of moderate power, such as an electric lamp, which uses almost pure resistance as a load. But the effect of counter emf is considerable in circuits (even of very low power) which use an inductance as part of the load, such as the primary of the power transformer in an ordinary radio receiver.

16. Generation of Counter Emf, Lenz' Law

a. It will be remembered that in the case of the simple generator, *motion* either of the conductor or of the magnet was necessary to an induced emf. In self inductance, the equivalent of motion, that is—a change in flux density of the magnetic lines of force about a conductor, is caused by the rise or fall of the current, since, as was seen in the study of electromagnetism, the intensity of a magnetic field about a conductor is directly proportional to the current through the conductor. The force setting up the flux lines is equal to 0.4π NI. Since $.4\pi$ is a constant, the factor NI is called the ampere turns. Any change in current changes the factor NI or ampere turns and, accordingly, the flux density. Thus, self inductance is present constantly in an a-c circuit because current is constantly changing, but it is present in a d-c circuit only at the moments of closing or opening the circuit.

b. That the emf self-induced in a conductor carrying current is a *counter emf* was deduced by H.F.E. Lenz from the principle of the conservation of energy. If the emf self-induced was not a counter emf, then an increase of current would aid the applied voltage, and this increase in applied voltage would in turn tend to increase the current. This process would continue, of course, until current reached an infinite amount, a condition not possible in the physical universe. *Lenz' Law* states: *An induced emf always has such a direction as to oppose the action that produces it.* Thus, when a current flowing through a circuit is varying in magnitude, it produces a varying magnetic field which sets up an induced emf that opposes the current change producing it. Or, it may be said that when the current in a circuit is increasing, the induced emf opposes the applied voltage and tends to keep the current from increasing; and when the current is decreasing, the induced emf aids the line voltage and tends to keep the current from decreasing.

c. The effect of counter emf may be observed experimentally in that an alternating current through an inductor is opposed by a force much greater than its simple d-c resistance. For example, the d-c resistance of the primary of an ordinary power transformer used in a typical radio receiver is approximately 5 ohms. As in figure 22, this primary is connected directly to the 110-volt, a-c outlet in the home. From Ohm's Law:

$$I = \frac{E}{R} = \frac{110}{5}$$

$$I = 22 \text{ amperes.}$$

Thus, the current is calculated to be 22 amperes, but when actually measured the current is found to be approximately 1 ampere. It may be seen that some opposition other than the 5-ohm resistance is present in an a-c circuit. This opposition is the counter emf. If by mistake such a radio set is connected to a 110-volt, *d-c* line, the current through the primary of the trans-

former is 22 amperes, and the transformer burns up. Hence, Ohm's Law as stated for d-c circuits must be modified, as will be shown later in this chapter, to include this effect of electrical inertia present in a-c circuits.

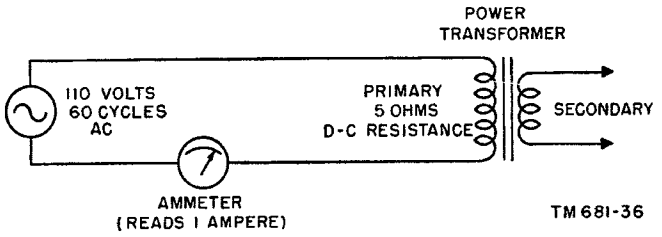


Figure 22. Effect of cemf on current flow.

17. Magnitude of a Counter Emf

a. The magnitude of a counter emf depends upon the same factors that govern any induced emf. In the analysis of an induced emf considered in the previous chapter, it was shown that the magnitude of the emf induced in a conductor of unit length depended on the number of flux lines cut per second. This principle may be restated as Faraday's Law of electromagnetic induction: *The emf induced in any circuit is dependent upon the rate of change of the flux linking the circuit.* Since there is no physical movement of the conductor or of the lines of force in self inductance, the rate of change of the flux density is equivalent to movement. But, as was shown above and in the study of electromagnetism in TM 11–661, the flux density about a conductor is directly proportional to the current in the conductor. Therefore, the magnitude of self-induced emf depends directly upon the rate of change of the current in the circuit. Thus, a rapidly changing current induces a greater counter emf than a slowly changing current. But for any ac the rate of change of current depends on the number of cycles per second, or the frequency. The counter emf then depends directly upon frequency.

b. The total magnitude of an induced emf depends also on the length of the conductor, since in the simple generator the length of the conductor and the number of conductors in a coil side, determine the total emf induced. Thus, a long conductor has greater counter emf induced, or has more self inductance, than a short one. If, however, a long conductor is wound on itself in the form of a coil, its self inductance is increased because of the increase in total flux density. Such a coil, or inductance, is a solenoid, and, as was shown in the study of electromagnetism, the flux density about it may also be increased by the addition of a core material of high permeability, such as soft iron. Figure 23 illustrates this type of inductance.

c. The magnitude of a counter emf, then, is determined by the following formula:

$$\text{cemf} = -\frac{.4\pi N^2 \mu A}{l} \times \frac{di}{dt} .$$

Since the physical characteristics of the solenoid, called the geometry of the coil, are all contained in the factor $\frac{.4\pi N^2 \mu A}{l}$, this expression may be isolated and given the name *inductance* and assigned the symbol L. Then:

$$L = \frac{.4\pi N^2 \mu A}{l} .$$

Substituting L for this factor:

$$\text{cemf} = -L \frac{di}{dt} .$$

The minus sign means that the voltage developed is a counter voltage and opposes the force producing it. Also, it should be noted that the separation of the physical character-

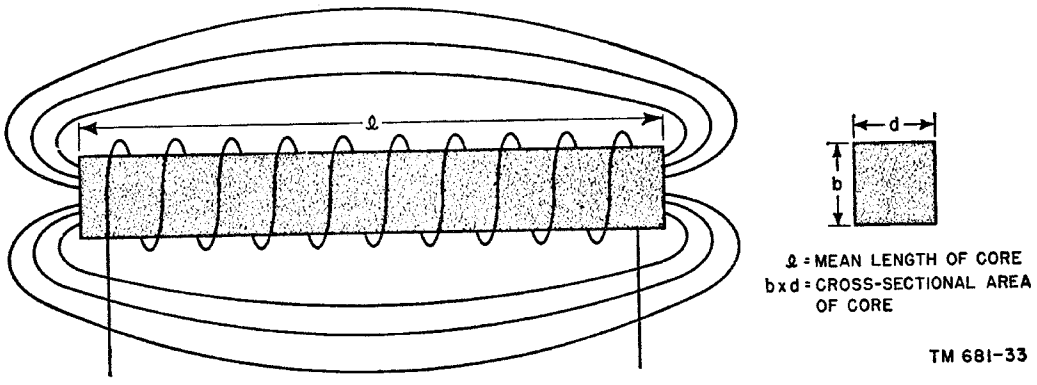


Figure 23. Inductance wound on iron core.

istics (inductance) from the rate of change of the current is done for convenience and is analogous to the separation usually made in the more familiar equation of the inertia of masses. The counter force, or reaction, is equal to the mass multiplied by the rate of change of velocity:

$$F = -m \frac{dv}{dt} .$$

Hence, inductance may be said to be analogous to mass, which is that property in nature which tends to oppose a change of velocity, and may be defined finally as that property of an electrical circuit which tends to prevent a *change* of current.

18. Measurement of Inductance: the Henry

a. The unit of measurement of inductance is the henry, named after Joseph Henry, the co-discoverer with Faraday of the principle of electromagnetic induction. A henry is defined as the inductance of a circuit in which a current change of 1 ampere per second causes a counter emf of 1 volt. Since the henry is defined in terms of practical units, the factor 10^{-8} must be used if the cemf is to be read in volts and the rate of change of the current in amperes per second. Then:

$$L = \frac{.4\pi N^2 \mu A}{l} \times 10^{-8} \text{ (henrys)}$$

where: L = self inductance of solenoid in henrys
N = number of turns of coil
μ = permeability of core in electromagnetic units
A = cross-sectional area of core in cm^2
l = mean length of core in cm.

b. This formula reveals the following important relationships:

(1) The inductance of a coil is proportional to the *square* of the number of turns.

(2) The inductance of a coil increases directly as the permeability of the material making up the core increases.

(3) The inductance of a coil increases directly as the cross-sectional area of the core increases.

(4) The inductance of a coil decreases as its length increases. In figure 24, A shows two coils of a fixed number of turns with different cross-sectional areas. The larger coil has a greater total flux, or less reluctance, and therefore greater inductance. B shows two coils of a fixed number of turns and the same cross-sectional area, but of different lengths. The longer coil has less total flux, or greater reluctance, and therefore, less inductance.

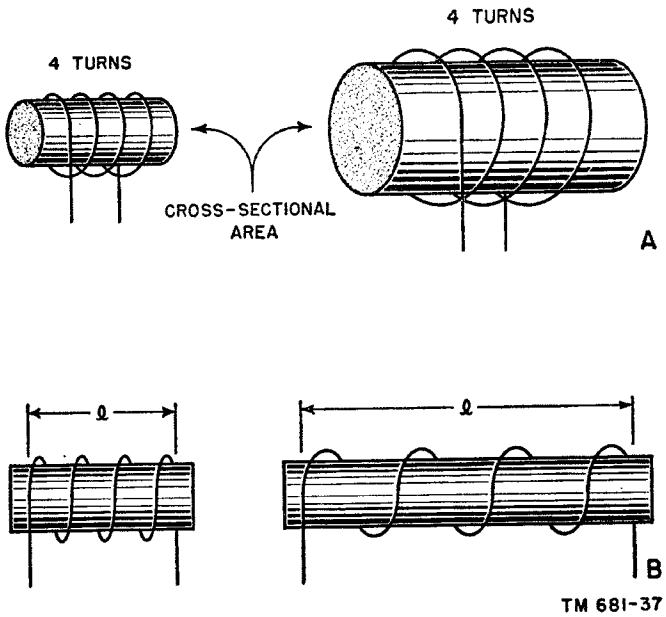
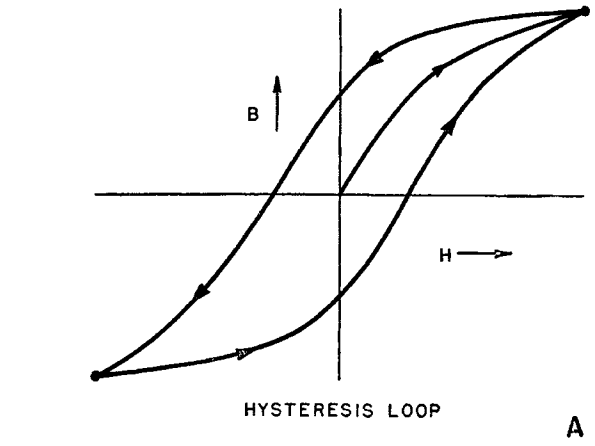


Figure 24. Variation of inductance with size of solenoid.

c. It should also be noted that for ferro-magnetic (iron-core) inductances, the permeability, μ, of the core material is not a constant, but depends on the magnitude of the magnetizing current. In a-c circuits, the current is constantly changing in magnitude and periodically in direction, and accordingly, an error is introduced in calculations of the magnitude of the inductance. In A of figure 25 the relationship between the flux density B and the field intensity H is shown in the form of a graph known as a hysteresis loop. But the ratio B/H is the definition of permeability. Therefore, it may be seen from this graph that the value of μ varies as the ratio of B to H varies for different points on the loop. For a fuller explanation of the hysteresis loop and the relationships among B, H, and μ, see TM 11–661. In B of figure 25 a permeability curve for cast steel is shown. Permeability for this material increases to a maximum value at approximately 7,000 lines per square centimeter and then falls off as the flux density increases. This variation of

μ invalidates calculations of inductance based on the formula given previously, and should be noted for reference in the discussions of transformers in chapter 6.



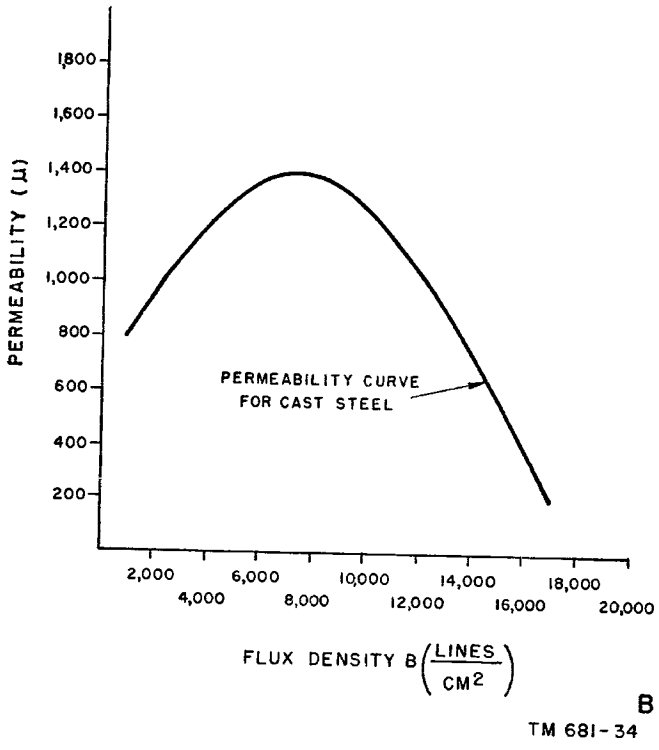


Figure 25. Variation of μ for iron-core materials.

19. Measurement of Counter Emf: Voltage

a. The formula derived previously for the magnitude of a cemf was found to be:

$$\text{cemf} = -L\frac{di}{dt}.$$

An examination of this formula reveals that the greater the inductance, or the faster the rate of change of the current, the greater is the cemf induced in the circuit. For example, a coil of 1-henry inductance has a current of 1 ampere flowing through it. If this current changes to 2 amperes in 1 second, the cemf will be:

$$\text{cemf} = -1 \times \frac{(2-1)}{1} = -1 \text{ volt.}$$

If the current change remains the same but the coil used has an inductance of 10 henrys, then:

$$\text{cemf} = -10 \times \frac{(2-1)}{1} = -10 \text{ volts.}$$

If the inductance remains, as in the first instance, at 1 henry, and the current change from 1 ampere to 2 amperes takes place in one-tenth of a second, then:

$$\text{cemf} = -1 \times \frac{(2-1)}{1/10} = -10 \text{ volts.}$$

b. From these examples it may be seen that a high value of opposition to the flow of current may be obtained either by increasing the inductance or the speed of the change of current in a circuit, or both. Thus, *low frequency* a-c circuits, because of the slow speed of change of the current, generally employ *high* values of inductance (iron cores) to obtain a high cemf. *High-frequency* a-c circuits, because of the great speed of the change of the current, often may generate sufficient cemf with *small* air core inductances. The following table illustrates the rise in cemf as the rate of change of the current increases:

L *henrys*	$\triangle$ i *amperes*	$\triangle$ t *seconds*	cemf *in volts*
1	1	1	—1
1	1	1/2	—2
1	1	1/4	—4
1	1	1/10	—10
1	1	1/20	—20
1	1	1/50	—50
1	1	1/100	—100
1	1	1/500	—500
1	1	1/1,000	—1,000
1	1	1/1,000,000	—1,000,000

where: $\triangle i$ (called delta i) = change in current
$\triangle t$ (called delta t) = time required for change in current.

c. From the table, it is apparent that if a change of 1 ampere were to take place instantaneously, that is, if $\triangle t = 0$, the induced

voltage e would become infinitely large. This would violate Kirchhoff's first law, which states that at any instant the applied voltage in a circuit must equal the sum of the voltage drops around the circuit. Certainly, if $\Delta t = 0$, the voltage drop across the inductance would be greater than any applied voltage could be. And by extension, it may be seen that at any instant, no matter how fast the change of current or how great the value of inductance, the induced voltage cannot be greater than the applied voltage. On the other hand, if there were no change of current, that is, if Δt were equal to infinity, the circuit would be a d-c circuit, and e would be zero.

20. Inductive Reactance

a. In the discussion above, the opposition offered to a specific change of current by an inductance was measured at any given instant in terms of counter emf, the voltage bucking the applied voltage. In d-c circuits, however, any opposition to current flow was termed resistance and measured in ohms. In a-c circuits, therefore, it is convenient to measure inductive opposition in ohms rather than in counter emf or in volts. This type of a-c opposition is called *inductive reactance* and is assigned the symbol X_L to distinguish it from a d-c resistance.

b. From the preceding table listing the magnitude of induced voltages, and from the examples given above, it will be seen that inductive reactance will depend directly upon the size of the inductance and upon the rate of change of the current. In an a-c circuit, the rate of change of the current is governed by the angular velocity of the applied voltage, and this velocity is measured in radians. Since each cycle is 2π radians, the angular velocity of any sine wave is equal to this factor multiplied by the frequency (par. 24, app. I). Therefore, the inductive reactance of any circuit is equal to the inductance multipled by the angular velocity $2\pi f$, or:

$$X_L = 2\pi fL$$

where: X_L = inductive reactance in ohms
$2\pi = 6.28$
f = frequency in cycles per second
L = inductance in henrys.

c. From this formula it may be seen that the higher the frequency, or the greater the inductance, the greater the inductive reactance. A of figure 26 is a graph of X_L plotted against frequency, and B is a graph of X_L plotted against inductance. The graphs are similar and illustrate that inductive reactance increases directly, or linearly, with frequency and inductance: the greater the frequency for any L, or the greater the inductance for any f, the greater the inductive reactance of the circuit.

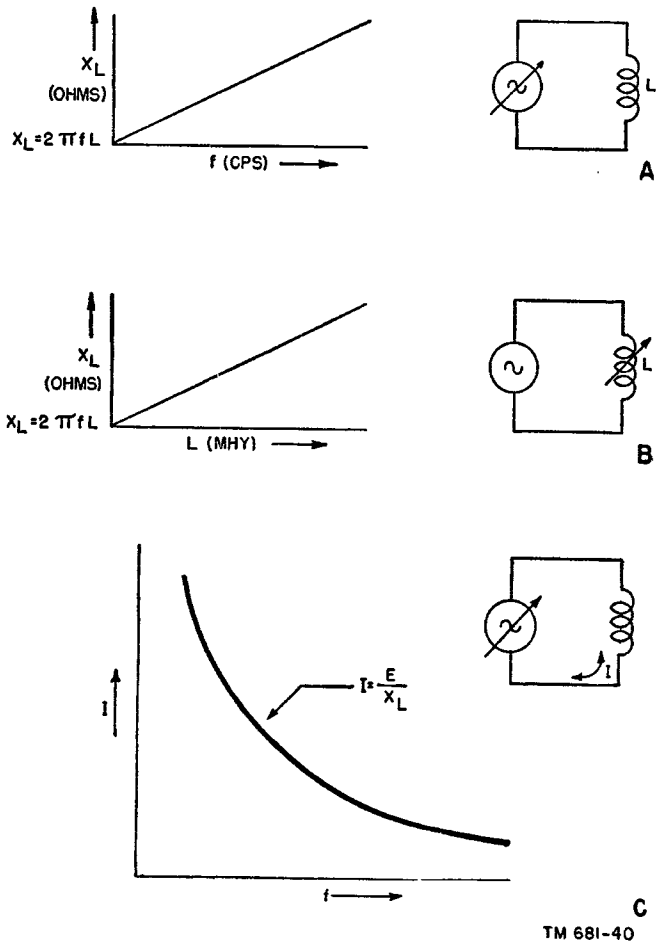


Figure 26. Variation of inductive reactance.

d. At the high frequencies used in radio communications, even small amounts of inductance may offer very great inductive reactance. C of figure 26 is a graph of current in an inductive circuit plotted against frequency. It may be said, then, that at high frequencies an inductance tends to act as an open circuit, since very little current flows in the circuit; and at low frequencies, an inductance tends to act as a simple conductor—a short circuit—since high current flows. Thus, inductances designed for use at low frequencies are generally large iron-core inductances. They are measured in full

units of henrys. Those used at high frequencies are generally small air-core inductances, which are measured in millihenrys (1/1,000 of a henry) or in microhenrys (1/1,000,000 of a henry).

e. For example, the opposition offered by a 20-henry inductance to a 110-volt, 60-cycle a-c source may be determined by use of the formula for inductive reactance:

$$X_L = 2\pi fL$$
$$X_L = 6.28 \times 60 \times 20$$
$$X_L = 7{,}536 \text{ ohms.}$$

The current in this a-c circuit is—

$$I = \frac{E}{X_L} = \frac{110}{7{,}536}$$
$$I = .014 \text{ ampere.}$$

If the frequency of this same circuit is increased to 400 cps (cycles per second), the inductive reactance is increased:

$$X_L = 6.28 \times 400 \times 20$$
$$X_L = 50{,}240 \text{ ohms.}$$

Current falls to: $I = \frac{110}{50{,}240} = .0021$ ampere.

f. In a transmitter operating on 5 megacycles (5 million cycles per second), a small inductance of 2.5 millihenrys (.0025 henry) is used to keep substantially all alternating currents out of a certain part of the transmitter while at the same time allowing the passage of dc. This type of inductance is called a *choke* since inductive reactance at that frequency is quite high and effectively chokes off the a-c currents. Thus:

$$X_L = 2\pi f L$$
$$X_L = 6.28 \times (5 \times 10^{+6}) \times (2.5 \times 10^{-3})$$
$$X_L = 78{,}500 \text{ ohms at 5 megacycles.}$$

21. Voltage and Current Across an Inductance: Phase Shift

a. Inductive reactance, as has been shown, not only limits the current flowing in an a-c circuit but also tends to retard the building-up and the falling-off of current. In A of figure 27, a sine wave of voltage is applied to a pure inductance. The current in the circuit also follows the form of the sine wave; but it is necessary to determine precisely the delay in time between the application of the maximum voltage and the moment the current reaches maximum value—that is, the phase shift between the voltage and current. By Kirchhoff's first law it is known that the algebraic sum of the voltage drops about any closed circuit is equal to zero. Thus:

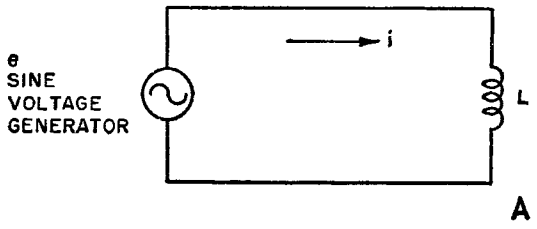
$$e \text{ (applied)} + \text{cemf} = 0$$
$$e = -\text{cemf.}$$

Since: $\text{cemf} = -L\frac{di}{dt}$

$$e = -\left(-L\frac{di}{dt}\right).$$

Therefore: $e = +L\frac{di}{dt}$.

This expression reveals that di/dt, the rate of change of current, is positive when the applied voltage is positive. Thus, when di/dt is positive, or greater than zero, *e* is positive; when di/dt is negative, or less than zero, *e* is negative; and when di/dt is equal to zero, *e* is equal to zero.



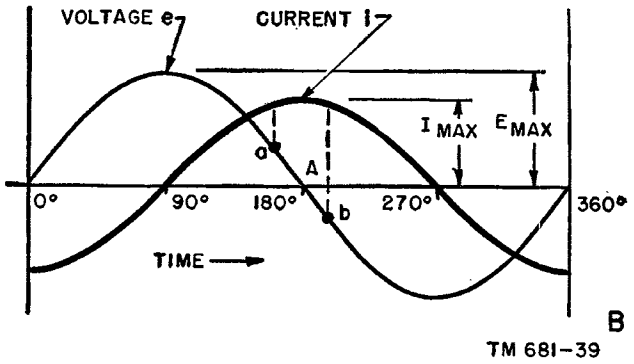


Figure 27. Current and voltage in an inductive a-c circuit.

b. B of figure 27 illustrates the voltage and current sine curves for this circuit. At a time corresponding to point *a*, the voltage *e* is positive, di/dt is positive, and therefore, the current is increasing with time. At point *b*, the voltage is negative, di/dt is less than zero, and current is decreasing with time. For some value of time, then, between points *a* and *b*, di/dt is equal to zero; that is, in going from a positive to a negative value, di/dt must go through zero. Thus, at point A (180° on the time axis), the curve for current must flatten out (neither increasing nor decreasing) as it passes through

its maximum value. In like manner, the rest of the current curve in B may be drawn. It may be seen, then, that current is zero when the applied voltage is maximum, or maximum when the applied voltage is zero. Consequently, the current is said to *lag* the applied voltage by one-quarter cycle or by 90°.

c. A further investigation of this phase shift across an inductance will show the relationships among the induced voltage, or cemf, the current, and the applied voltage. From Lenz' Law and the analysis in the previous chapter of the generation, magnitude, and direction of the induced voltage, it is known that the cemf is, by definition, a voltage opposite in phase to the applied voltage, or 180° out of phase with the applied voltage. In figure 28, A illustrates this condition graphically. Thus, when the applied voltage is zero, there is no opposition, and when the applied voltage is maximum, there is a maximum opposition, an induced voltage in the opposite direction.

d. An examination of the sine wave of current through the inductance will show its phase relationship to the induced voltage. In B, it will be seen that the rate of change of the current is greatest (either negative or positive) as the current passes through zero—that is, the slope of the curve at this point is steepest. This means that if a small time interval were isolated at this point, as in C, it would show a relatively large amount of current change. Therefore, the induced voltage or cemf is greatest at the point at which the current passes through zero. In like manner, a further examination of the curve of the current, in B, will show that the rate of change of current is least at the point at which the current reaches a maximum value; that is, the slope of the curve is, at that point, least steep. A small time interval isolated at that point would show a relatively small amount of current change, or no current change, as in C. Therefore, the counter emf would be least, or zero. The same conditions exist for the other half-cycle.

e. The sine wave of induced voltage then is maximum when current through the inductor is zero, and zero when the current is maximum. Moreover, as the current passes through zero moving in a positive direction, the slope of its curve is positive, and therefore, the induced voltage is a negative maximum, since the induced voltage is always in such a direction as to oppose the action producing it (the change in current). From the point of view of phase, then, the current is said to *lead* the induced voltage by 90°. However, since the induced voltage is, by definition, 180° out of phase with the applied voltage, the current goes through zero moving in either a positive or a negative direction one-quarter cycle or 90° *after* the applied voltage. The current then is said to *lag* the applied voltage by 90°. Figure 29 illustrates these three relationships in graph form.

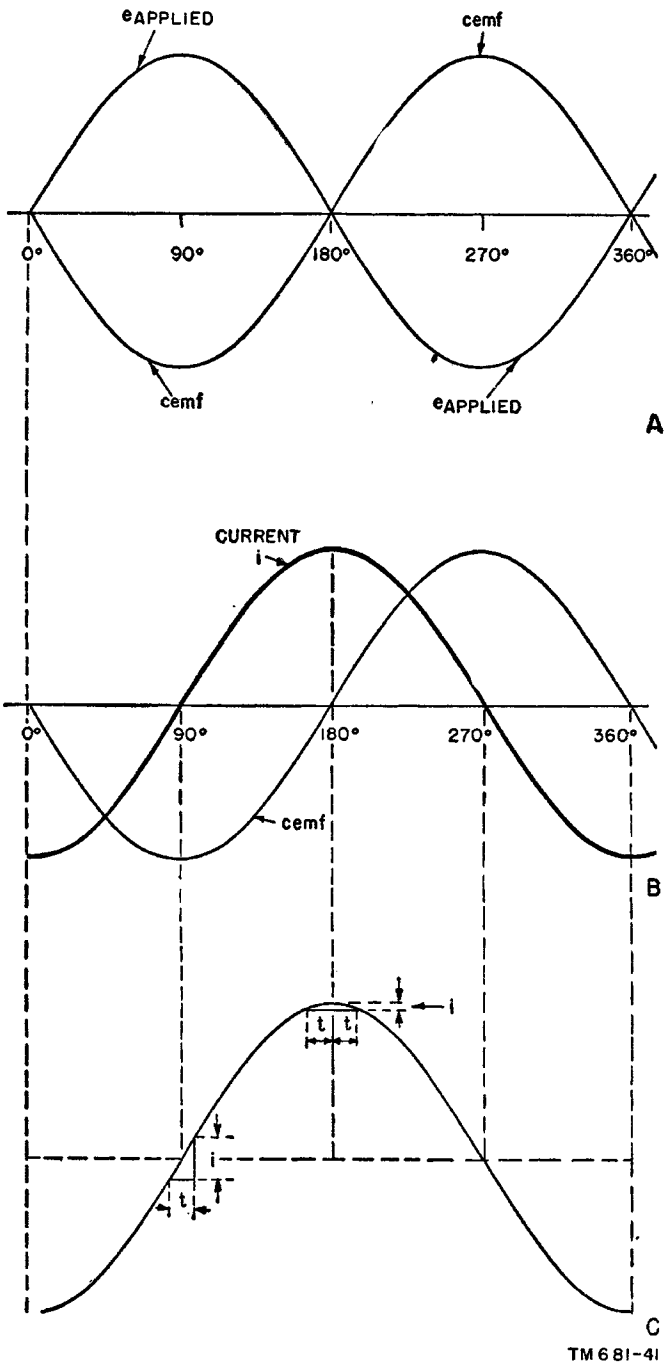


Figure 28. Phase shift in an inductive a-c circuit.

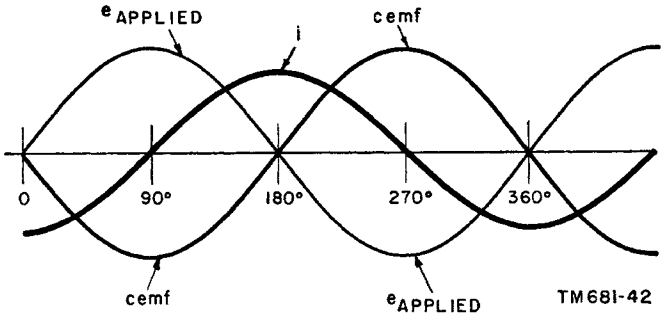


Figure 29. Voltage, current, and cemf in an inductive a-c circuit.

22. Voltage and Current in an L-R Circuit: Phase Angle

a. In any resistive circuit without inductance (L equal to zero), the voltage and current are said to be *in phase*. But it should be noted that in no circuit is the value of the inductance ever actually zero, since, by definition, it would then be possible to change the value of the current instantaneously. However, for all practical purposes, those circuits which do not contain inductors, or appreciable amounts of inductance, can be considered as pure resistive circuits. The time lag of the current in such a circuit is so small as to be negligible. Figure 30 shows this circuit and illustrates graphically the in-phase relationship of a sine wave of voltage and current across a resistance. From the graph, it may be seen that the current and the voltage are alternating in polarity at the same frequency. Current rises as the applied voltage rises and is maximum when the applied voltage is maximum; current falls off as the voltage falls and is zero when the voltage is zero. The magnitude of the current may be determined by Ohm's Law for maximum, effective, or average value, since the frequency of the current change has no effect on the in-phase relationship. Thus:

$$I_{max} = \frac{E_{max}}{R}$$

$$I_{rms} = \frac{E_{rms}}{R}$$

$$I_{av} = \frac{E_{av}}{R} .$$

b. In any circuit containing both inductance and resistance, there is a 90° phase shift of voltage and current across the inductance alone, and no phase shift across the resistance. But it must be emphasized that in any series circuit, *current is everywhere the same.* Since current

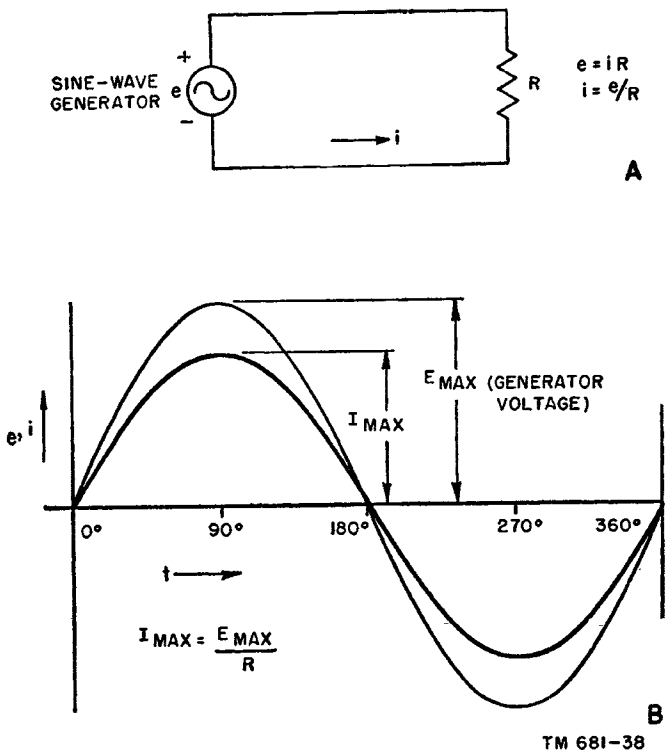


Figure 30. Current and voltage in a resistive a-c circuit.

is the line of reference for both the inductance and the resistance, it may be seen that the voltage developed across the resistance (by the current through it) will be 90° out of phase with the voltage across the inductance. A of figure 31 shows an *L-R* circuit, and B the relationships among the current, the voltage across the inductance, and the voltage across the resistance. Thus, it may be seen that the presence of resistance in an inductive circuit results in two separate voltage drops 90° out of phase with each other. The resultant voltage of these two voltages is the voltage drop in the whole circuit and is, by Kirchhoff's Law, equal to the applied voltage. The amount of phase shift of the current in such a circuit is measured, not with relation to the voltage across the inductance alone (always 90°), but with relation to the resultant voltage, which is the applied voltage.

c. The resultant voltage and the phase angle (generally ascribed the symbol θ) of any *L-R* circuit may be determined by means of vectors. (A complete discussion of vectors may be found in sec. IV, app. I.) In A of figure 32, the voltage across the resistance is laid off on the horizontal vector, and the voltage across the inductance on the vertical vector. Since these two voltages are 90° out of phase, the angle between them

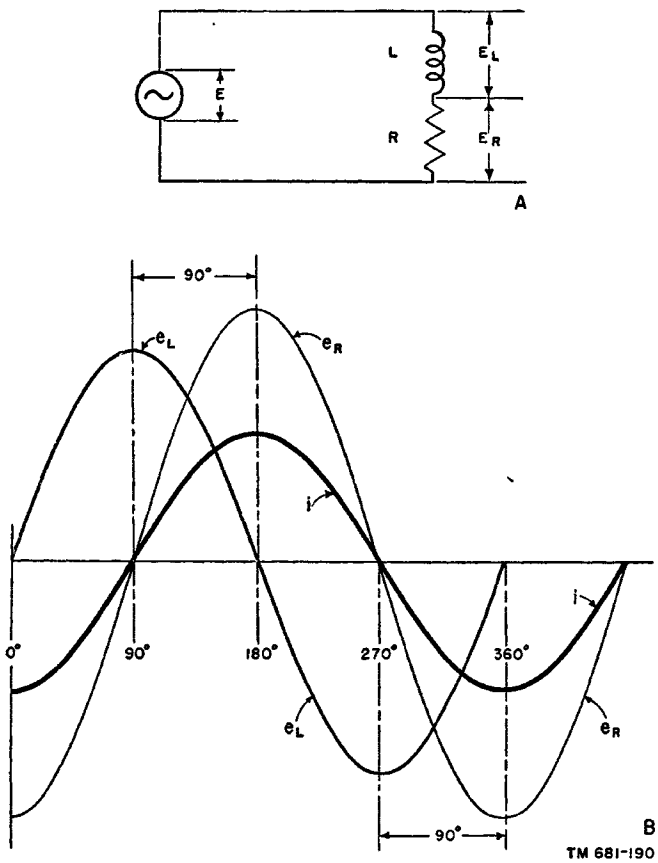


Figure 31. Current and voltages in a series L-R circuit.

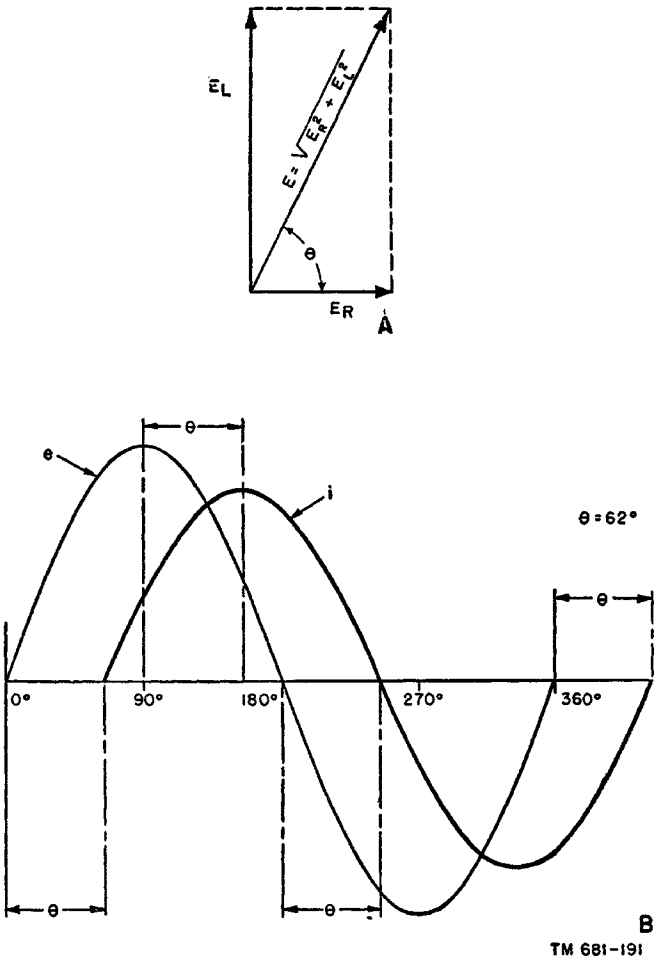


Figure 32. Applied voltage and current in an L-R circuit.

is a right angle. By drawing in a parallelogram based on these two sides, the resultant vector E is seen to be the hypotenuse of a right triangle. Then, by the theorem of Pythagoras, the square on the hypotenuse is equal to the sum of the squares on the other two sides, or:

$$E^2 = E_R^2 + E_L^2$$
$$E = \sqrt{E_R^2 + E_L^2}.$$

d. From the previous discussion, it is known that the current in the circuit is in phase with the voltage across the resistance; therefore, the position of the current with relation to the applied voltage is the same as the vector E_R, the voltage across the resistance. The phase angle θ then is the angle that the applied voltage vector E makes with the vector E_R, as shown in A of figure 32. The angle θ may be measured in terms of any of its trigonometric functions, depending on the values known. If the voltage across the resistance is large with relation to that across the inductance, the resultant vector will approach the horizontal and the phase angle will be small; and in like manner, if the voltage across the resistance is small, the resultant vector will approach the vertical and the phase angle will approach 90°. Hence, the presence of resistance in an inductive circuit causes current to lag the applied voltage by some angle *less* than 90°. B of figure 32 illustrates in graph form the relative positions of voltage and current and the phase angle θ.

23. Inductive Reactance and Resistance: Impedance

a. In any circuit containing both inductance and resistance, the total opposition offered by the circuit is not the simple arithmetical sum of the inductive reactance X_L and the resistance R. The inductive reactance must be added to the resistance in such a manner as to take into account the 90° phase difference between the two voltages in the circuit. This total opposition is termed *impedance* and assigned the symbol Z. Since the voltage across the inductance is determined by the inductive reactance and the current through the inductance, then:

$$E_L = IX_L .$$

The voltage across the resistance is determined by the resistance and the current through it:

$$E_R = IR \ .$$

Then the resultant voltage of these two, or the applied voltage, is determined by the current and the total opposition of the circuit:

$$E = IZ \ .$$

But as previously shown:

$$E = \sqrt{E_R^2 + E_L^2}$$

or

$$E = \sqrt{(IR)^2 + (IX_L)^2} \ .$$

Then:

$$IZ = \sqrt{I^2 (R^2 + X_L^2)}$$

$$IZ = I\sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{R^2 + X_L^2} \ .$$

Thus, *the impedance of an L-R circuit is equal to the square root of the sum of the squares of the resistance and the inductive reactance.*

b. The same result may be obtained more readily by means of vectors. The voltage across the resistance E_R is equal to IR, and the voltage across the inductance E_L to IX_L. Since each vector represents a product of which current is a common factor, the vectors may be laid off proportional to R and X_L and separated by 90°. Figure 33 shows these vectors. The resultant vector Z is the hypotenuse of a right triangle and represents the impedance of the circuit. Then:

$$Z = \sqrt{R^2 + X_L^2}.$$

It will be seen also that the angle θ is the phase angle because the direction of the impedance vector is actually the same as that of the applied voltage vector. This angle is generally determined in terms of its tangent, X_L/R, or in terms of its cosine, R/Z.

c. From the vector diagram of figure 33, it may be seen that if the resistance is large with relation to the inductive reactance, the circuit tends to act as a pure resistive circuit, the phase angle approaches 0°, and the impedance approches R. If the inductive reactance is large with relation to the resistance, the circuit acts as a pure inductive circuit, the phase angle approaches 90°, and the impedance approaches X_L. For practical purposes, then, the impedance of the circuit may be taken to be substantially equal to the larger quantity, if the ratio of the reactance to the resistance, or of the resistance to the reactance, is 10 to 1 or greater.

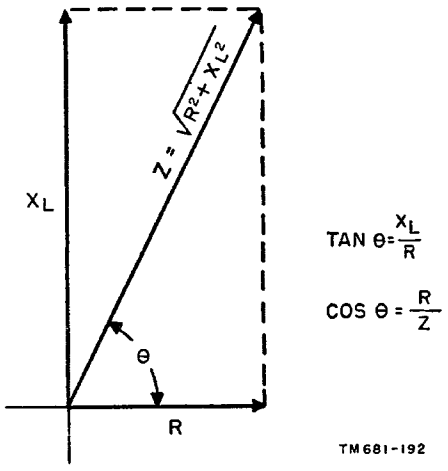


Figure 33. Impedance of an L-R circuit.

d. The current in an a-c circuit containing inductance and resistance may be determined by substituting the impedance Z for the resistance R used in the formula applied to d-c circuits. Thus:

$$I = \frac{E}{Z}$$

$$E = IZ$$

and

$$Z = \frac{E}{I} \ .$$

These formulas are called Ohm's Law for a-c circuits and they apply to solutions for maximum, effective, or average values but not to any instantaneous value. Therefore:

$$I_{max} = \frac{E_{max}}{Z}$$

$$I_{rms} = \frac{E_{rms}}{Z}$$

$$I_{av} = \frac{E_{av}}{Z} \ .$$

e. For example: a 110-volt 60-cycle a-c line is connected across an inductance of 5 henrys and a resistance of 1,000 ohms in series. The inductive reactance alone is:

$$X_L = 2\pi f L = 6.28 \times 60 \times 5$$

$$X_L = 1{,}884 \text{ ohms}$$

and as given: $= 1{,}000$ ohms .

Then the total impedance Z of the circuit is:

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{(1000)^2 + (1884)^2}$$

$$Z = \sqrt{1{,}000{,}000 + 3{,}547{,}256}$$

$$Z = \sqrt{4{,}547{,}256}$$

$$Z = 2{,}132 \text{ ohms.}$$

The effective or rms current in this circuit is:

$$I = \frac{E}{Z} = 110/2132$$

$$I = .051 \text{ ampere.}$$

The phase angle θ of the lag of the current behind the applied voltage is:

$$\text{Cos } \theta = \frac{R}{Z} = 1000/2132 = .4695$$

$$\theta = 62° \ .$$

f. The following conclusions may now be drawn relative to the characteristics of a series *L-R* circuit:

(1) The current through the inductance lags the voltage across it by 90°.

(2) The current through the resistance is always in phase with the voltage across it.

(3) The voltage across the inductance and the voltage across the resistance are 90° out of phase (E_L leading E_R).

(4) The resultant voltage across both inductance and resistance is equal to the square root of the sum of the squares of E_R and E_L.

(5) The resultant voltage is equal to the applied voltage E (Kirchhoff's Law).

(6) The current in the circuit lags the applied voltage by the angle θ, called the phase angle, which is determined by geometric construction ((10) below).

$$\text{Tan } \theta = \frac{E_L}{E_R}, \quad \text{Cos } \theta = \frac{E_R}{E}.$$

(7) The opposition offered by the inductance, X_L, is measured in ohms and called *inductive reactance.* It is determined by the size of the inductance and the angular velocity of the applied voltage.

$$X_L = 2\pi fL.$$

(8) The opposition offered by the resistance, R, is the same as the resistance in a d-c circuit.

(9) The total opposition offered by the circuit, Z, is measured in ohms and is the resultant of the goemetric addition of X_L and R.

$$Z = \sqrt{R^2 + X_L^2}.$$

(10) The phase angle θ may also be determined in terms of R, X_L, and Z ((6) above).

$$\text{Tan } \theta = \frac{X_L}{R}, \quad \text{Cos } \theta = \frac{R}{Z}.$$

(11) The total current in the circuit is found by Ohm's Law re-stated for a-c circuits:

$$I = \frac{E}{Z}$$

where I and E are either maximum, effective, or average values.

24. Parallel L-R Circuits

a. A of figure 34 shows an inductance L and a resistance R connected in parallel across an a-c source. In a circuit of this type it will be seen that, by Kirchhoff's Law for parallel circuits, the voltage across the inductance is equal to the voltage across the resistance, and that this voltage is the same as the applied voltage. Therefore, all voltages in this circuit, being the same voltage, are in phase with each other. However, the current through the inductance lags the applied voltage by 90°, and the current through the resistance is in phase with the applied voltage (B of fig. 34). Then the current in the inductance lags the current in the resistance by 90°. The resultant current, or line current, is the vector sum of these two currents.

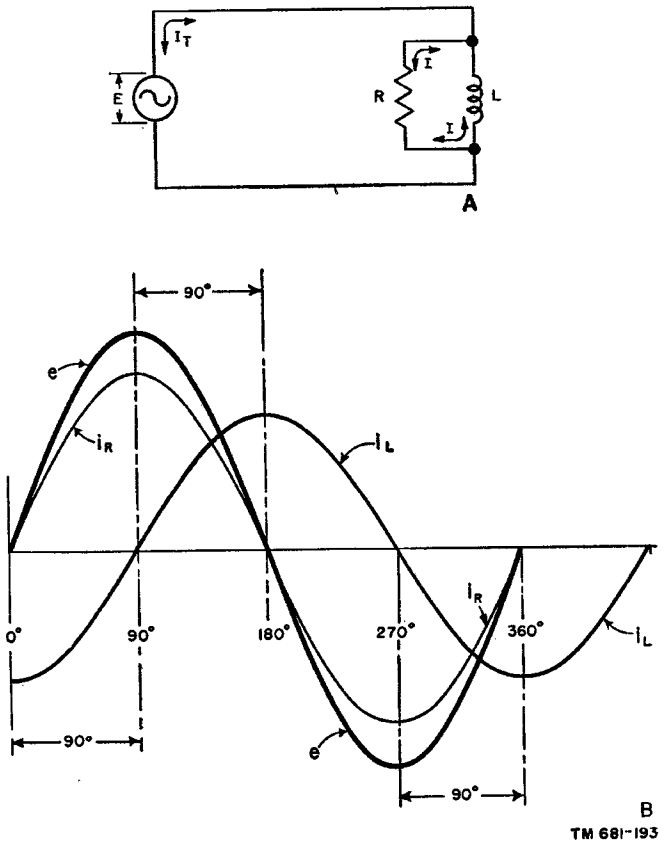


Figure 34. Voltage and currents in a parallel L-R circuit.

b. In figure 35, the current through the resistance I_R is laid off on the horizontal vector, and the current through the inductance I_L on the vertical vector. The I_L vector is laid off in the negative direction because this current lags the current in the resistance, which is taken as the

reference vector, since it is in phase with the applied voltage and represents also the direction of the applied voltage. Now, as in any parallel circuit, the current in the resistance is equal to the voltage divided by the resistance:

$$I_R = \frac{E}{R}.$$

The current in the inductance is equal to the voltage divided by the X_L, the inductive reactance:

$$I_L = \frac{E}{X_L}.$$

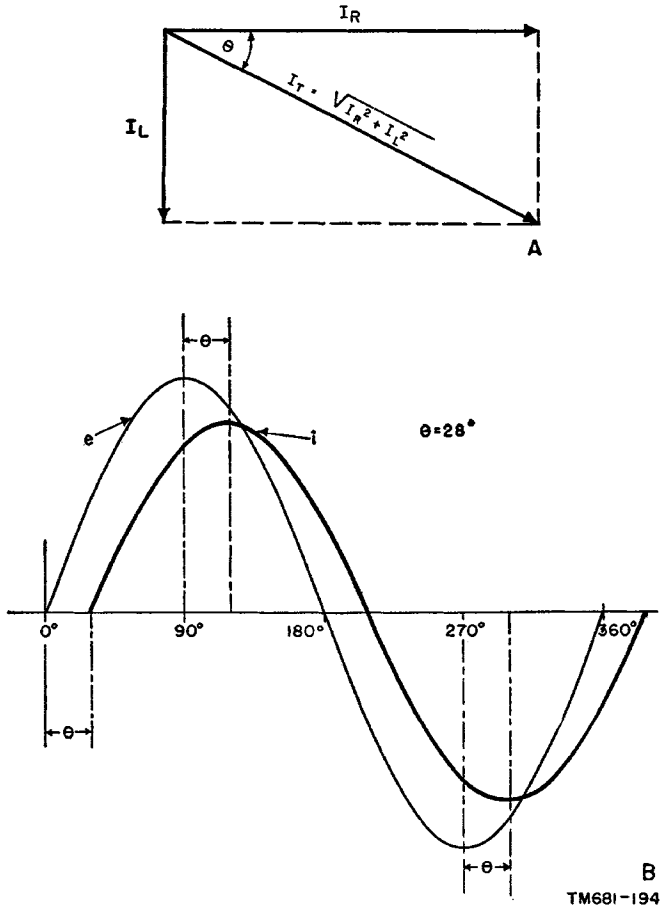


Figure 35. Total current and voltage in a parallel L-R circuit.

c. The resultant vector I_T represents the total current in the circuit, and the angle this vector makes with the horizontal is the phase angle θ. Therefore, the angle θ is the angle the line current makes with relation to the applied voltage, since, as shown above, the direction of the applied voltage is the same as that of the vector I_R. By convention, vectors are rotated in a counter-clockwise direction; since the I_T vector follows the E vector, the line current is said to *lag* the applied voltage by the angle θ. The tangent of this angle is then I_L/I_R. But I_L is equal to E/X_L and I_R is equal to E/R. By substitution and cancellation:

$$\text{Tan } \theta = \frac{R}{X_L}$$

$$\text{Cos } \theta = \frac{Z}{R}.$$

The magnitude of the line-current vector I_T must always be greater than either I_R or I_L because it is the hypotenuse of a right triangle.

Then: $I_T = \sqrt{I_R^2 + I_L^2}$.

Thus, it may be seen that, as in d-c circuits, total current in a parallel L-R circuit is always greater than the current in either branch; and, by extension, the impedance of the circuit is less than the opposition of either branch. By Ohm's Law: $Z = \frac{E}{I_T}$.

d. The impedance of a parallel *L-R* circuit may also be obtained by the use of a formula similar to that used for resistances in parallel. It will be remembered that the total resistance of two resistances in parallel is equal to their product divided by their sum:

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}.$$

Then, by analogy: $Z = \frac{R \times X_L}{R \times X_L}$.

But, as has been shown previously, the addition of two vector quantities $(R + X_L)$ may not be made directly; therefore:

$$Z = \frac{RX_L}{\sqrt{R^2 + X_L^2}}.$$

e. For example: If the values used previously for the series *L-R* circuit are transferred to the parallel *L-R* circuit of figure 34, then as before:

$R = 1{,}000$ ohms

$X_L = 1{,}884$ ohms

$$Z = \frac{RX_L}{\sqrt{R^2 + X_L^2}} = \frac{1{,}884{,}000}{2{,}132}$$

$Z = 884$ ohms.

The line current $I_T = \frac{E}{Z} = \frac{110}{884}$

$I_T = .122$ ampere.

The line current lags the applied voltage by the angle θ:

$$\text{Cos } \theta = \frac{Z}{R} = \frac{884}{1{,}000} = .884$$

$\theta = 28°$.

f. The following comparisons may now be made between series and parallel L-R circuits:

(1) The tangent of the phase angle of the series circuit θ_s is the reciprocal of the

tangent of the phase angle of the parallel circuit θ_p.

$$\text{Tan } \theta_s = \frac{1}{\text{Tan } \theta_p} = \frac{1}{\frac{R}{X_L}} = \frac{X_L}{R}.$$

(2) If the values of inductance and resistance for the two circuits are the same, the two angles are complimentary:

$$\theta_s + \theta_p = 90°.$$

(3) The *greater* the resistance added to a series circuit, the more resistive the circuit becomes; θ_s approaches 0°.

(4) The greater the resistance added across a parallel circuit, the more inductive the circuit becomes; θ_p approaches 90°.

25. Time Constant of L-R Circuit

a. The property of inductance, as has been shown, introduces a delay in time between the applied voltage and the current produced by this voltage in any circuit in which a change of current occurs. In a-c sine-wave circuits, the current is a sine wave also, following the applied voltage by the phase angle θ. In such a circuit, θ depends on the ratio X_L/R—that is, on the frequency, inductance, and resistance of the circuit. *In d-c circuits (or circuits using pulses of dc),* however, the time lag resulting from the starting and stopping of the voltage does not depend on the frequency, since frequency is zero, but on the inductance and the resistance of the circuit.

b. Thus, when a dc is applied to a pure resistance, the current is said to rise immediately to its E/R or maximum value. But if an inductance is added to the circuit, the current is held back by the counter emf and does not rise to its E/R value immediately. A of figure 36 shows the graph of the rise of current in a d-c circuit containing resistance and inductance. This curve is an exponential curve; that is, the current rises rapidly at first and then gradually tapers off to its maximum value, thus describing the characteristic curve shown. An analysis of this curve reveals that in regular units of time, the current rises in decreasing amounts. Thus, in the first unit of time, current rises to 63.2 percent of its final value; and in the second unit to 63.2 percent of the 36.8 percent remaining (86.4 percent of maximum); and in like manner for each succeeding unit of time. Theoretically, in such a progression, the current would never reach a maximum because there would always be a remainder, but for practical purposes current is considered at a maximum after five units of time.

c. Conversely, the current in a d-c inductive circuit cannot fall immediately to zero, as it would in a pure resistive circuit from which the voltage was removed. B of figure 36 shows the graph of the fall of the current. This curve is the same as that described for current rise, but is turned inside out. That is, current falls rapidly at first and then tapers gradually to a minimum. In the first unit of time, it falls 63.2 percent from its maximum E/R value, or to 36.8 percent of its maximum value; in the next unit of time, it falls an additional 63.2 percent of the remainder. This process continues, as described above, until, after five units of time, the current is considered to be at zero.

d. This recurring 63.2 percent of the maximum rise or fall of current in a fixed unit of time is called the *time constant* of a circuit. The more inductance in a circuit, the longer the unit of time required to reach this initial 63.2 percent of the E/R value; and the more resistance in a circuit, the shorter the time required to reach this value, since the greater the resistance of an *L-R* circuit, the less the effect of inductance. Therefore, it may be said that the time constant is equal to inductance divided by resistance, or:

$$t \text{ (time constant)} = \frac{L}{R}.$$

If L is in henrys and R in ohms, then t is in seconds and is the time required for the current to reach 63.2 percent of its d-c E/R value.

e. For example, if 100 volts dc is applied to an L-R circuit containing a 20-henry inductance and a 10-ohm resistance, the E/R value of the current is 10 amperes. The time constant is L/R, 20/10 or 2 seconds. Therefore, it will require 2 seconds for the current to reach 6.32 amperes, and an additional 8 seconds before it reaches approximately its maximum of 10 amperes. If the resistance is lowered, the time constant is increased. In a circuit of 20-henry inductance and 4-ohm resistance, the time con-

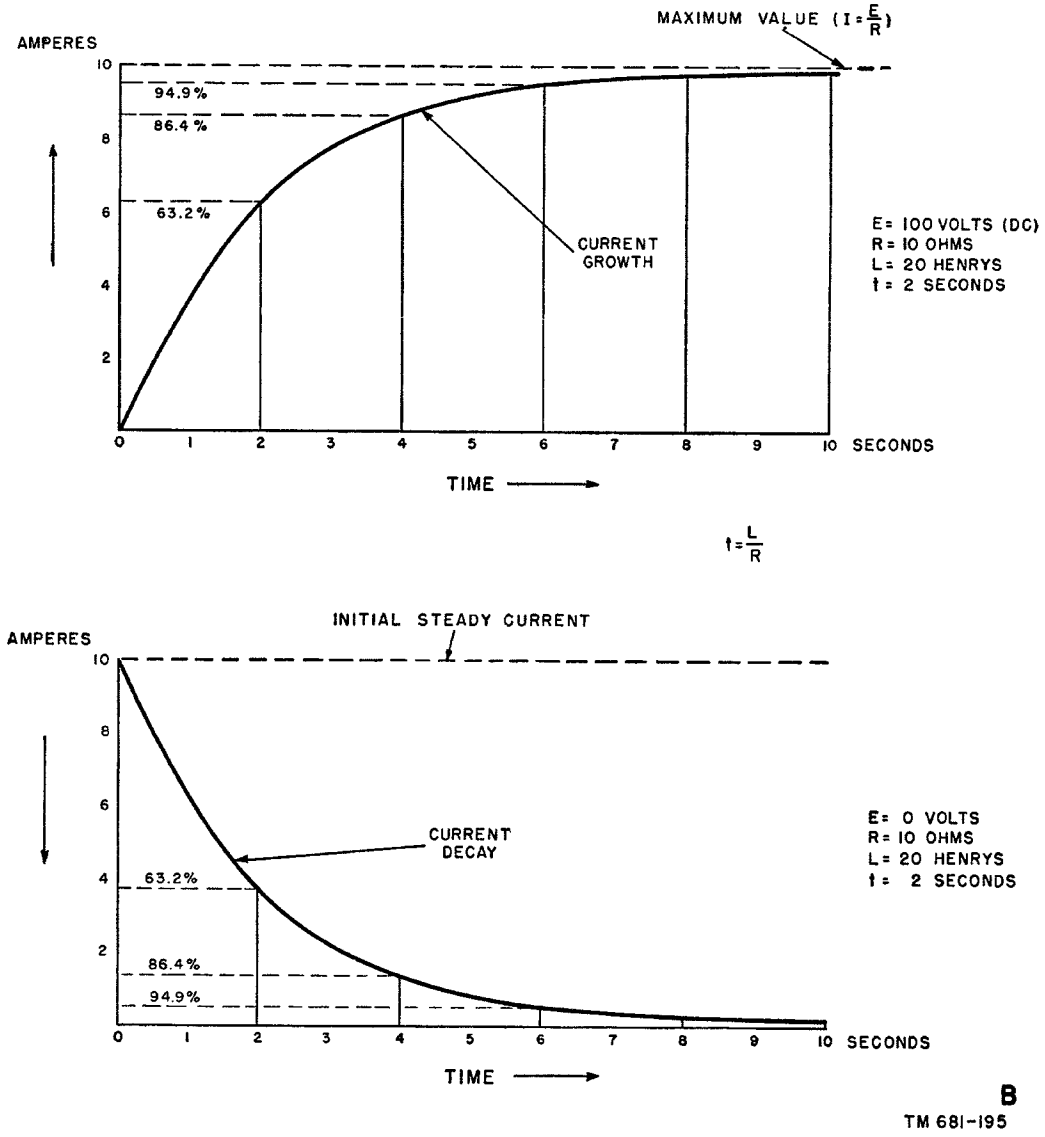


Figure 36. Time Constant in L-R circuit.

stant is 5 seconds. However, it must be remembered that since resistance is lowered, the maximum current in the circuit is now 25 amperes. At the end of the first 5 seconds, the current will have reached a value of 63.2 percent of 25, or 15.8 amperes; and 20 more seconds will elapse before the maximum of 25 amperes is reached. Since the amount of current in the circuit is determined by the resistance alone, it is necessary to vary the time constant by varying the inductance, if it is desired to keep maximum current at a certain fixed value. Thus, in the original example, if resistance is kept at 10 ohms, current remains at 10 amperes, and the time constant of 2 seconds may be increased to 4 seconds by doubling the inductance (L equal to 40 henrys). Then:

$$t = \frac{L}{R} = \frac{40}{10}$$

$$t = 4 \text{ seconds.}$$

If the applied voltage is removed from this circuit, then at the end of 4 seconds current is 3.68 amperes, and at the end of 20 seconds it is almost zero.

f. In the examples above, the circuit values and the time constant were made deliberately large for purposes of explanation, but it should be understood that in high-frequency transmitting and receiving circuits, the time constant may be measured in milliseconds or even in microseconds. Small inductances and high resistances are generally used in such cases. Even though these circuits are primarily a-c circuits, the time constant determines the level of the dc

in those circuits containing both ac and dc. In addition, it is often necessary to use waveforms other than sine waves in a-c circuits. These complex waveforms are shaped in various ways. A of figure 37 shows a current sawtooth waveform which was shaped by first applying a d-c voltage to an L-R circuit and then removing the voltage. In B of figure 37, the voltage has not been removed until after the current reached a maximum and remained there for a given time, resulting in the flat-top form shown.

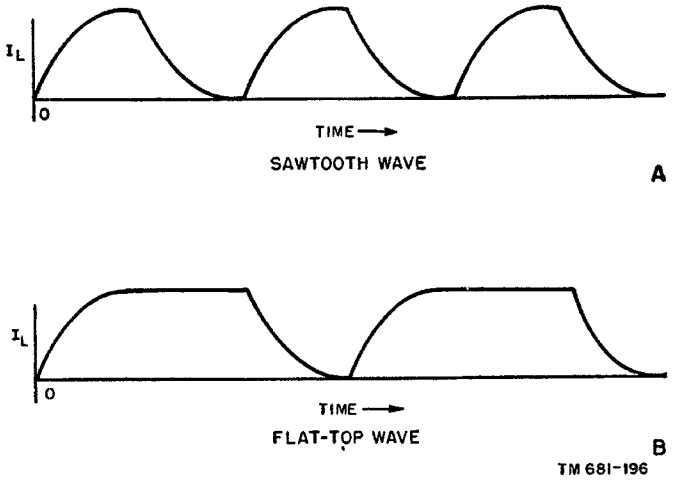


Figure 37. Complex waveforms based on time constant.

26. Mutual Inductance

a. If two inductances are placed physically near each other, the effect of a varying magnetic field caused by a changing current in the first coil induces not only a counter emf in that coil, as has been shown, but also an induced emf in the second coil. The emf induced in the second coil is opposite in direction to the applied voltage, since it is caused by the same forces setting up the counter emf in the first coil. In addition, the changing current in the second coil (caused by the emf induced in it) sets up an induced emf in the first coil. The two coils are then said to be inductively coupled, and this effect of both coils acting one upon the other is called *mutual inductance.* The symbol of mutual inductance is M, and it is measured in henrys, the unit of self-inductance.

b. The amount of mutual inductance present in a circuit depends on the amount of inductance in each coil and the coupling between them. The amount of coupling depends on the physical position of one coil with relation to the other and on the magnetic permeability of the material between them. If all of the flux lines set up by one coil link the other coil, the coupling between them is said to be unity, and the mutual inductance is expressed by the following formula:

$$M = \sqrt{L_1 L_2}\ .$$

If the coupling is less than unity, its extent is expressed by a coefficient of coupling K, which is the ratio of the mutual inductance present compared with the maximum obtainable—that is, when coupling is unity (K equal to 1).

c. Figure 38 shows the mutual coupling of two inductances with the flux linkages indicated by the symbol ϕ. All of the flux lines set up by coil 1 do not cut the turns of coil 2 because the high reluctance of the magnetic circuit causes most of the flux lines to follow paths which do not link coil 2. Under these conditions, mutual inductance is expressed by the following formula:

$$M = K\sqrt{L_1 L_2}$$

where: M = mutual inductance in henrys
K = coefficient of coupling (expressed as a decimal factor)
L_1 = self inductance in henrys of coil 1
L_2 = self inductance in henrys of coil 2.

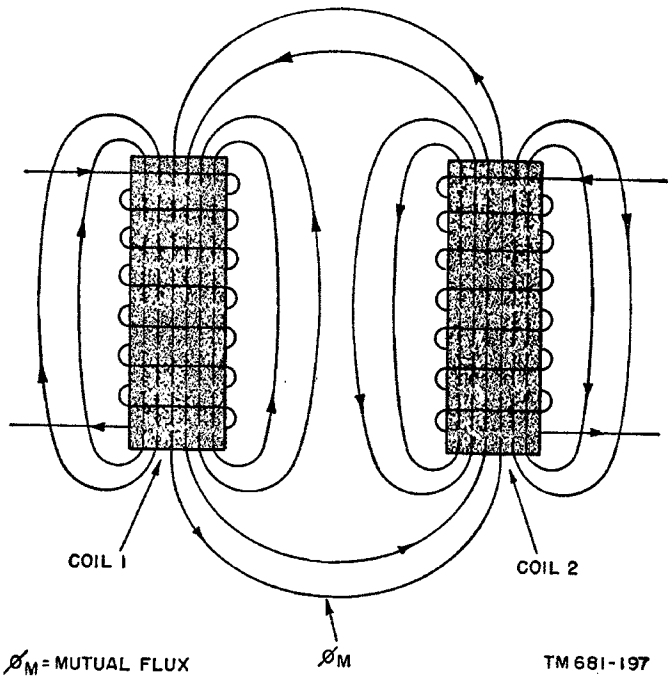


Figure 38. Mutual inductance.

d. The total inductance of a circuit containing more than one inductance connected *in series* is calculated in the same manner as that

used for resistors in series. The total inductance is the sum of the separate inductances:

$$L_T = L_1 + L_2 + L_3 \ldots + L_n .$$

This formula, however, holds true only for circuits in which the inductances are shielded from each other, or so situated physically with relation to each other that no mutual inductance exists between them. If, on the other hand, two inductors are connected in series and so arranged that the flux lines link each other, the resultant total inductance must include the mutual inductance present. Then:

$$L_T = L_1 + L_2 \pm 2M.$$

The plus-or-minus sign must be used in this general expression because the coils may be so placed that the induced voltages either aid or oppose each other, increasing or decreasing total inductance. In figure 39, A shows the two inductances connected in series aiding, for which the formula is:

$$L_T = L_1 + L_2 + 2M.$$

B shows the same inductances connected in series opposing. Hence:

$$L_T = L_1 + L_2 - 2M.$$

If the coils are so arranged that one can be rotated relative to the other, changing the coefficient of coupling K, the total inductance in the circuit can be varied. This is the principle of the variocoupler, or variometer, illustrated and explained in greater detail in TM 11–661.

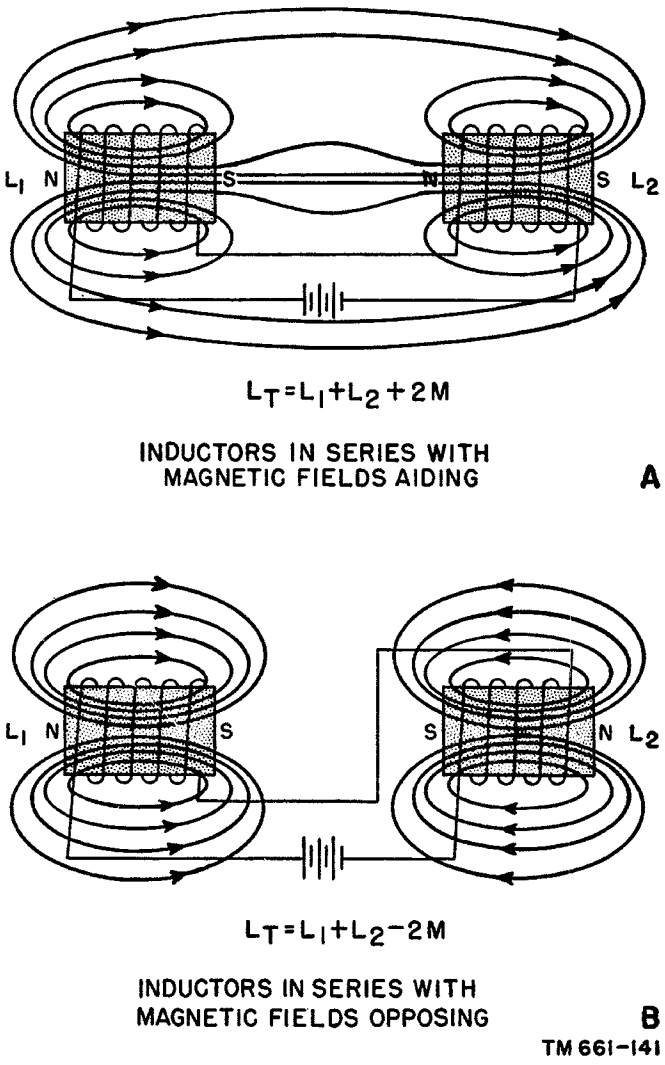


Figure 39. Inductances in series.

27. Inductances in Parallel

a. The total inductance of a circuit containing more than one inductance connected *in parallel* is calculated in the same manner as that used for resistors in parallel. The total inductance is the reciprocal of the sum of the reciprocals of the individual inductances (provided the coils are shielded from each other, M equal to zero). Then:

$$L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots + \frac{1}{L_n}} .$$

If the coils are of equal inductance, total inductance may be obtained immediately by dividing the inductance of one coil by the number of inductances in the circuit. Thus, four 1-henry inductors total one-fourth henry when connected in parallel.

b. For any two inductances, the simplified formula used with resistances may also be employed:

$$L_T = \frac{L_1 \times L_2}{L_1 + L_2} .$$

As noted above, these formulas hold true only when the coils are shielded from each other. Any mutual inductance existing between inductances in parallel tends to reduce the total inductance. Thus, there can be no gain in total inductance and hence little practical use for unshielded inductances in parallel.

28. Types of Inductors

a. A-C RESISTANCE.

(1) As described in TM 11–661, direct current distributes itself uniformly throughout the cross-sectional area of a conductor. Hence the d-c or ohmic resistance of the wire is directly pro-

portional to the specific resistance and the length, and inversely proportional to the cross-sectional area. However, alternating currents flowing in a conductor meet additional resistance (as distinct from the reactance) because of *eddy currents* and *skin effect.* Because alternating currents induce voltages in conductors, small currents, called eddy currents, result. Since these currents represent a loss of power, they are effectively an added resistance in the circuit. In particular, the iron core of a low-frequency inductor, being a conductor, shows a high eddy current loss. For this reason, iron cores are generally made up of thin strips of iron or steel packed together to form a *laminated* core. Such a laminated core reduces the eddy current loss.

(2) The voltage induced at the center of a conductor repels the flow of electrons outward to the surface of the conductor. Hence, skin effect is the name given to the tendency of alternating currents to show a greater density of current flow at the surface of a conductor than at its center. In any given conductor, this tendency of the current to concentrate at the surface reduces the effective cross-sectional area and raises the resistance of the conductor. Both skin effect and eddy currents become more pronounced the higher the frequency of the applied ac, and skin effect increases at a greater rate for large conductors than for small conductors. Thus, a number of small conductors each insulated from the other and stranded into a cable is a common type of a-c conductor. Because copper tubing is not a conductor throughout its cross-sectional area, it also tends to decrease skin effect and is widely used in high-frequency transmitting circuits. The total a-c resistance, then, of an inductor includes the ohmic or d-c resistance of the wire, eddy current loss, and skin effect.

b. IRON-CORE INDUCTORS. In the general analysis of inductance earlier in this chapter, two types of inductors were listed: the iron-core inductor for low-frequency work and the air-core inductor for high-frequency work. However, various other factors besides frequency enter into the design and construction of inductors, and numerous subtypes have been developed. Thus, iron-core inductances, generally called *chokes* because of the high reactance they offer to power-line frequencies, are rated according to the current and voltage they are designed to withstand. In the photograph of figure 40, the large pitch-encased inductor fitted with porcelain feedthrough insulators is an 8.5-henry power choke for filter work on 60-cycle ac and designed to pass as much as .5 ampere of current up to 5,000 volts. The other chokes shown are designed to pass smaller amounts of current (100 to 300 milliamperes) at lesser voltage, and are designed for the power-supply units of heavy-duty transmitting and receiving equipment.

c. AIR-CORE INDUCTORS. In the photograph of figure 41, an assortment of air-core inductors is shown. The largest unit is a low-inductance coil of silver-plated $\frac{3}{8}$-inch copper tubing designed for high-frequency work. Tubing instead of solid wire is used because at the higher frequencies *skin effect* becomes more pronounced. Tubing is also lighter and less expensive than solid wire of the same cross-sectional area. The small amounts of inductance in the larger units in the photograph indicate their use in higher-frequency circuits. The section-wound inductors are common types of radio-frequency chokes in values from 2 millihenrys to 5 millihenrys. This type of construction lessens the *distributed capacitance* of the coil. (For a detailed analysis of this effect see par. 45.) The compactly wound inductor in the lower right-hand side of the photograph is a radio-frequency choke of 100-millihenry inductance designed for lower-frequency work.

29. Summary

a. The velocity of a mass cannot be changed instantaneously, whether that mass be an automobile, an electron, or an electrostatic charge. In electrical circuits, this law results in the effect of electrical inertia.

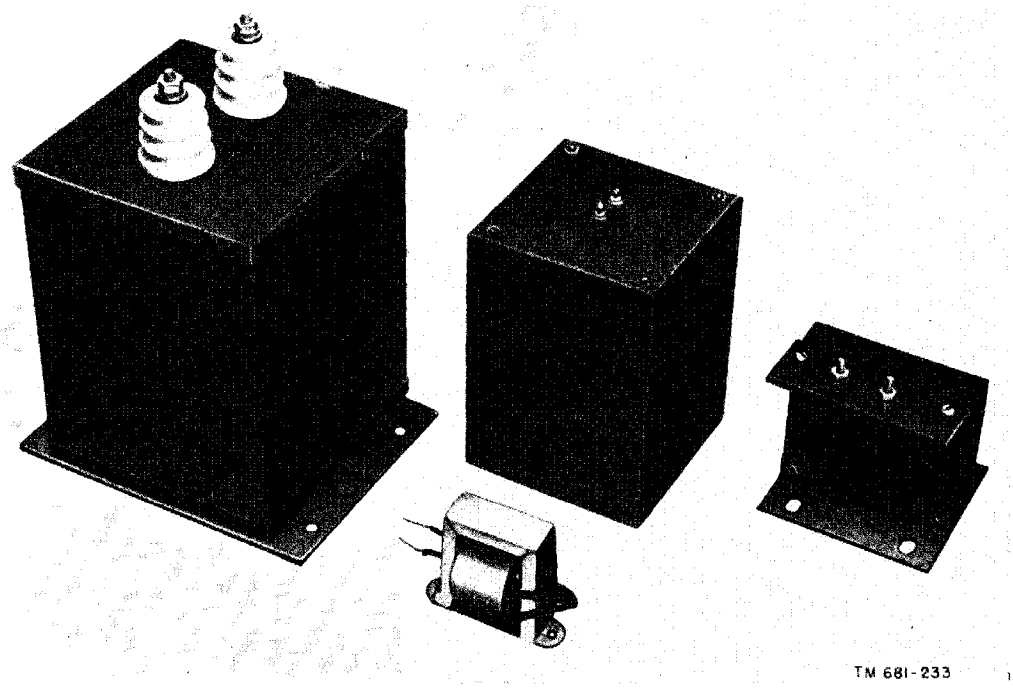


Figure 40. Assortment of iron-core chokes.

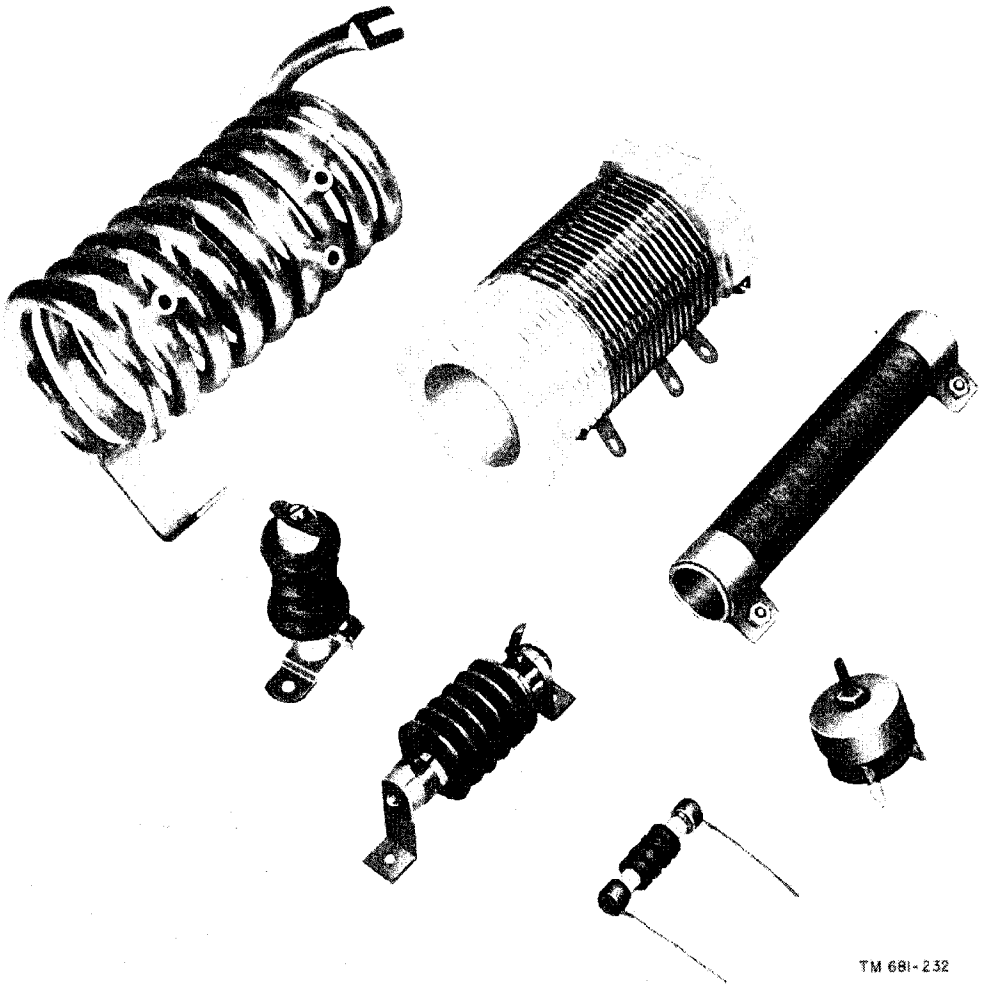


Figure 41. Assortment of air-core chokes.

b. Electrical inertia prevents the instantaneous rise or fall of any electric current. The rise or fall of current is delayed a definite time interval after the rise or fall of the voltage causing the current. This delay results from the presence in the circuit of an induced voltage which opposes the applied voltage. The induced voltage is called a counter or back emf.

c. Counter emf is a result of the changing intensity of the magnetic field about any conductor carrying current. If current changes, the intensity of the magnetic field about the conductor changes and produces a counter emf in the conductor carrying the current. This is an example of Faraday's Law, which states that *the emf induced in any circuit is dependent on the rate of change of the flux linking the circuit.*

d. Counter emf is determined as a counter or back emf by Lenz' Law, which states that *an induced emf always has such a direction as to oppose the action that produces it.*

e. The *magnitude* of a counter emf depends upon the same factors that govern any induced emf—that is, upon the rate of change of the flux density, which in turn depends upon the rate of change of the current in the circuit. Also, initial flux density may be increased, and therefore, the magnitude of the counter emf, by winding the conductor as a coil on a core of high permeability.

f. Thus, the magnitude of a counter emf is expressed in the following formula:

$$\text{cemf} = \frac{-.4\pi N^2 \mu A}{l} \times \frac{di}{dt}.$$

g. In the formula above, the static characteristics, or geometry of the coil, are isolated, given the name *inductance*, and assigned the symbol *L*.

$$L = \frac{.4\pi N^2 \mu A}{l}.$$

h. The formula for counter emf may then be written:

$$\text{cemf} = -L\frac{di}{dt}.$$

This formula, which represents the effect of electrical inertia, is similar to the formula for the inertia of masses:

$$F = -m\frac{dv}{dt}.$$

i. Inductance L is similar to mass *m* and is defined as that property of an electrical circuit which tends to prevent a change of current.

j. The unit of measurement of inductance is the *henry*. A henry is defined as the inductance of a circuit in which a current change of 1 ampere per second causes a counter emf of 1 volt. To express *L* in henrys:

$$L = \frac{.4\pi N^2 \mu A}{l} \times 10^{-8}.$$

k. The following generalizations may be made about inductance:

(1) The inductance of a coil is proportional to the square of the number of turns.
(2) The inductance of a coil increases directly as the permeability of the material making up the core increases.
(3) The inductance of a coil increases directly as the cross-sectional area of the core increases.
(4) The inductance of a coil decreases as its length increases.

l. The permeability μ of iron-core inductances is not a constant, but depends on the magnitude of the magnetizing current. The hysteresis loop represents the variation of μ for changing values of *B* and *H*.

m. The formula cemf $= -L$ di/dt reveals that the greater the inductance, or the faster the rate of change of the current, the greater the cemf induced in the circuit.

n. By reducing cemf to units of opposition, the opposition offered by an inductance to a sine wave of voltage is measured in ohms, called *inductive reactance*, and assigned the symbol X_L. It is proportional to the angular velocity $2\pi f$ and the inductance L.

$$X_L = 2\pi fL.$$

Thus, the higher the frequency, or the greater the inductance, the greater the inductive reactance.

o. The current through a pure inductance *lags* the applied voltage by 90°. This is the *phase shift* of current and voltage, generally indicated by the angle θ. The current through a pure inductance *leads* the cemf produced by 90°. Thus, the applied voltage and the cemf are 180° out of phase.

p. The current through a pure resistance is in phase with the voltage across it.

$$I_{max} = \frac{E_{max}}{R}$$
$$I_{rms} = \frac{E_{rms}}{R}$$
$$I_{av} = \frac{E_{av}}{R}.$$

q. In a *series L-R* circuit, *current* is everywhere the same, and so the voltage across the inductance is 90° out of phase with the voltage across the resistance. The resultant voltage of these two voltages is equal to the applied voltage. Thus:

$$E = \sqrt{E_R^2 + E_L^2}.$$

r. The total opposition offered to current flow by a series L-R circuit is termed *impedance,* given the symbol *Z,* and measured in ohms. The impedance *Z* is equal to the square root of the sum of the squares of the resistance R and the inductive reactance X_L.

$$Z = \sqrt{R^2 + X_L^2}.$$

s. The direction of the impedance vector is the same as that of the applied voltage, and the direction of the resistance vector is the same as that of the current. Therefore, the current in a series L-R circuit *lags* the applied voltage by the angle θ, the angle the Z-vector makes with the R-vector.

$$\text{Tan } \theta = \frac{X_L}{R}$$
$$\text{Cos } \theta = \frac{R}{Z}.$$

t. Ohm's Law for a-c circuits is as follows:

$$I = \frac{E}{Z}.$$

where *I* and *E* are maximum, effective, or average values.

u. In a *parallel L-R* circuit, *voltage* is everywhere the same, and so the current through the inductance is 90° out of phase with the current through the resistance. The resultant current is the line current I_T.

$$I_T = \sqrt{I_R^2 + I_L^2}.$$

Since the total current is greater than the current in either branch, the impedance of a parallel *L-R* circuit is less than the opposition of either branch:

$$Z = \frac{E}{I_T}.$$

Also, the angle that the total-current vector makes with the resistance-current vector is the phase angle of the circuit. Therefore, the total current *lags* the applied voltage by the angle θ.

$$\text{Tan } \theta = \frac{R}{X_L}$$
$$\text{Cos } \theta = \frac{Z}{R}.$$

v. Also, by analogy with parallel resistances, the impedance is:

$$Z_T = \frac{R X_L}{\sqrt{R^2 + X_L^2}}.$$

w. The time required for the rise or fall of current in a d-c L-R circuit is a function of the inductance and resistance in the circuit. Current rises to 63.2 percent of its maximum value, and falls 63.2 percent from its maximum value, in the first unit of time. In each succeeding unit of time, there is an additional 63.2 percent rise or fall of the remainder. This regularity of percentage increase or decline is called the *time constant* of a circuit.

$$t = \frac{L}{R}$$

where *t* is the time constant in seconds, *L* is in henrys, and *R* is in ohms.

x. When two coils are placed physically close to one another in such a manner that the flux lines set up by one coil link the other coil, *mutual inductance* exists between them. The symbol of mutual inductance is *M,* and it is measured in henrys.

$$M = \sqrt{L_1 \times L_2}.$$

If all of the flux lines from coil 1 do not link coil 2, the extent of the coupling between the coils is expressed by the coefficient *K*. Then:

$$M = K\sqrt{L_1 \times L_2}.$$

y. The total inductance of inductances in series is equal to the sum of the inductances.

$$L_T = L_1 + L_2 + L_3 \ldots + L_n.$$

If mutual coupling exists between the inductances, then:

$$L_T = L_1 + L_2 \pm 2M.$$

z. The total inductance of inductances in parallel is equal to the reciprocal of the sum of the reciprocals of the individual inductances.

$$L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots + \frac{1}{L_n}}$$

Or, as in the form made familiar by calculations involving two parallel resistances:

$$L_T = \frac{L_1 \times L_2}{L_1 + L_2}.$$

tors, on the other hand, are capable of supplying electrical energy in almost any desired amount. The practical generator may use one or more electromagnets instead of the permanent magnet, an iron-core coil composed of many turns of wire instead of the single-turn loop, and a superior though similar slip-ring and brush assembly. For d-c motors, the slip-ring and brush assembly is replaced by a commutator and brush assembly, as will be explained in the following paragraphs. The housing and parts of practical generators are designed for maximum speed consistent with use and mechanical limits of safety.

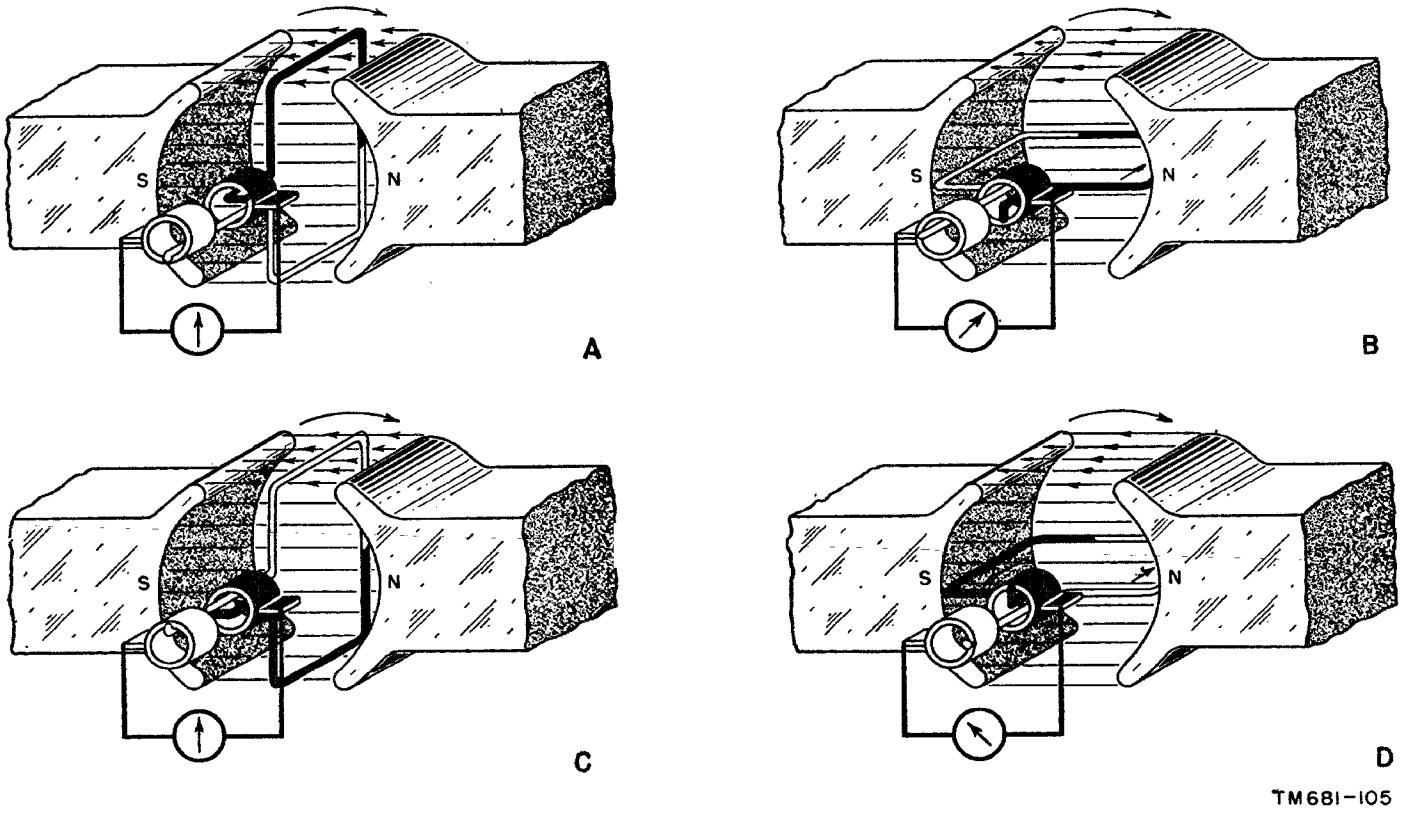


Figure 152. Generation of an emf.

93. Principle of Operation

Consider the four positions of the single-turn loop and slip rings as they rotate clockwise in a uniform magnetic field produced by the poles of a magnet (fig. 152).

a. When the loop rotates through the position shown in A, the black coil side is moving toward the north pole and the white side is moving toward the south pole. Because the coil sides are moving parallel to the direction of the field, no flux lines are cut and the emf induced in the loop is zero. This condition can be proved by connecting a suitable meter across the output terminals as shown (center-scale position of needle indicates zero volts). Note that the coil ends are connected to the slip rings, which rotate simultaneously with the loop. The stationary carbon brushes make contact with the slip rings and are used to conduct the generated voltage to the external load—in this case, a meter.

b. After the loop has rotated 90° from its initial position and is passing through the position shown in B, the black coil side is moving downward and the white side upward. Both sides are cutting a maximum number of flux lines and the induced emf, as indicated by the meter, is at a positive maximum.

c. As the loop passes through an angle of 180° in C, the coil sides are again cutting no flux lines and the generated emf is again zero.

d. As the loop passes through an angle of 270°, as in D, the coil sides are cutting a maximum number of flux lines and the generated emf is at a negative maximum.

e. The next 90° revolution of the loop completes the 360° revolution and the generated emf falls to zero.

f. The preceding facts may be summarized as follows: As the loop makes one complete revolution of 360°, the generated emf passes from zero to a positive maximum, to zero, to a negative

CHAPTER 3
CAPACITANCE

31. Capacitance

a. Capacitance is that property of an electric circuit which tends to oppose a change in voltage.

b. In the previous chapter, the inertia of an electric circuit was analyzed in terms of the property of inductance and its reactive effect. This inductive inertia is very much like the inertia manifested by rigid bodies in mechanical systems and may be contrasted to the inertia manifested by elastic bodies. Consequently, a force applied directly to a rigid mass meets immediate opposition or reaction, but a force applied to an elastic body, such as a steel spring, meets little or no opposition in the first moments of application and then an increasing opposition as the elastic or spring is compressed or extended. Thus, it may be seen that the inertia of an elastic body produces an effect opposite to that of a rigid body and complementary to it. Furthermore, if an elastic material, such as a rubber band, is stretched and then fixed in position, or if a steel spring is compressed and fixed in position, the work done in stretching or compressing is stored indefinitely in the elastic and is returned when the rubber band or the steel spring is released.

c. The counterpart of these mechanical effects characteristic of elastic bodies manifests itself in electrical circuits as the property of *capacitance,* which produces an effect opposite to that of inductance and complementary to it. Thus, if a voltage is applied to an inductance, opposition is immediate, and there is a delay in current rise through it; but if a voltage is applied to a circuit containing capacitance, current flows at a maximum almost instantaneously and then gradually falls to zero as opposition to it builds up. Furthermore, if the applied voltage is removed, the current caused by the capacitance of the circuit is stored indefinitely in that circuit and may be used at some later time. These effects of capacitance appear particularly in a *capacitor,* or *condenser,* which, physically, is any two conductors separated by an insulating material (fig. 42); but capacitance, like inductance, is ever-present in all types of electrical circuits. Random or stray capacitive and inductive effects may be observed in any circuit, just as the most rigid bodies may be said to be elastic to some extent, and the most elastic bodies display some rigidity.

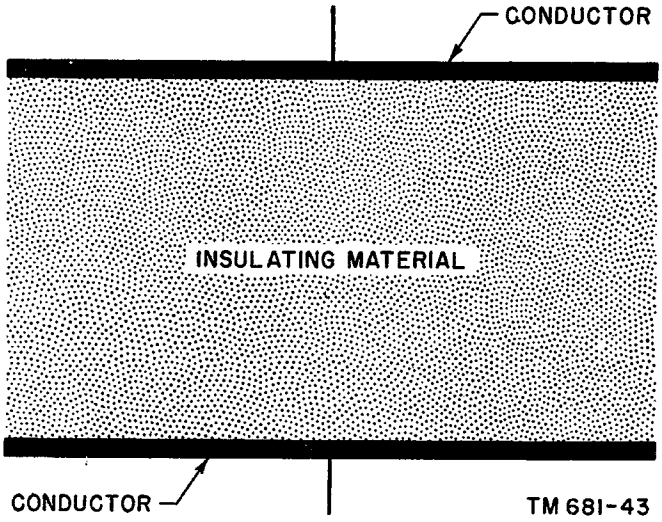


Figure 42. Cross section of basic capacitor.

d. An examination of capacitance in a *d-c* circuit, in which its effects are noticeably present only at the moments of opening or closing the circuit, reveals the basic storage action of a capacitor and its delayed reaction to an applied voltage. In A of figure 43, a simple capacitor is shown in series with a battery and a switch. The switch is open and the capacitor uncharged—that is, no difference in potential exists between the plates. At the moment of closing the switch, shown in B, plate G is at zero potential with relation to plate H, and both plates are at different potentials from the terminals of the battery. Therefore, the free electrons

on plate G are attracted to the positive terminal of the battery, and those on plate H are repelled by the negative terminal of the battery. Then, since in a series circuit current is everywhere the same, the number of electrons flowing out of plate G equals the number flowing into plate H. Thus, plate G is positive because of a deficiency of electrons and plate H negative because of an excess of electrons. The capacitor is said to be fully *charged* when the difference in potential existing across the plates of the capacitor equals the battery voltage. In C, the switch is open, and the capacitor remains charged, since there is no possible way in which the electrons on plate H can reach plate G. Thus, the capacitor stores the energy received from the battery and holds it until needed. If a conductor is placed across the plates of the capacitor, the electrons will flow from plate H to plate G, returning the capacitor to its original neutral uncharged condition.

e. Capacitance then is seen to be a kind of electrical inertia opposite in effect to inductance and similar to the property of elasticity in mechanical systems. Elasticity is defined as that property in nature which tends to oppose a change in force. In like manner, since a capacitor offers no immediate opposition or reaction to an applied voltage (current flows), and offers a maximum reaction when the applied voltage is removed (capacitor charged), a capacitor may be said to offer always a delayed reaction to voltage. Capacitance then may be defined as that property of an electric circuit which tends to oppose a *change* in voltage.

32. Measure of Storage Action: the Farad

a. In the analysis above of the basic storage action of a capacitor, it will be seen that the number of electrons entering and leaving the plates depends on the free electrons available and on the force applied—that is, on the physical size of the plates and on the difference in potential between the battery terminals. If the

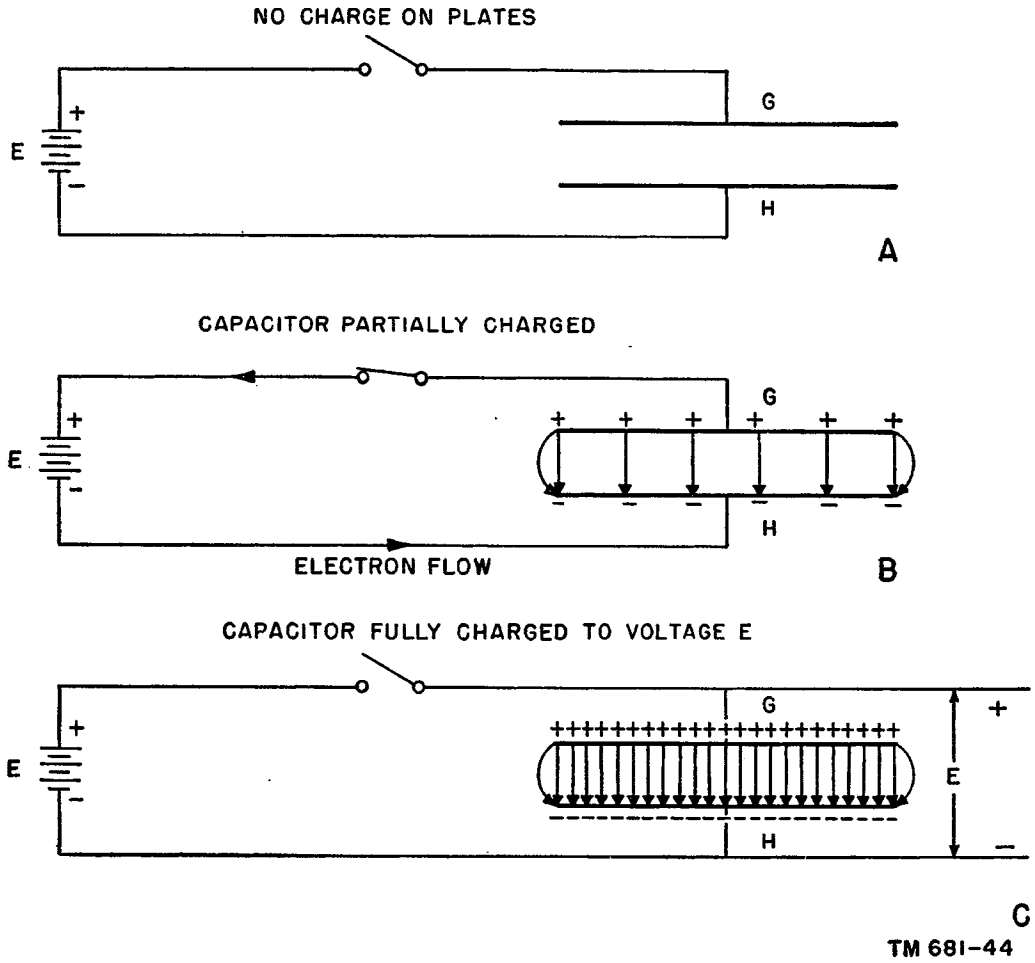


Figure 43. Storage action of capacitor.

applied voltage is high, the forces of attraction and repulsion are great, and the charge deposited on the plate of the capacitor is also great. Thus, experimentally it was discovered that for *a given capacitor* the *ratio* between the amount of this charge and the voltage causing it is always a *constant*. Also, since the electrostatic charge on one plate of a capacitor is equal and opposite to the charge on the other plate, the charge on one plate may be taken as the amount of this charge, which is designated as Q. Therefore, the ratio of the charge Q to the voltage E is taken to be a measure of capacitor action, which is called *capacity* and labeled C. Thus:

$$C = \frac{Q}{E}.$$

b. The unit of measurement of capacity C is the *farad,* named in honor of Michael Faraday, whose contribution to the theory of electromagnetic induction has already been described. A farad then is defined as the capacity of a capacitor on one plate of which a charge of 1 coulomb (6.24 times 10^{18} electrons) is deposited by a difference in potential of 1 volt. This unit of measurement, however, is too large for practical circuits, and so the μf (*microfarad*), or one millionth of a farad (10^{-6} farads), is generally used. In radio communication circuits, even the microfarad unit often proves too large, and so the $\mu\mu$f (*micromicrofarad*), or one millionth of a millionth of a farad (10^{-12} farads), is used.

c. For example, a capacitor connected across a 10-volt battery shows a charge on one plate of one-thousandth of a coulomb. Then:

$$C = \frac{Q}{E} = \frac{.001}{10} = .0001 \text{ or } 100 \times 10^{-6} \text{ farad.}$$

$C = 100$ microfarads.

If the battery is increased to 100 volts, the charge increases to one-hundredth of a coulomb, and the capacity remains the same. Thus:

$$C = \frac{.01}{100} = .0001 \text{ farad} = 100\ \mu\text{f.}$$

Since capacity for a given capacitor is a constant, the magnitude of the charge may be computed when the voltage is increased to 1,000 volts. Thus:

$$Q = CE = .0001 \times 1{,}000 = .1 \text{ coulomb.}$$

Again, since capacity for a given capacitor is a constant, the voltage across the capacitor may be computed if the charge is doubled. Thus:

$$E = \frac{Q}{C} = \frac{.2}{.0001} = 2{,}000 \text{ volts.}$$

33. The Parallel Plate Capacitor

a. In the experimental determination of the formula for capacity, the charge on two concentric spheres in a vacuum was measured and compared to the voltage difference between the spheres. Since, however, this shape of capacitor is impractical, the parallel-plate capacitor, or some variation of it, is generally used (fig. 44). The plates of this capacitor are conceived to be sections of the surfaces of the two spheres of infinite radius, since the surface of a sphere of infinite radius is a plane. Then, the charge Q on one plate is seen to be a part of the total charge on the sphere of infinite radius, a part proportional to the physical size of the plate. Also, the distance between the plates is proportional to the difference in potential between them, since the closer the plates are one to another the less is the force necessary to set up a given charge; and the greater the distance between the plates, the greater the force necessary to maintain that charge. Thus, if the distance between the plates is zero (plates touching), the voltage E is zero; and if the distance were infinite, the voltage would be infinite. Therefore, for a parallel-plate capacitor of linear dimensions which are large in comparison with the distance between the plates, the formula $C = \frac{Q}{E}$ may be written:

$$C = \frac{A}{d}$$

where A is the area of one plate, and d the distance between the plates. However, in order to use this formula for values of capacity in microfarads and linear dimensions in centimeters, a conversion factor of 8.84 times 10^{-8} must be used to bring all quantities under the same system of units. Then:

$$C = 8.84 \times 10^{-8} \frac{A}{d}$$

where: A = the area of one plate in square centimeters
d = the distance between the plates in centimeters
C = the capacitance in microfarads.

The same formula for values in inches and square inches becomes:

$$C = 22.45 \times 10^{-8} \frac{A.}{d}$$

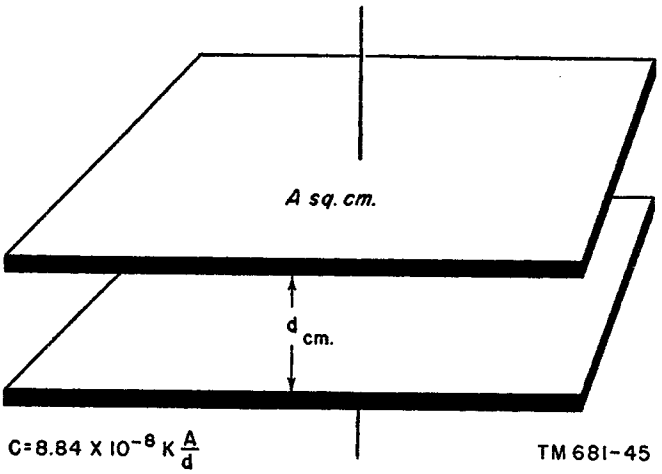


Figure 44. Parallel-plate capacitor.

b. The formulas above hold for a parallel-plate capacitor in a vacuum, that is, with nothing between the plates. The introduction of some insulating substance between the plates intensifies the electrostatic field, and hence a greater charge is held on the plates for the same difference in potential. Thus, by proper selection of this insulating material, known as the *dielectric,* capacitance may be increased, since the ratio of Q to E is increased. Various materials have been tested experimentally to determine the ratio of this increase compared to a vacuum, which is taken as 1. This ratio is called the *dielectric constant, K,* of the material and is a factor independent of area or thickness, since in any single case it is a comparison with a vacuum of the same dimensions. The formulas above then become:

$$C = 8.84 \times 10^{-8} K \frac{A}{d}$$

and: $$C = 22.45 \times 10^{-8} K \frac{A}{d}$$

where K is the dielectric constant. This formula reveals that the greater the dielectric constant K, the greater will be the capacitance for a given set of dimensions.

c. The difference between the dielectric constant for air and for a vacuum is very small (air being 1.00059), and therefore for practical purposes, air may be taken to be 1, the same as a vacuum. Following are some typical values of K, for the more common solid dielectrics:

Material	K (*dielectric constant*)
Air	1.0
Resin	2.5
Asbestos paper	2.7
Hard rubber	2.8
Dry paper	3.5
Isolantite	3.5
Common glass	4.2
Quartz	4.5
Mica	4.5–7.5
Porcelain	5.5
Flint glass	7.0
Crown glass	7–9

For example, a parallel-plate air capacitor has plates 10 times 6 centimeters in dimensions and .1 centimeter apart. The capacity is:

$$C = 8.84 \times 10^{-8} K \frac{A}{d} = \frac{8.84 \times 10^{-8} \times 1 \times 60}{.1}$$

$$C = 8.84 \times 6 \times 10^{-6} = 53 \times 10^{-6}$$

$$C = .000053\ \mu f = 53\ \mu\mu f.$$

If a sheet of mica is placed between the plates, the capacity is increased. Since K for mica is approximately 6, then:

$$C = 6 \times 53 = 318\ \mu\mu f.$$

If a parallel-plate capacitor of .02 μf uses paper .01 inch thick, the dimensions of the plates may be computed. Since:

$$C = 22.45 \times 10^{-8} K \frac{A}{d}$$

Then: $$A = \frac{Cd \times 10^8}{22.45\,K} = \frac{.02 \times .01 \times 10^8}{22.45 \times 3.5}$$

$$A = \frac{2 \times 10^4}{78.60} = 248.4 \text{ square inches.}$$

If the plates are square, the dimension on a side is equal to the square root of 248.4, or 15.8 inches.

d. For larger capacities, the form of the basic parallel-plate capacitor is altered so that a series of plates and sheets of dielectrical material may be stacked or piled one upon the other for greater compactness of construction. Figure 45 shows such a capacitor pile. Each group of alternate plates, which are connected, forms a plate of the capacitor, and both surfaces of the internal plates are used in calculations of total area.

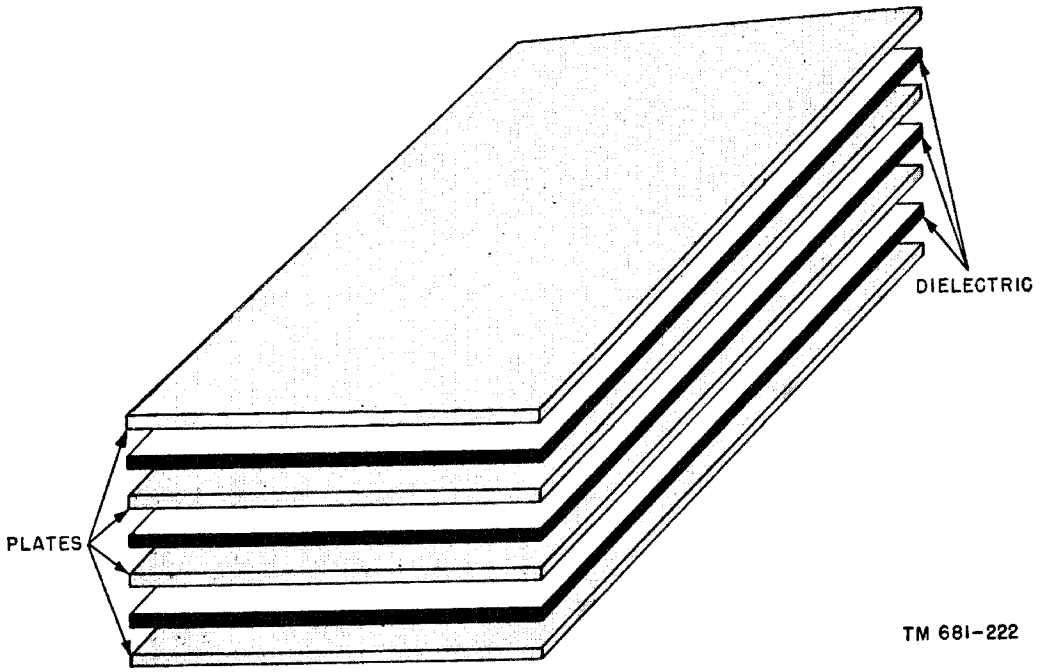


Figure 45. Multiple-plate capacitor.

34. Breakdown Voltage

a. From the preceding, it will be seen that the capacity of a capacitor varies directly with the dielectric constant of the insulating material used. Therefore, for given dimensions and a given voltage, a capacitor using mica has 6 times the capacity of one using air. The charge for a given voltage is increased, since:

$$Q = CE.$$

The use of various dielectrics introduces an additional important consideration in the practical use of capacitors. Since the voltage appearing across the plates of a capacitor is often quite high, particularly in high-power communication circuits, the problem of breakdown or arcing over from one plate to the other arises. If the dielectric breaks down and becomes a conductor, the capacitor is useless, since it no longer can hold a charge and is said to be short-circuited. The voltage required to break down the dielectric varies with the material used and with its thickness. Thus, a great voltage would be required to break down a near perfect vacuum, but a parallel-plate capacitor using as a dielectric air one-thousandth of an inch thick breaks down at 80 volts. If the air space is widened to one-hundredth of an inch, the breakdown voltage increases, in proportion, to 800 volts; but as noted from the formulas above, the capacity has been lessened, since d, the distance between the plates, has increased. Other dielectric materials, however, not only increase capacity for given dimensions, but also increase the voltage necessary to cause a breakdown. Thus, mica one-thousandth of an inch thick withstands 2,000 volts before breaking down, compared to 80 volts for air, and increases capacity 6 times over the capacitor using air.

b. Following is a table of common dielectric substances and their approximate breakdown voltages per one-thousandth of an inch of thickness. It should be noted, however, that no necessary relationship exists between the dielectric constant and the dielectric strength, or breakdown voltage.

Dielectric	K	*Dielectric strength (Volts per .001 inch)*
Air	1	80
Fiber	6.5	50
Bakelite	6	500
Glass	4.2	200
Mica	6.0	2000
Castor oil	4.7	380
Paper		
(1) beeswaxed	3.1	1800
(2) paraffined	2.2	1200
Porcelain	5.5	750

c. Thus, the dielectric strength of a capacitor is an important factor in the practical construction of capacitors. Commercial capacitors, therefore, are marked to indicate capacity and *working voltage.* By working voltage is meant the limit of safe rms voltage that may be applied continuously across the capacitor. The capacitor will withstand higher *surge* voltages, that is, voltages present for a short time. It should be remembered also that a capacitor marked for a given working voltage, for instance .5 μf at 600 volts, actually is designed to withstand the peak voltage present in every a-c cycle, or 600 times 1.414 = 848.4 volts. In addition, a considerable margin of safety has been allowed by the manufacturer.

35. Generation of Conductance

a. In the paragraphs above, the storage action of a simple capacitor was described by noting its behavior in a d-c circuit. The reactive effect of the capacitor and its similarity to the inertia of elastic bodies may now be considered. It was noted that, although current flows at a maximum almost instantaneously when a capacitor is connected across a d-c source, this current flow soon falls to zero as the capacitor becomes fully charged. A capacitor, then, unlike an inductance, may be said to be an open circuit, that is, to offer infinite resistance to a d-c source. Counter or back emf is *immediately* present at a maximum to voltage applied across an inductance, but counter emf appears across a capacitor at maximum *after* it has become charged. A of figure 46 shows by the direction of its arrows the current flow as the capacitor charges. If the battery is then reversed, as in B, and the switch closed, as in C, the current flow is at a maximum in the opposite direction, since the counter voltage developed across the capacitor as it charged now aids the battery voltage to send current in the opposite direction. In like manner, the compression in a spring would aid its own extension if the force compressing it were reversed.

b. This effect of the delay in the building up of the counter voltage is of great importance in a-c circuits, since ac, like the reversed battery above, is periodically changing direction. Thus, a capacitor, which offers a path of infinite resistance to dc, is found to offer a conducting path to an ac,

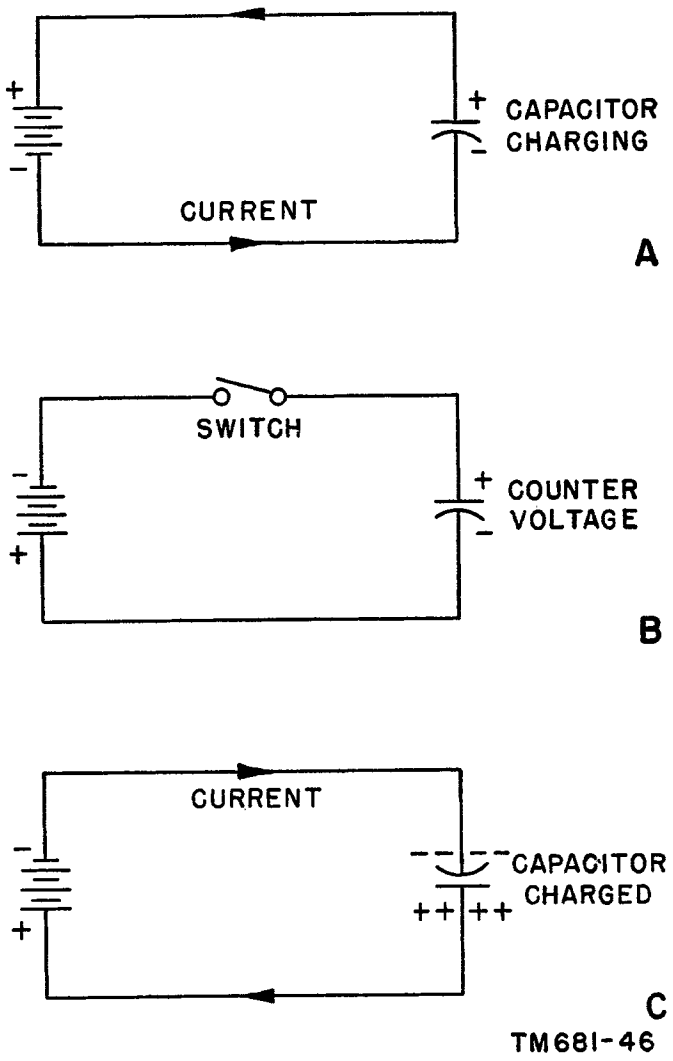


Figure 46. Charge and recharge of a capacitor.

current flowing with relative ease first in one direction and then in the other. For example, the d-s resistance of a 2-microfarad wax paper capacitor is very high (millions of ohms), yet this same capacitor placed across a 400-cycle a-c source shows an appreciable current flow. Figure 47 illustrates this circuit; the voltage applied is 200 volts. Judged by its high d-c resistance, the current through the capacitor should be practically zero; the ammeter, however, reads 1 ampere. It is evident, then, that the capacitor represents a conducting path to an a-c source. It should be noted, however, that although a-c *energy* finds the capacitor a conducting path, the electrons in the circuit do *not* pass across the capacitor from one plate to another. But, since an alternating current is defined as one periodically changing direction, a capacitor effectively passes such a current.

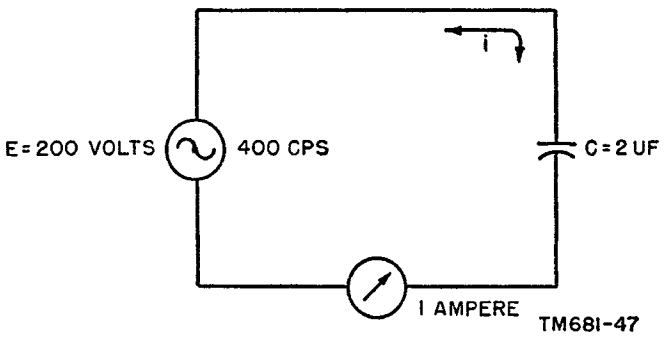


Figure 47. A-c current flow in a capacitive circuit.

c. From the observations above, it will be seen that a capacitor, which offers infinite opposition to dc, offers less opposition to ac. In the circuit of figure 47, the applied voltage is 200 and the current is 1 ampere. Therefore, by Ohm's Law, the ooposition offered by the capacitor is only 200 ohms. This opposition is the effect produced by counter emf, the voltage periodically built up across the capacitor. By voltage across the capacitor is meant the *voltage drop* across the capacitor, which is generated by the building-up of the charge on the plates of the capacitor and is a voltage always equal and opposite in direction to the applied voltage, a counter emf. As with inductance, the counter emf of a capacitor must be understood to be a voltage *generated* by the voltage applied across the reactive element; the effect of this generated voltage is not to cancel the applied voltage (since it is caused by the applied voltage) but to offer an opposition to current flow and thereby produce a *voltage drop.* As with inductance, the magnitude of this counter voltage at any instant determines the opposition to current flow.

36. Coulomb's Law

a. In the study of electrostatic charges in TM 11–661, it was shown that the actual physical force between two charges separated by a distance d depended on the magnitude of the charges and the square of the distance between them. Thus:

$$F = \pm \frac{Q_1 Q_2}{d^2} .$$

This is the expression for Coulomb's Law of force between charges, and it indicates that the greater the charges, and the smaller the distance between them, the greater the force between them. For the special case of a parallel-plate capacitor, Coulomb's Law takes the previously described form:

$$E = \frac{Q}{C} .$$

As above, this formula indicates that the greater the charge, and the smaller the capacity, the greater the voltage across the capacitor. For any given capacitor, then, it will be seen that the voltage across the capacitor is directly dependent on the magnitude of the charge Q. In an a-c circuit, however, the charge Q on the capacitor is constantly changing in magnitude, and hence the voltage across the capacitor, which varies directly with the charge, is constantly changing in magnitude. This variation of charge, or flow of charge, is current, which, as defined in TM 11–661, is the time rate of change of charge. Hence:

$$I = \frac{Q}{t} .$$

where: Q = the charge on one plate of the capacitor in coulombs
t = the time in seconds required to build up that charge
I = the current in amperes.

b. Thus, if .2 of a coulomb of electrons is deposited in 1 second on the negative plate of a capacitor, the current in the circuit is .2 of an ampere. As above, the magnitude of the voltage depends on the magnitude of the charge, but it should be noted that the magnitude of the current depends on the *rate of change* of the charge. However, since Q is equal to CE, the expression above may be written:

$$I = \frac{CE}{t} .$$

Since the physical factors expressing the magnitude of the charge are contained under the capacitance, then C may be isolated and the rate of change of the charge expressed in terms of the rate of change of voltage. Then:

$$I = C\frac{E}{t} .$$

Thus, the magnitude of the current through a capacitor is greater for a rapidly changing voltage (t is small) than for a slowly changing voltage (t is large). Therefore:

$$I = C\frac{de}{dt}$$

where: de = the change in voltage, in volts
dt = the change in time, in seconds

C = the capacitance in farads
I = the current in amperes.

The formula above reveals that the greater the capacity, or the faster the rate of change of the voltage, the greater the current in the circuit. Also, the farad may now be redefined as follows: *A farad is the capacitance of a circuit in which a voltage change of 1 volt per second causes a current flow of 1 ampere.*

37. Measurement of Capacitor Current

a. A high current through a capacitor may be obtained either by increasing the capacitance or the speed of the change of the voltage across it, or both. For example a 1-microfarad capacitor has 1,000 volts across the plates. If this voltage changes to 2,000 volts in 1 second, the current will be:

$$I = C\frac{de}{dt} = 1 \times 10^{-6} \times \frac{2000-1000}{1}$$

$I = 1000 \times 10^{-6} = .001 = 1$ milliampere.

If the change of voltage remains the same, but the capacitor unit is changed to 10 microfarads, then:

$I = .010 = 10$ milliamperes.

If the capacitance remains, as in the first instance, at 1 microfarad, and the change of charge takes place in 1/10 of a second, then:

$I = .010 = 10$ milliamperes.

b. From the examples above, it may be seen that *high frequency* a-c circuits, because of the great speed of change of the voltage, pass sufficient current with small values of capacitance; *low frequency* a-c circuits, because of the slow speed of change of the voltage, generally employ large values of capacitance in order to pass sufficient current. The following table illustrates, for a given capacitor, and a fixed change in voltage, the rise in current as the rate of change of voltage increases:

C *microfarads*	$\triangle e$ *volts*	$\triangle t$ *milliseconds*	I *milliamperes*
1	100	100	1
1	100	50	2
1	100	25	4
1	100	10	10
1	100	5	20
1	100	1	100
1	100	1/2	200
1	100	1/5	500
1	100	1/8	800
1	100	1/10	1,000

where: $\triangle e$ (delta e) = amount of change in voltage.

$\triangle t$ (delta t) = amount of time required for change in voltage.

c. From the table it is apparent that if a change of voltage were to take place instantaneously, that is, if $\triangle t$ were equal to 0, the current in the circuit would become infinitely large, an impossible condition. If no change of voltage were to take place, that is, if $\triangle t$ were equal to infinity, the circuit is a d-c circuit and the current would be zero. For any a-c voltage, however, the rate of change depends on the number of cycles per second, or frequency. An increase in frequency means that electrons must be taken from one plate and deposited on the other at a faster rate; then, since current is the rate at which charges are deposited, an increase in frequency brings about an increase in current. The current through a given capacitor, then, is directly proportional to frequency, and therefore the opposition offered by a capacitor is inversely proportional to frequency. Thus, a capacitor reveals characteristics opposite in effect to an inductance: as frequency increases, an inductance generates a greater and greater counter emf, whereas a capacitor under the same circumstances becomes increasingly a conductor. Figure 48 illustrates this contrast in graph form. Furthermore, the formula $I = C\frac{de}{dt}$ may be transposed to the form: $C = I\frac{dt}{de}$.

Then it will be more readily apparent that the larger the capacitance, C, of a given capacitor the smaller the change in voltage, de. Therefore, capacitance is defined as that property of an electric circuit which tends to oppose a *change* in voltage.

d. Some characteristics of the property of capacitance may now be summarized:

(1) A capacitor stores electrical energy and offers a delayed reaction to a change in voltage.

(2) The ratio between the charge on a capacitor and the voltage causing it is a constant, called capacity:

$$C = \frac{Q}{E}.$$

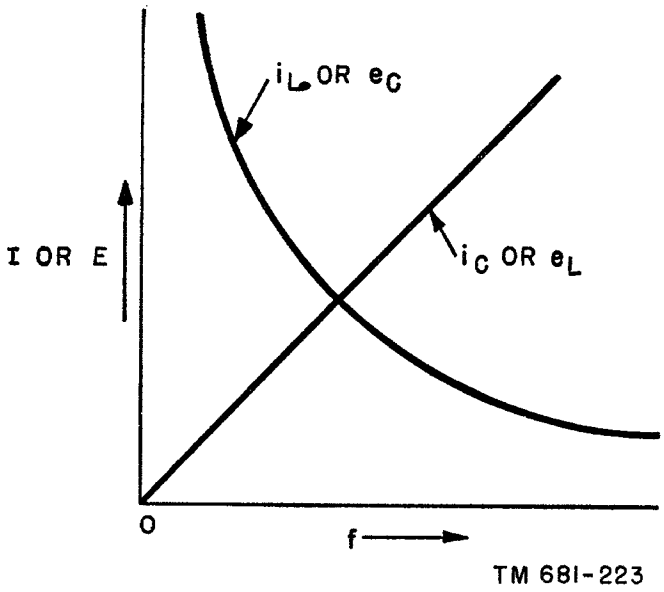


Figure 48. Variation of current or voltage with frequency.

(3) Different dielectrics show varying ratios of electrostatic permittivity as compared to air.

(4) Different dielectrics show varying breakdown resistance to high voltages.

(5) A capacitor is a conducting path to alternating current.

(6) The current through a capacitor is determined by its capacity and the rate of change of voltage across it:

$$I = C\frac{de}{dt}.$$

(7) A capacitor produces an effect opposite to that of an inductance, opposition to ac decreasing with frequency.

38. Capacitive Reactance

a. In the discussion above, the conductance of a capacitor at any instant was realized in terms of the current flowing in the circuit at that instant. The unit of measure of conductance, however, is the *mho* and its symbol is *G*. In TM 11–661, it was shown that the word *mho* is the word *ohm* spelled backwards, and that the concept of conductance—that is, the ease with which current flows in a circuit—is the inverse of the concept of resistance, the opposition to current flow. Thus, conductance is the reciprocal of resistance and resistance is the reciprocal of conductance. Then:

$$G\ (\text{mhos}) = \frac{1}{R\ (\text{ohms})} \qquad \text{or } R\ (\text{ohms}) = \frac{1}{G\ (\text{mhos})}$$

b. From the preceding table listing the magnitude of current and from the examples above, it is seen that the conductance of a capacitor depends directly on the size of the capacitor and the rate of change of voltage. In an a-c circuit, the rate of change of voltage is governed by the angular velocity of the applied voltage, and this velocity is measured in radians. Since each cycle is 2π radians, the angular velocity of any sine wave is equal to this factor multiplied by the frequency. Therefore, the conductance of any capacitor is equal to the capacitance in farads multiplied by the angular velocity $2\pi f$, or:

$$G_C = 2\pi fC\ (\text{mhos}).$$

c. Then, since the reaction of an inductance to alternating current is measured in ohms, it is convenient to measure the reaction of a capacitor in ohms also. Therefore, inductive reactance X_L finds its counterpart in *capacitive reactance,* which is given the symbol X_C and measured in ohms. Capacitive reactance, then, is equal to the reciprocal of the conductance. Thus:

$$X_C = \frac{1}{G_C}.$$

Substituting $2\pi fC$ for G_C

$$X_C = \frac{1}{2\pi fC}.$$

where: $2\pi = 6.28$
f = frequency in cycles per second
C = capacitance in farads
X_C = capacitive reactance in ohms.

From this formula, it may be seen that the higher the frequency, or the greater the capacitance, the less the capacitive reactance. A of figure 49 is a graph of X_C plotted against frequency, and B is a graph of X_C plotted against capacitance. The graphs are similar and indicate that capacitive reactance decreases nonlinearly with frequency or with capacitance: the greater the frequency for any C, or the greater the capacitance for any f, the less the inductive reactance of the circuit.

d. At the very high frequencies used in radio communications, even small stray amounts of capacitance offer a path of such low reactance to the signal voltage that often a desired signal is short-circuited. At the other extreme, the separate d-c circuits are effectively isolated from each other by either small or large capacitors, since either offers a path of maximum

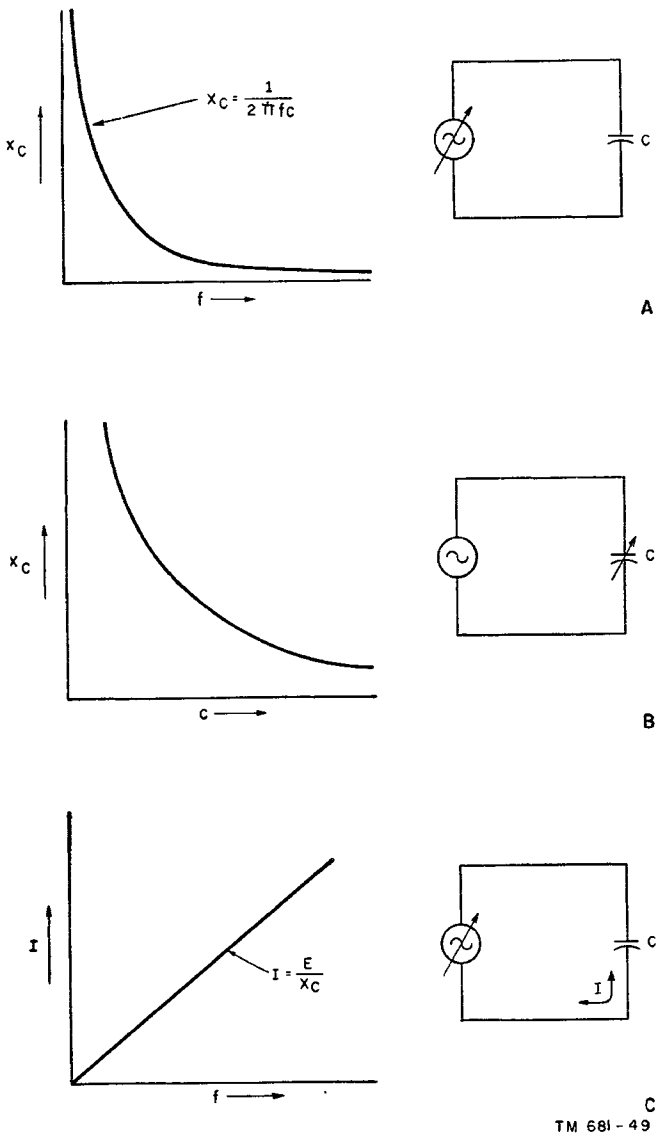


Figure 49. Variation of capacitive reactance.

reactance to direct current. C of figure 49, a graph of current in a capacitive circuit plotted against frequency, illustrates these relationships. It may be said, then, that at low frequencies, a capacitor tends to act as an open circuit, since current is very low; and at high frequencies a capacitor tends to act as a simple conductor, or short circuit. Thus, capacitors designed for use at low frequencies are generally large capacitors, which are measured in full units of microfarads (one-millionth of a farad). Those used at high frequencies are generally small capacitors, which are measured in micromicrofarads (one-millionth of a *microfarad*).

e. For example, the opposition offered by a 2-microfarad capacitor to a 110-volt, 60-cycle a-c source may be determined by use of the formula for capacitive reactance:

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{6.28 \times 60 \times 2 \times 10^{-6}} = \frac{10^6}{6.28 \times 60 \times 2}$$

$X_C = 1333$ ohms.

The current in this a-c circuit is:

$$I = \frac{E}{X_C} = \frac{110}{1333}$$

$I = .0825$ ampere.

If the frequency of this same circuit was increased to 1,000 cycles per second, the capacitive reactance is decreased:

$$X_C = \frac{1}{6.28 \times 1000 \times 2 \times 10^{-6}}$$

$X_C = 79.7$ ohms.

Current increases to: $I = \frac{110}{79.7} = 1.38$ amperes.

f. In a transmitter operating on 7 megacycles (7 million cycles per second), a small capacitance of 250 $\mu\mu$f (.00025 μf) is used to block direct currents out of a certain part of the transmitter while at the same time allowing the passage of the a-c signal. Such a capacitor is called a d-c blocking or a-c coupling capacitor, since it offers infinite resistance to the dc and is a conducting path to the a-c signal. Thus:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 7 \times 10^6 \times 250 \times 10^{-12}}$$

$$X_C = \frac{10^6}{6.28 \times 8 \times 250} = \frac{10^4}{6.28 \times 7 \times 2.5}$$

$X_C = 91.0$ ohms at 7 megacycles.

39. Current and Voltage across a Capacitor: Phase Shift

a. Capacitive reactance, as has been shown, not only increases the current flowing in an a-c circuit but also tends to retard the building-up and the falling-off of voltage. In A of figure 50, a sine wave of voltage is applied to a pure capacitance. The variation in charge on the plates of the capacitor also follows the form of the since wave and is in phase with the voltage, since for any given capacitor the voltage across the capacitor depends directly on the charge (B and C). The current in the circuit also follows the form of the sine wave, but since a capacitor tends to retard voltage change it is necessary to determine precisely the delay in time between the current flow and the building up of voltage, that is, the phase shift between current and voltage.

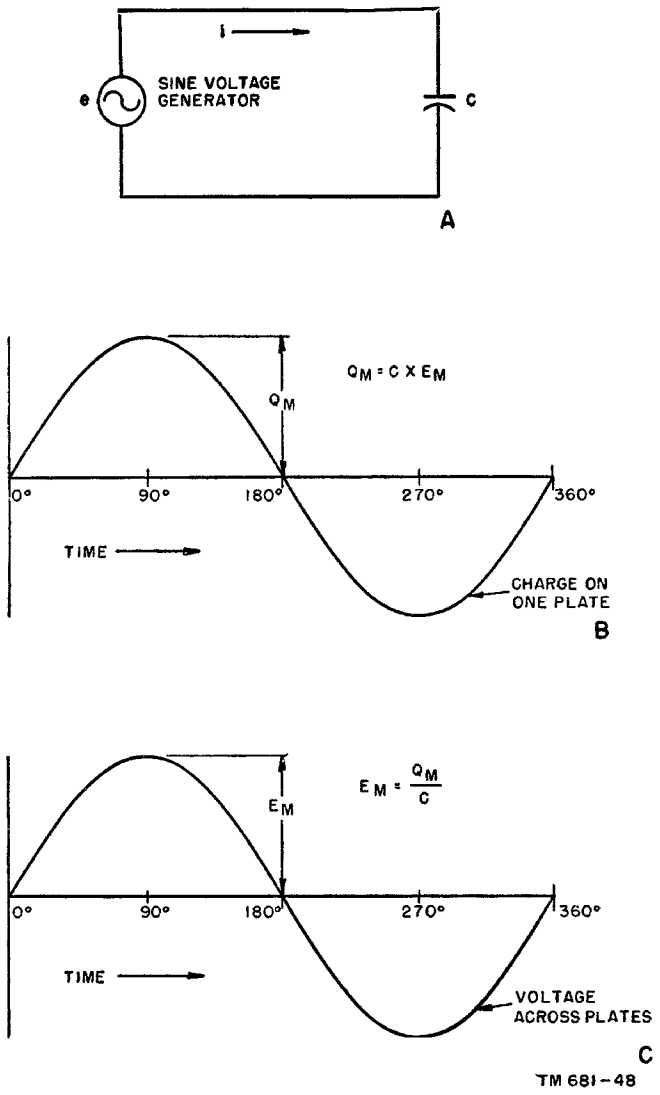


Figure 50. Graphs of charge and voltage in capacitive circuit.

b. By definition, current in any circuit is the time rate of change of charge. Thus:

$$I = \frac{Q}{t} .$$

And current at any instant may be expressed:

$$i = \frac{dq}{dt}$$

where: dq = the change in charge in coulombs
dt = the change in time (in seconds) required for that change
i = the average current during that time.

This expression reveals that dq/dt, the rate of change of charge, is positive when the current is positive. Thus, when dq/dt is positive, or greater than zero, i is positive; when dq/dt is negative, or less than zero, i is negative; and when dq/dt is equal to zero, i is equal to zero.

c. In figure 51, A illustrates the sine curves of current and variations in charge for the circuit of figure 50. At a time corresponding to point *a*, the current i is positive, dq/dt is positive, and therefore, the charge is increasing with time. At point *b*, the current is negative, dq/dt is negative, or less than zero, and the charge is decreasing with time. For some value of time, then, between points *a* and *b*, dq/dt is equal to zero; that is, in going from a positive to a negative value, dq/dt must go through zero. Thus, at point A (180° on the time axis), the curve for charge must flatten out (neither increasing nor decreasing) as it passes through its maximum value. In like manner, the rest of the curve for charge may be drawn. It may be seen, then, that the charge is zero when the current in the circuit is maximum, and maximum when the current is zero. Therefore, the charge on the plate of a capacitor is said to *lag* the current through it by 90°. Since, however, the building-up and falling-off of charge is the building-up and falling-off of voltage, the voltage across the capacitor is said to *lag* the current through it by 90°, or the current is said to *lead* the voltage by 90° (B of fig. 51).

d. A further investigation of this phase shift across a capacitance will show the relationships among the applied voltages, the voltage drop or counter emf across the capacitor, and the current. From Kirchhoff's Law it is known that the algebraic sum of the voltages around any closed circuit is zero. Therefore, the voltage across the capacitor is, by definition, a voltage opposite to the applied voltage, or 180° out of phase with the applied voltage. A of figure 52 illustrates this condition graphically. Thus, when the applied voltage is zero there is no opposition, and when the applied voltage is maximum, there is a maximum opposition, a voltage produced by the charge on the capacitor.

e. The voltage across the capacitor produces the current through it, and an examination of the sine wave of voltage across the capacitor will show its phase relationship to the current produced. In B, it will be seen that the rate of change of voltage is greatest (either negative or positive) as the voltage passes through zero (for instance, at 180°); that is, the slope of the curve at this point is steepest. This means that if a small time interval were isolated at this

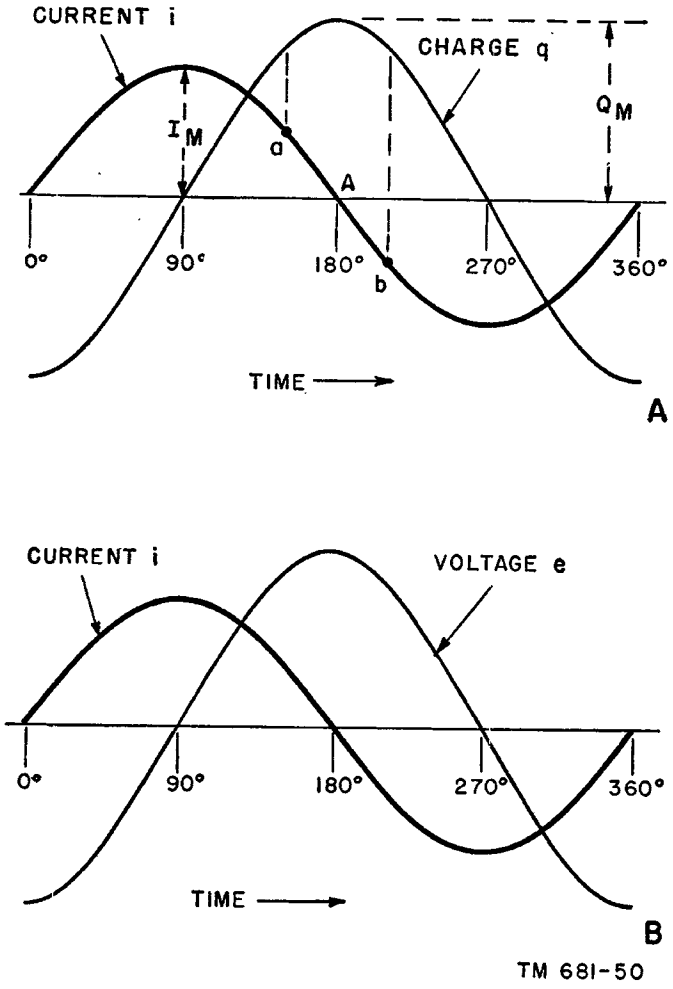


Figure 51. Current, charge, and voltage in a capacitive circuit.

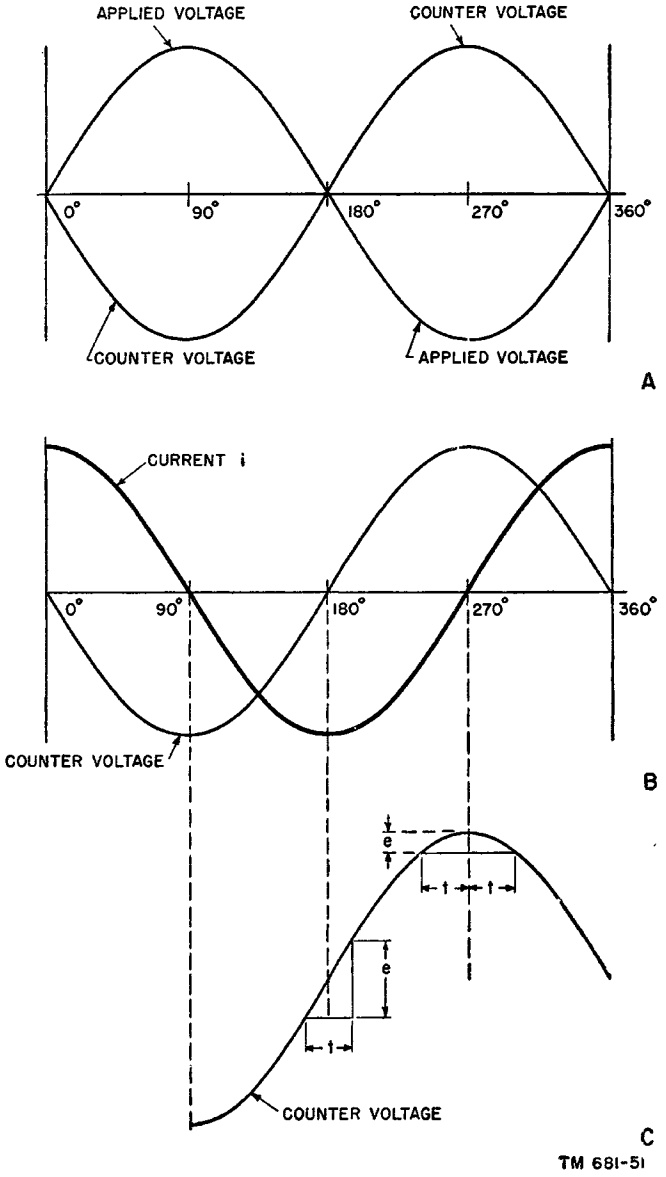


Figure 52. Phase shift in a capacitive circuit.

point, as in C, it would show a relatively large amount of voltage change. Therefore, the current is greatest at the point at which the voltage passes through zero. In like manner, a further examination of the curve of counter voltage will show that the rate of change is least at the point at which the current reaches zero value; that is, the slope of the curve is, at that point, least steep. A small time interval isolated at that point would show a relatively small amount of change in voltage, or no change in voltage, as in C. Therefore, the current would be least, or zero. The same conditions exist for the other half-cycle.

f. The sine wave of current then is a maximum when the voltage across the capacitor is zero, and zero when the voltage is maximum. Moreover, as the counter voltage passes through zero moving in a positive direction, the slope of its curve is positive, and therefore the current is a negative maximum, since by Lenz' Law the current is in such a direction as to oppose the action producing it (the change in voltage). From the point of view of phase, then, the counter voltage is said to *lead* the current by 90°. However, since the voltage across the capacitor is, by definition, 180° out of phase with the applied voltage, the applied voltage goes through zero, moving in either a positive or a negative direction 90° *after* the current. The current in a capacitor then *leads* the applied voltage by 90°. Figure 53 illustrates these relationships in graph form.

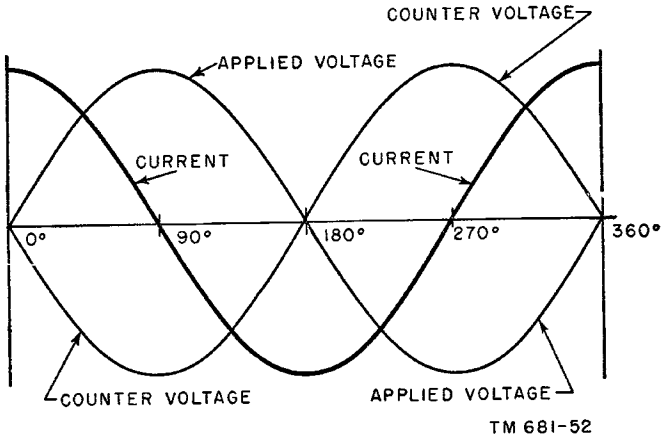


Figure 53. Voltage, current, and cemf in a capacitive circuit.

40. Current and Voltage in an R-C Circuit: Phase Angle

a. In any circuit containing capacitance and resistance, there is a 90° phase shift of current and voltage across the capacitance, and no phase shift across the resistance. As noted in detail in the discussion of inductance, current in a series circuit is everywhere the same and is therefore taken as the line of reference for both the capacitance and the resistance. Since the voltage across the resistance is in phase with the current through it, and the voltage across the capacitance is 90° out of phase with this same current, it will be seen that these two voltages are 90° out of phase with each other. In figure 54, A shows a series *R-C* circuit and B the relationship between the current and the voltages across the elements. The voltage which results from the two voltage drops that are 90° out of phase is the voltage drop in the whole circuit and is, by Kirchhoff's Law, equal to the applied voltage. The phase shift of the current in the circuit, measured in respect to the applied voltage, is called the phase angle of the circuit and assigned, as previously, the symbol θ.

b. The relationship between the applied voltage and the voltage drops, and the phase angle, of any series *R-C* circuit may be determined by means of vectors. In A of figure 55, the voltage across the resistance is laid off on the horizontal vector, and the voltage across the capacitance on the vertical vector. Since these two voltages are 90° out of phase, the angle between them is a right angle. By drawing in a parallelogram based on the two sides, the resultant vector E is seen to be the hypotenuse of a right triangle. Then, by the theorem of Pythagoras, the square of the hypotenuse is equal to the sum of the squares of the other two sides, or:

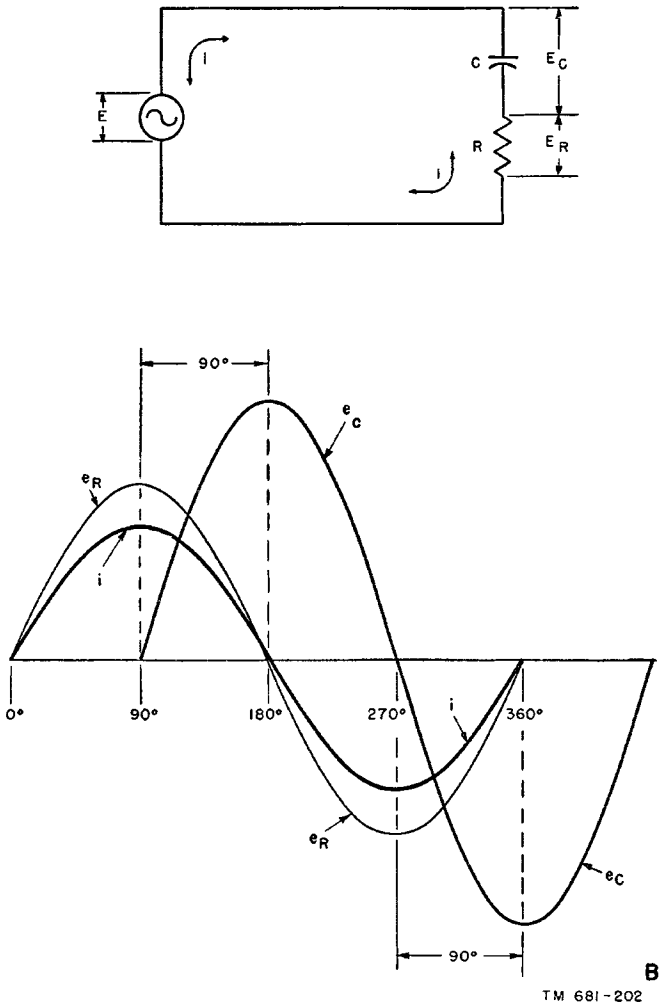


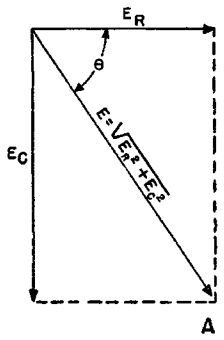
Figure 54. Current and voltages in a series R-C circuit.

$$E^2 = E_R^2 + E_C^2$$

$$E = \sqrt{E_R^2 + E_C^2} .$$

c. Since it is known that the current in the circuit is in phase with the voltage across the resistance, the direction of the current vector is the same as the vector E_R, the voltage across the resistance. The phase angle θ then is the angle that the applied voltage E makes with the vector E_R, as shown in A. If the voltage across the resistance is large in respect to that across the capacitance, the resultant vector will approach the horizontal and the phase angle will be small; and in like manner, if the voltage across the resistance is small, the resultant vector will approach the vertical, and the phase

angle will approach 90°. Hence, the presence of resistance in a capacitive circuit causes the current to lead the applied voltage by some angle *less* than 90°. B of figure 55 illustrates in graph form the relative positions of current and voltage and the phase angle θ.



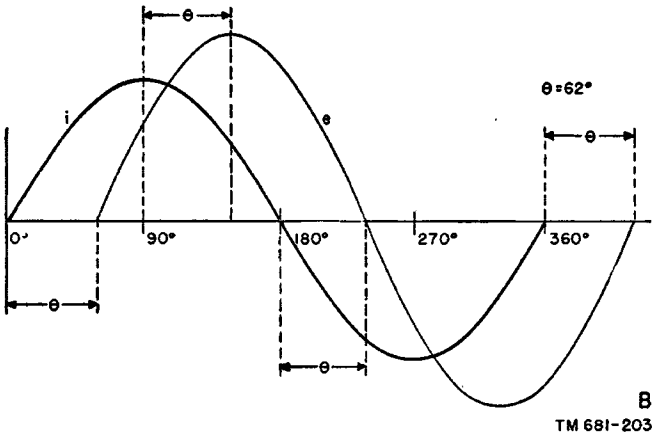


Figure 55. Current and applied voltage in an R-C circuit.

41. Capacitive Reactance and Resistance: Impedance

a. As noted previously in the calculation of the impedance of an *L-R* circuit, the total opposition offered by a circuit containing both a reactive element and a resistance is not the simple arithmetical sum of the reactance X and the resistance R. The capacitive reactance is added to the resistance in such a manner as to take into account the 90° phase difference between the two voltages in the circuit. Since the impedance Z, or total opposition of an *R-C* circuit, is equal to the applied voltage divided by the current in the circuit, then:

$$E = IZ.$$

From above: $E = \sqrt{E_R^2 + E_C^2}$.

Or: $E = \sqrt{(IR)^2 + (IX_C)^2}$.

Then: $IZ = \sqrt{I^2 (R^2 + X_C^2)}$

$$IZ = I\sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{R^2 + X_C^2}\ .$$

Thus, *the impedance of an R-C circuit is equal to the square root of the sum of the squares of the resistance and the capacitive reactance.*

b. The same result may be obtained more readily by means of vectors. The voltage across the resistance E_R is equal to IR, and the voltage across the capacitor E_C to IX_C. Since each vector represents a product of which current is a common factor, the vectors may be laid off proportional to R and X_C, which are separated by 90°. Figure 56 shows these vectors. The resultant vector Z is the hypotenuse of a right triangle and represents the impedance of the circuit. Then:

$$Z = \sqrt{R^2 + X_C^2}\ .$$

It will be seen also that the angle θ is the phase angle because the direction of the impedance vector is the same as that of the applied voltage vector. This angle is generally determined by its tangent, X_C/R, or in terms of its cosine, R/Z.

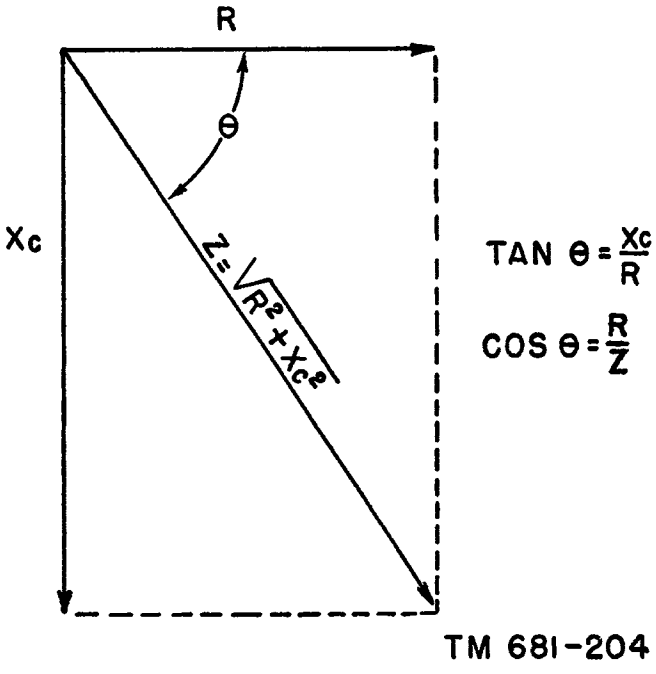


Figure 56. Impedance of an R-C circuit.

c. From the vector diagram of figure 56, it may be seen that if the resistance is large in respect to the capacitive reactance, the circuit tends to act as a pure resistive circuit, the phase angle approaches 0°, and the impedance approaches R. If the capacitive reactance is large in respect to the resistance, the circuit tends to act as a pure capacitive circuit, the phase angle approaches 90°, and the impedance approaches X_C. For practical purposes, then, the impedance of the circuit may be taken to be substantially equal to the larger value, if the ratio of the reactance to the resistance, or of the resistance to the reactance, is 10 to 1 or greater.

d. From Ohm's Law for a-c circuits and from the formulas above, a series R-C circuit may be solved for its important values. For example, a 110-volt 60-cycle a-c line is connected across a 1-μf capacitor and a 1,000-ohm resistor in series. The capacitive reactance alone is:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 60 \times 1 \times 10^{-6}}$$

$$X_C = \frac{10^6}{6.28 \times 60} = \frac{10^6}{376}$$

$$X_C = 2{,}660 \text{ ohms.}$$

And as given: $R = 1{,}000$ ohms.

Then the impedance of the circuit is:

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{(1000)^2 + (2660)^2}$$

$$Z = \sqrt{1{,}000{,}000 + 7{,}120{,}000}$$

$$Z = \sqrt{8{,}120{,}000}$$

$$Z = 2840 \text{ ohms.}$$

The current in this circuit is:

$$I = \frac{E}{Z} = \frac{110}{2840}$$

$$I = .0386 \text{ ampere.}$$

The phase angle θ of the lag of the applied voltage behind the current is—

$$\text{Cos } \theta = \frac{R}{Z} = \frac{1000}{2840} = .352$$

$$\theta = 69.4°.$$

e. The following conclusions may now be drawn in respect to the characteristics of a series R-C circuit:

(1) The current through the capacitor *leads* the voltage across it by 90°.

(2) The voltage across the capacitor and the voltage across the resistance are 90° out of phase (E_C lagging E_R).

(3) The applied voltage is equal to the square root of the sum of the squares of E_R and E_C.

(4) The current in the circuit *leads* the applied voltage by some angle θ, called the phase angle:

$$\text{Tan } \theta = \frac{E_C}{E_R}$$

$$\text{Cos } \theta = \frac{E_R}{E}.$$

(5) The opposition offered by the capacitance (X_C) is measured in ohms and called *capacitive reactance.* It is determined by the size of the capacitance and the angular velocity of the applied voltage:

$$X_C = \frac{1}{2\pi fC}.$$

(6) The impedance of the circuit is the result of the geometric addition of R and X_C:

$$Z = \sqrt{R^2 + X_C^2}.$$

(7) The phase angle may be determined in terms of R, X_C, and Z:

$$\text{Tan } \theta = \frac{X_C}{R}$$

$$\text{Cos } \theta = \frac{R}{Z}.$$

(8) The total current in the circuit is found by Ohm's Law for a-c circuits:

$$I = \frac{E}{Z}.$$

42. Parallel R-C Circuits

a. A of figure 57, shows capacitance C and resistance R connected in parallel across an a-c source. Since this is a parallel circuit, voltage is everywhere the same, and all voltages are therefore in phase with each other. However, the current through the capacitor leads the applied voltage by 90°, and the current through the resistance is in phase with the applied voltage as in B. Thus, the capacitive current leads the resistive current by 90°, and the resultant current, or line current, is the vector sum of these two currents. In A of figure 58, the current through the resistance I_R is laid off on the horizontal vector, and the current through the capacitance I_C on the vertical vector. The I_C vector is laid off in the positive

direction because this current leads the resistive current, which is taken as the reference vector, since it is in phase with the applied voltage and represents the direction of the applied voltage. The resultant vector I_T represents the total current in the circuit, and the angle this vector makes with the horizontal is the phase angle θ.

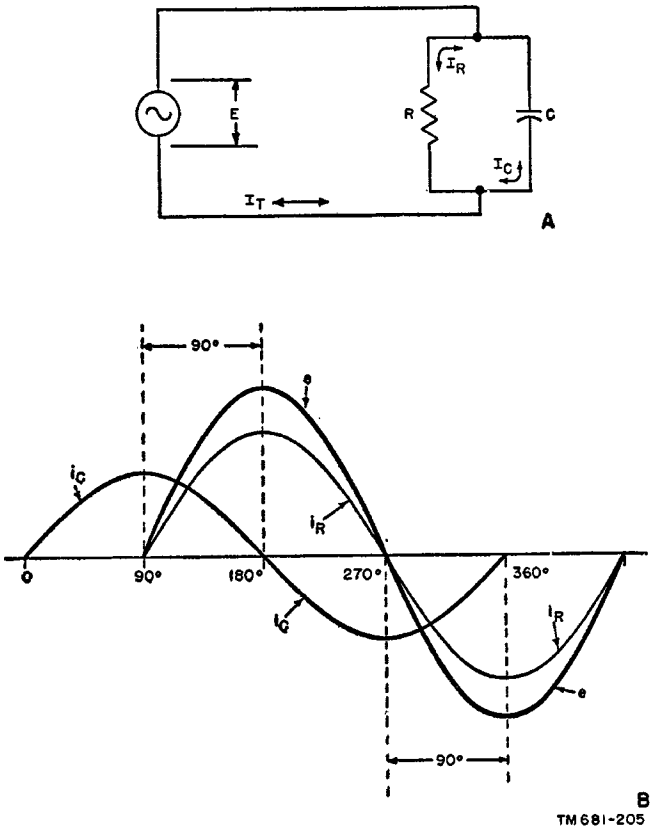


Figure 57. Voltage and currents in a parallel R-C circuit.

b. The line current, then, is said to lead the applied voltage by the angle θ, since by conventional rotation the I_R vector follows the I_T vector (B of fig. 58). The tangent of this angle is I_C/I_R, but as in any parallel circuit:

$$I_C = \frac{E}{X_C}$$

$$I_R = \frac{E}{R} .$$

And:

$$I_T = \frac{E}{Z} .$$

Then by substitution and cancellation:

$$\text{Tan } \theta = \frac{R}{X_C}$$

$$\text{Cos } \theta = \frac{Z}{R} .$$

The magnitude of the line-current vector I_T must always be greater than either I_R or I_L because it is the hypotenuse of a right triangle. Then:

$$I_T = \sqrt{I_R^2 + I_C^2} .$$

Thus, it may be seen that, as in d-c circuits, total current in a parallel *R-C* circuit is always greater than the current in either branch, and, by extension, the impedance of the circuit is less than the opposition of either branch.

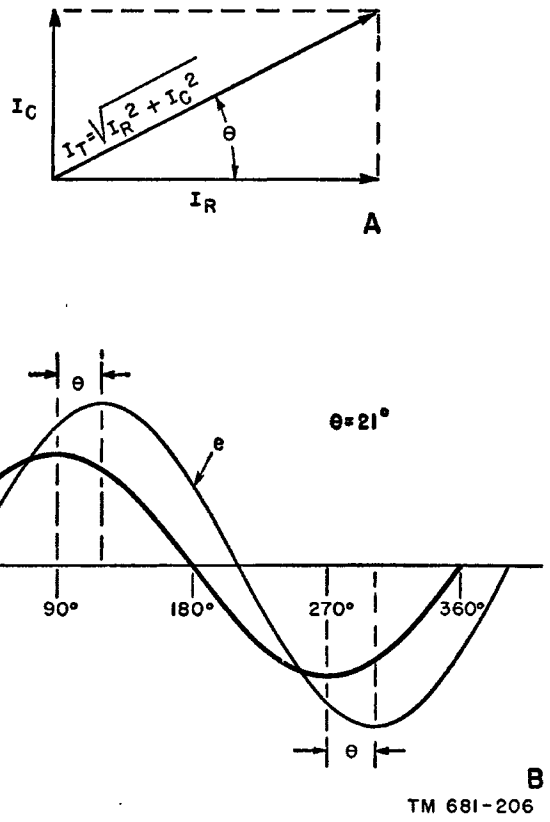


Figure 58. Total current and voltage in a parallel R-C circuit.

c. The impedance of a parallel R-C circuit may also be obtained by the use of a formula similar to that used for resistances in parallel.

Since:

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

then:

$$Z = \frac{R \times X_C}{R + X_C} .$$

But, as has been shown previously, the addition of two vector quantities ($R + X_C$) may not be made directly. Thus:

$$Z = \frac{R\,X_C}{\sqrt{R^2 + X_C^2}} .$$

d. For example, if the values used previously for the series *R-C* circuit are transferred to the parallel *R-C* circuit in A of figure 57, then as before:

$$R = 1{,}000 \text{ ohms}$$
$$X_C = 2{,}660 \text{ ohms.}$$

Then:

$$Z = \frac{R X_C}{\sqrt{R^2 + X_C^2}} = \frac{2{,}660{,}000}{2{,}840}$$
$$Z = 935 \text{ ohms.}$$

The line current is:

$$I_T = \frac{E}{Z} = \frac{110}{935}$$
$$I_T = .118 \text{ ampere.}$$

The line current leads the applied voltage by the angle θ:

$$\text{Cos } \theta = \frac{Z}{R} = \frac{935}{1{,}000} = .935$$
$$\theta = 21°.$$

e. The following comparisons may now be made between series and parallel *R-C* circuits:

(1) The phase angles are complementary if the values of frequency, capacitance, and resistance are the same:

$$\theta_S + \theta_p = 90°.$$

(2) The greater the resistance added to a series circuit, the more resistive the circuit becomes: θ_S approaches 0°.

(3) The greater the resistance added across a parallel circuit, the more capacitive the circuit becomes: θ_p approaches 90°.

43. Time Constant of R-C Circuit

a. The property of capacitance, as has been shown, introduces a delay in time between the circuit current and the appearance of voltage in the circuit. In a-c sine-wave circuits, the voltage is a sine wave also, following the current by the phase angle θ. In such a circuit, the phase angle depends on the ratio X_C/R, that is, on $\frac{1}{2\pi f}$CR, the frequency, capacitance, and resistance of the circuit. In d-c circuits (or circuits using pulses of dc), as was shown similarly in connection with the analysis of the time constant of inductive circuits, the time lag resulting from the starting and stopping of the current depends not on frequency, since frequency is zero, but on the capacitance and resistance of the circuit.

b. Thus, when a dc is applied to an *L-R* circuit, the current is held back by the counter emf and does not rise immediately to its *E/R* value. However, when a dc is applied to an *R-C* circuit, the current reaches its maximum *E/R* value almost immediately and then gradually falls to zero as the counter emf offers greater and greater opposition. A of figure 59 shows the graph of current rise and fall in a d-c circuit containing capacitance and resistance. It will be noticed that after the almost instantaneous rise of current to its maximum, the curve of falling current is an exponential curve and is the characteristic curve of current rise in an *L-R* circuit turned inside out. Thus, the current falls rapidly at first and then gradually tapers off to its zero value. An analysis of this exponential curve reveals that in regular units of time, the current falls in decreasing amounts. Thus, in the first unit of time, current falls 63.2 percent from its maximum value, or to 36.8 percent of its maximum value; and in the second unit current falls 63.2 percent of the remaining 36.8 percent (86.4 percent from maximum); and in like manner for each succeeding unit of time. Theoretically, in such a progression, current would never reach zero value, but for practical purposes current is considered to be zero and the capacitor fully charged after five units of time.

c. Conversely, if the d-c source is removed and the capacitor is allowed to discharge, current flows again almost instantaneously at a maximum in the opposite direction and then gradually returns to zero. B of figure 59 is the graph of the discharge current. This curve is the same as that described for the charging current but turned upside down. In the first unit of time, current returns rapidly 63.2 percent from its maximum; in the next unit of time, it returns an additional 63.2 percent of the remainder. This process continues as described above, until, after five units of time, current is considered to be zero and the capacitor discharged.

d. This recurring 63.2 percent fall from maximum rise of current in a fixed unit of time is called the *time constant* of an *R-C* circuit. The more capacitance in a circuit, the longer time required to reach 63.2 percent of full charge; also, the more resistance in an R-C circuit, the longer the time required to reach

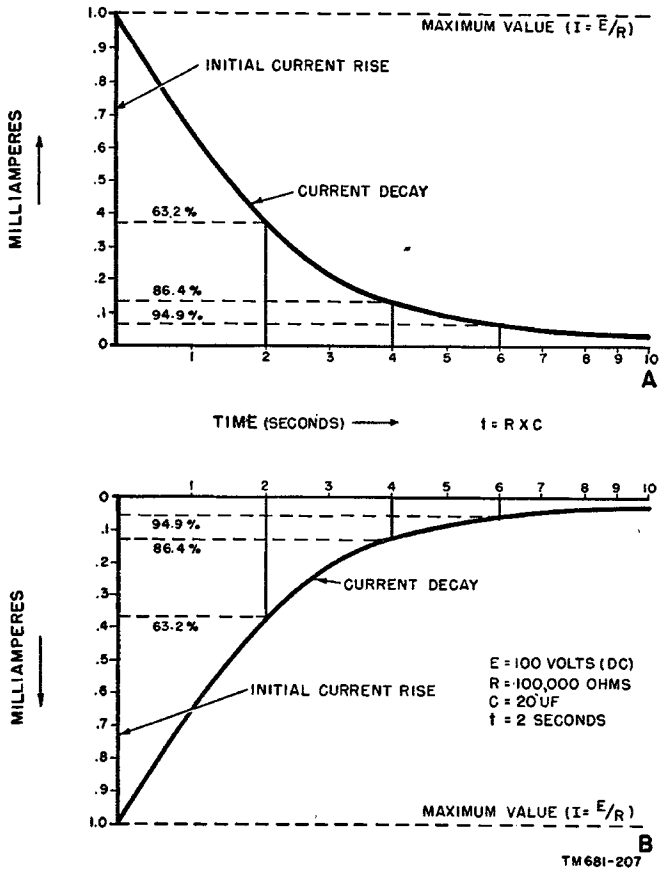


Figure 59. Time constant in an R-C circuit.

the full amount of charge and discharge, since the *R-C* circuit tends to act as a resistive circuit and hold current to its E/R maximum for a longer period. Therefore:

$$t \text{ (time constant)} = R \times C \ .$$

If C is in microfarads and R in megohms, then t is in seconds and is the time required for current to fall 63.2 percent from its E/R maximum.

e. For example, if 100 volts dc is applied to an *R-C* circuit containing a 20-microfarad capacitor and a 100,000-ohm resistance, the E/R value of the current is 1 milliampere. The time constant is:

$$t = RC = .1 \text{ megohm} \times 20 \ \mu\text{f}$$
$$t = 2 \text{ seconds.}$$

It will therefore require 2 seconds for current to fall to .368 milliampere, and an additional 8 seconds before it reaches approximately zero. If the resistance is decreased to 50,000 ohms, the time constant is decreased to 1 second. However, it must be remembered that since resistance is decreased, the maximum current in the circuit increases to 2 milliamperes. At the end of 1 second, then, the current will have reached a value of 36.8 percent of 2 milliamperes, or .736 milliampere; and 4 seconds more will elapse before the current reaches zero. If the amount of current in the circuit is an important factor, it is necessary to vary the time constant by varying the amount of capacitance. Thus, in the example above, if the resistance is kept at 100,000 ohms, maximum current remains at 1 milliampere, and the time constant of 2 seconds may be lowered to 1 second by using 10 microfarads of capacitance. Then:

$$t = RC = .1 \times 10 = 1.$$

In discharging this capacitor the current drops to .368 milliampere at the end of 1 second, and the capacitor becomes fully discharged 4 seconds later.

f. In the examples above, the circuit values and the time constant were made deliberately large for purposes of explanation, but it should be understood that in high-frequency transmitting and receiving circuits, the time constant is measured in milliseconds or even in microseconds. Small capacitors and low resistance are generally used in such cases. Even though these circuits are primarily a-c circuits, the time constant determines the level of the dc in those circuits containing both ac and dc. In addition, complex waveforms based on the charging and discharging of a capacitor are often used. A of figure 60 shows such a pulse waveform obtained by placing the waveform of discharge next to the waveform of charge. B shows the rise and fall of voltage across a capacitor. Since voltage rise and fall across a capacitor is delayed in a manner similar to current rise and fall through an inductance, it will be seen that the sawtooth waveform in B is the same as the sawtooth waveform of current through an inductance. In like manner, since voltage across an inductance is immediately present and then falls as current reaches a maximum, the voltage waveform across an inductance is the same as the capacitive current waveform in A.

44. Capacitors in Series and Parallel

a. The total capacitance of a circuit containing more than one capacitor connected in *series* is calculated in the same manner as that for resistors in *parallel.* The total capacitance is

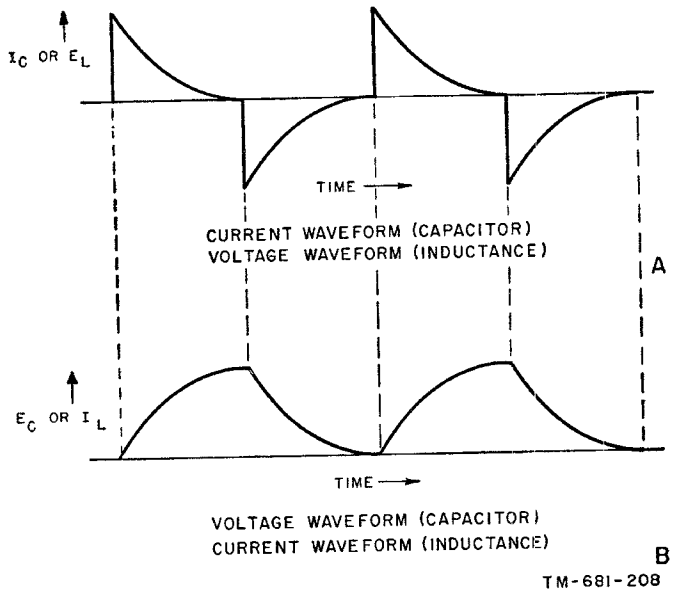


Figure 60. Complex waveforms based on a time constant.

the reciprocal of the sum of the reciprocal of the individual capacitors. Thus:

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots + \frac{1}{C_n}}.$$

An examination of this formula reveals that other generalizations true of resistors in *parallel* may be applied to capacitors in *series*; the total capacitance is less than the capacity of the smallest element; if capacitors are of equal value, the total capacitance may be obtained immediately by dividing the capacity of one by the number of capacitors; for any two capacitors, the simplified formula below may be used:

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2}.$$

That two capacitors in series give a total capacity less than one capacitor is evident from the schematized diagram of A of figure 61. It will be noted that the addition of another capacitor in effect widens the distance between the plates while the area of the plates remains the same. Thus, total capacity is less, since:

$$C = \frac{A}{d}.$$

b. On the other hand, the total capacitance of a circuit containing capacitors connected in *parallel* is calculated by the simple addition of the individual capacities, as with resistance in *series*.

Thus: $C_T = C_1 + C_2 + C_3 \ldots + C_n$.

B of figure 61 shows two capacitors connected in parallel. It will be noted that the addition of the second capacitor in effect increases the area of the plates without changing the distance between them. Thus, by the formula above, it is evident that total capacity increases and is always greater than the larger capacity.

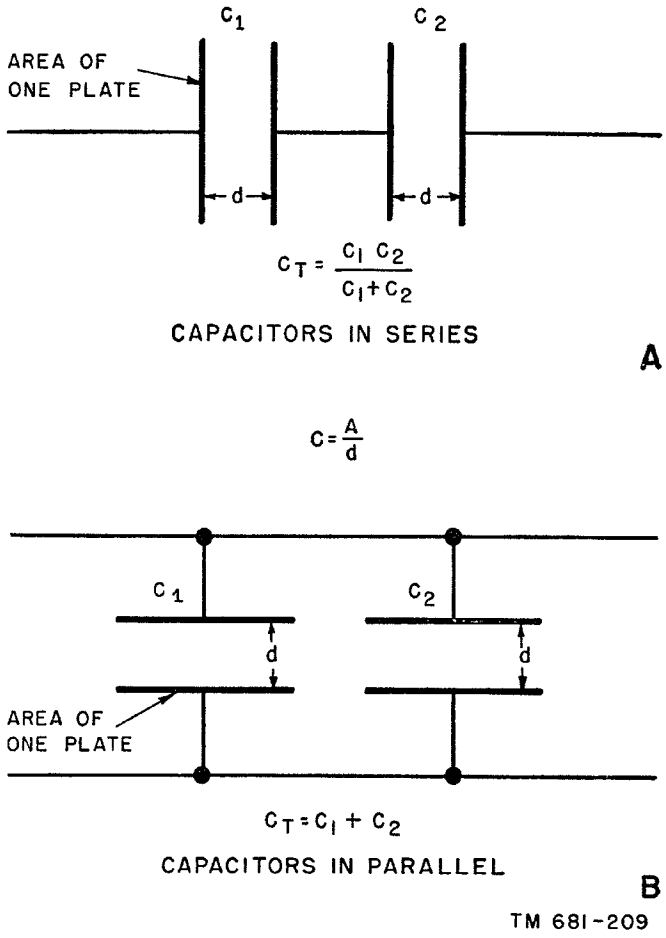


Figure 61. Capacitors in series and parallel.

45. Distributed Capacitance

a. Stray or random capacitance caused by the intricate circuit wiring and the physical construction of components is present everywhere throughout a-c circuits. Such stray capacitance is called the *distributed capacitance* of the circuit as distinguished from the *lumped capacitance* of the capacitor elements as such. Since a capacitor is, by definition, any two conductors separated by a dielectric, small amounts of capacitance exist between connecting wires, between components located physically near each other, and between different parts of a given component. Generally, such distributed capacitance is undesired, and at low frequencies is such

a small amount compared to the lumped capacitance of the circuit that it may be ignored. At high frequencies, however, the distributed capacitance may well equal or exceed the estimated lumped capacitance of the circuit, and therefore must be considered in circuit design. In some instances, the distributed capacitance is used as the capacitance of the circuit, but for the most part, every effort is made to cut down its total by careful wiring, positioning of components, and special physical construction of certain components.

b. Since the voltage drop across an inductance is distributed proportionately along its length and therefore exists between turns, turns adjacent to each other appear as plates of a capacitor. A of figure 62 illustrates this effect across an inductance, the dotted line capacitors representing the effective distributed capacitance. It will be noticed that the capacitors as shown are effectively in *series* from one end of the inductance to the other, resulting in a total capacitance less than the smallest. In large, tightly wound inductances, however, the turns fall one upon the other in layers in such a manner that the small distributed capacitances are effectively in parallel and the total amount is considerable. B of figure 62 shows a cross section of such an inductance and the accumulated distributed capacitance.

c. Inductances using a large number of turns are often specially constructed to avoid the effect of accumulated distributed capacitance. A of figure 63 shows a pie-type inductance and its distributed capacitance. This design is based on the fact that although the distributed capacitance for any one layer is cumulative, the capacitances between layers are effectively in series, and thus the total capacitance is less than that of one layer. B shows a refinement of this constructional technique. Succeeding sections are graduated by size, which cuts down the accumulated capacitance of the smaller sections and effectively inserts a smaller final distributed capacitance in the series circuit. Then, since the total capacitance is less than the smallest capacitor, the total distributed capacitance is less than that of the smallest section.

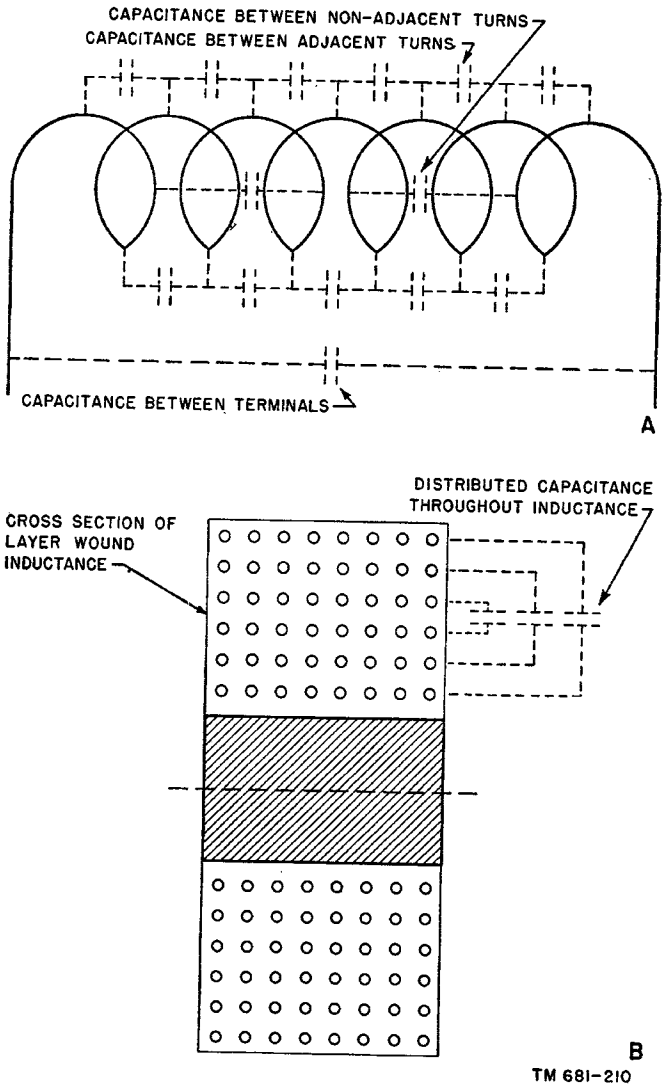


Figure 62. Distributed capacitance.

46. Types of Capacitors

a. GENERAL. The basic parallel-plate capacitor described earlier in this chapter is modified in a variety of ways to meet the numerous requirements of circuits for power supplies and communication equipment. The two principal classes evolved are the fixed capacitors and the variable capacitors; the former are used if a definite lumped capacitance is needed and the latter if different amounts of capacitance must be readily available at different times. Fixed capacitors are subdivided generally according to the nature of the dielectric, variable capacitors according to their functions. Thus, fixed types include paper, oil, mica, ceramic, and electrolytic capacitors; variable types include tuning, trimmer, padder, and neutralizing capacitors. The color coding of paper, mica, and ceramic capacitors is described in detail in appendix III.

b. PAPER CAPACITORS.

(1) The fixed waxed paper capacitor is a common component in most communi-

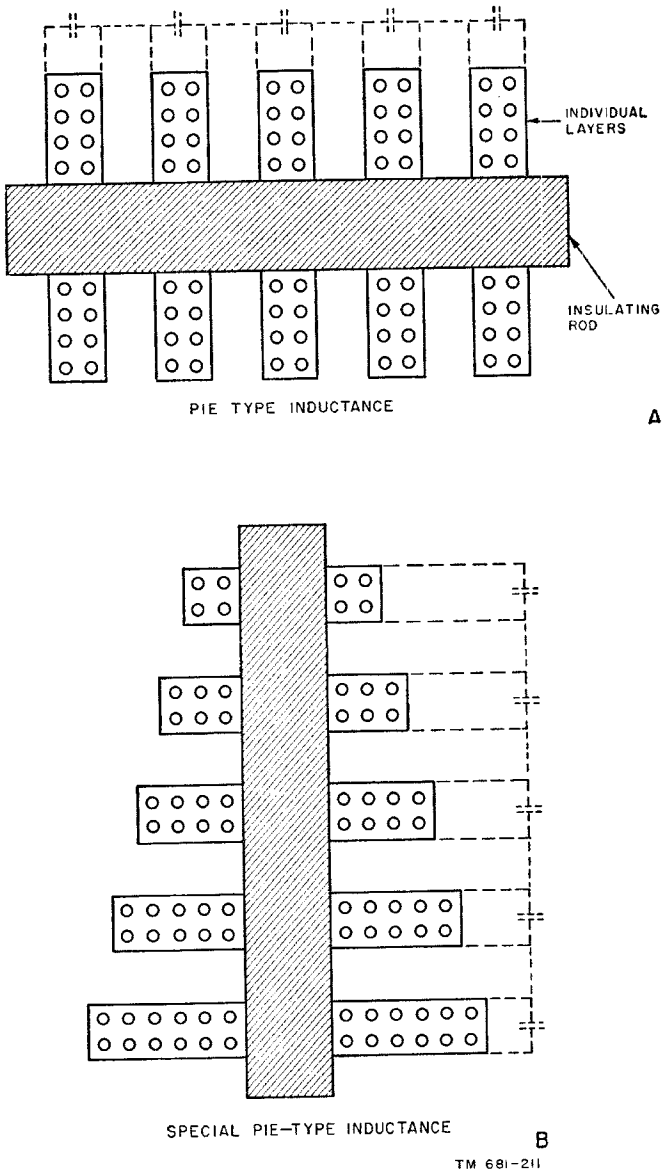


Figure 63. Distributed capacitance of pie-type inductances.

cation equipment and finds its widest uses for capacities between .0005 μf and 8 μf. As noted previously, wax paper has a dielectric constant of 3.5 and a relatively high breakdown voltage of 1,200 to 1,800 volts per .001 inch thickness. In this type of capacitor, the plates are long flat strips of tin foil separated by thin sheets of waxed paper and all rolled together in a compact cylinder or flat package. More than one sheet of paper is used as the dielectric so that tiny holes or flaws in the paper will not invite breakdown. Such capacitors are commonly used in circuits not exceeding 600 volts. It should be noted also that because the plates of the capacitor are rolled, one plate will have two active surfaces, and therefore the active area of this type of capacitor is twice the area of one plate. Figure 64 shows the details of the internal construction of the paper capacitor and figure 65 is the photograph of a cutaway section which shows this construction in a typical capacitor. The rolled strips of tin foil project slightly on either side so that the connecting bond may be soldered to each turn and not to the end of the strip only. Such an arrangement prevents the tin foil conductors from becoming inductances (by shunting the turns), since physically the capacitor resembles two coils wound one within the other. A capacitor so constructed is called a *noninductive* capacitor. The cylinder or flat package is then put into a cardboard or metal container and sealed with wax or pitch to keep out moisture.

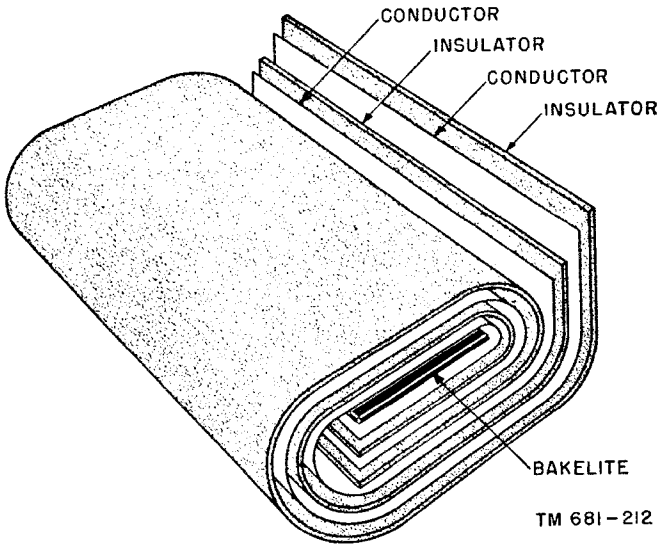


Figure 64. Internal construction of a paper capacitor.

(2) In general, only electrolytic capacitors show polarity; all other capacitors may be connected across ac or across pulsating dc without regard to polarity. However, it will be observed that those paper capacitors in cardboard containers and designed for use in relatively high-frequency a-c circuits

carry a notation such as *negative* or *outside foil* for a particular lead, or a single black band near one end. This notation is to insure that the outside foil is connected to ground, since it then becomes a shield for the entire capacitor; otherwise, the inside foil is grounded, and the whole area of the outside foil carries the a-c voltage and may affect circuits in its immediate vicinity. This notation is, of course, unnecessary for a paper capacitor in a metal container, which itself acts as an effective shield. Figure 66 is a photograph of an assortment of paper capacitors, showing types of containers, leads, and markings.

c. OIL CAPACITORS. Capacitors designed for high power at medium high voltages (above 600 volts), as in radio transmitters, are often oil capacitors. From the table previously shown of the dielectric strength of various materials, it will be noted that the breakdown voltage of oil is considerably less than that of paper. However, *oil-impregnated paper* shows a high dielectric strength and is used for medium high voltages (between 600 and 2,000 volts). When such capacitors carry heavy current they tend to operate at a lower temperature. Physically, oil capacitors resemble paper capacitors and must be marked for identification. Some of the larger units of figure 66 are oil capacitors. It is also of interest that relatively expensive paper capacitors designed for medium high voltage use are *immersed* in oil to prevent the infiltration of moisture which causes corrosion and breakdown. These capacitors are commonly called *oil capacitors*; however, oil is the sealing agent, not the dielectric, and effectively preserves the rated capacity and resistance to breakdown against salt water and tropical climates. Large transmitting capacitors using glass as a dielectric are sometimes immersed in oil to prevent arcing over (in air) between plates and between terminals.

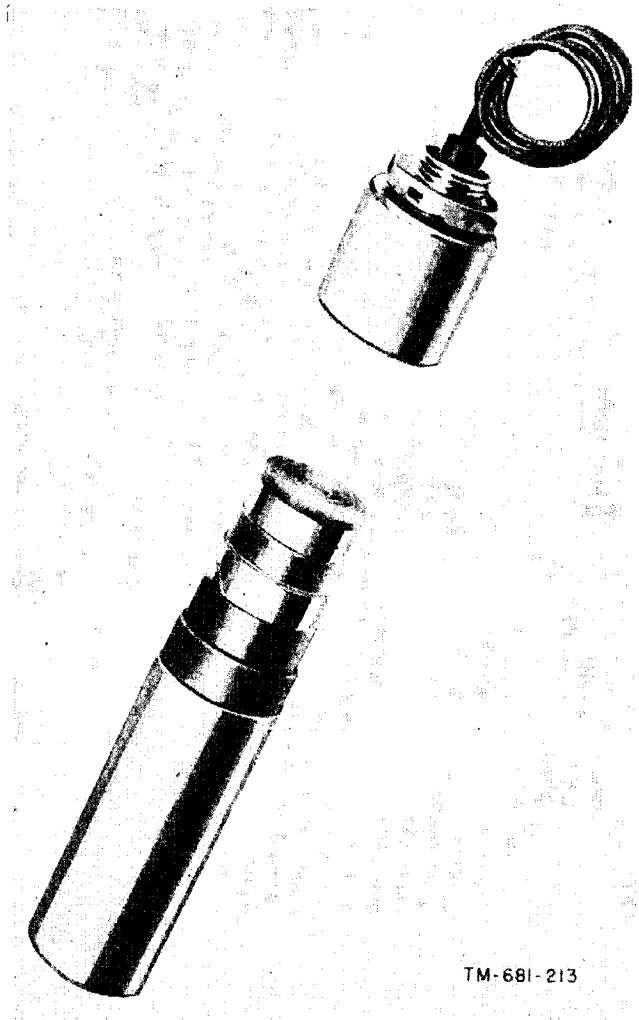


Figure 65. Cutaway section of a paper capacitor.

d. MICA CAPACITORS. Mica capacitors are used generally for capacities between 5 micromicrofarads and 50,000 micromicrofarads (.000005 μf to .05 μf). These relatively low values of capacitors are used for higher-frequency circuits. Since the dielectric constant of mica is more than five or six times that of air and its breakdown voltage much higher (about 2,000 volts per .001 inch), this relatively expensive material is used for high-frequency, high-voltage capacitors (up to 7,500 volts) which are physically small in size in comparison to the same capacity and breakdown voltage of a paper capacitor. Figure 67 is a photograph of a cutaway section which shows the internal construction of a typical mica capacitor. Alternate layers of tin foil and mica are molded together in a bakelite or plastic case, achieving compact design and durability. Figure 68 is a photograph of an assortment of mica capacitors, showing types and positioning of leads.

e. CERAMIC CAPACITORS. Increasing use of the higher frequencies, particularly in television and UHF equipment, has occasioned the development of the ceramic capacitor. These units are generally used for very small capacitances, those between .5 of a micromicrofarad and 250 micromicrofarads (.0000005 μf and .00025 μf).

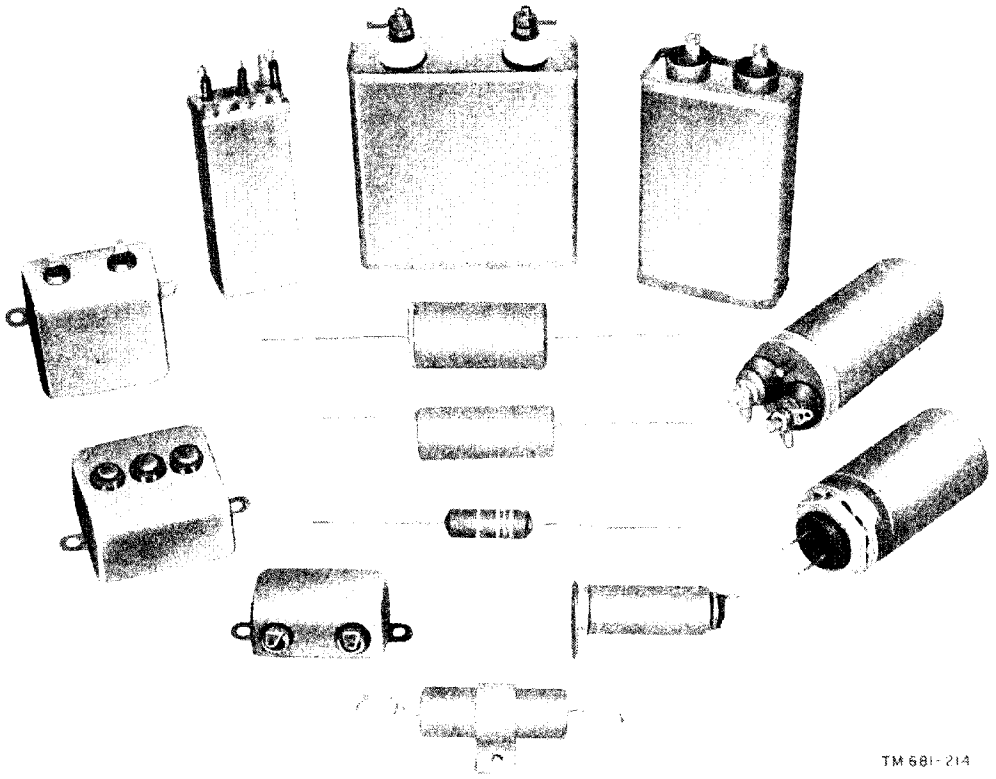


Figure 66. Assortment of paper and oil capacitors.

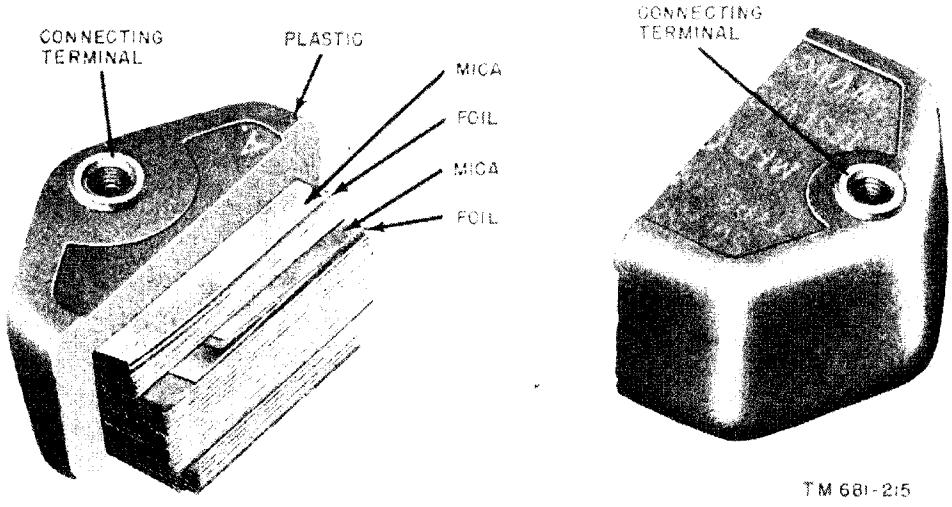


Figure 67. Internal construction of a mica capacitor.

Usually, a small hollow ceramic cylinder is used both as the form of the capacitor and as its dielectric. Leads are attached to either end of the cylinder, which is coated inside and out with silver paint. These silver coatings act as the conductor and are separated from each other by the dielectric ceramic cylinder. These capacitors are quite small physically and ideally adapted for VHF and UHF work. Because of their size, shape, and color coding (app. III), ceramic capacitors are sometimes mistaken for composition resistors, which they resemble. Ceramic capacitors also lend themselves for use in low-power, very-high-voltage circuits (up to 30,000 volts). These are extensively used in television high-voltage power supply units. The cylindrical capacitors of figure 68 are types of ceramics.

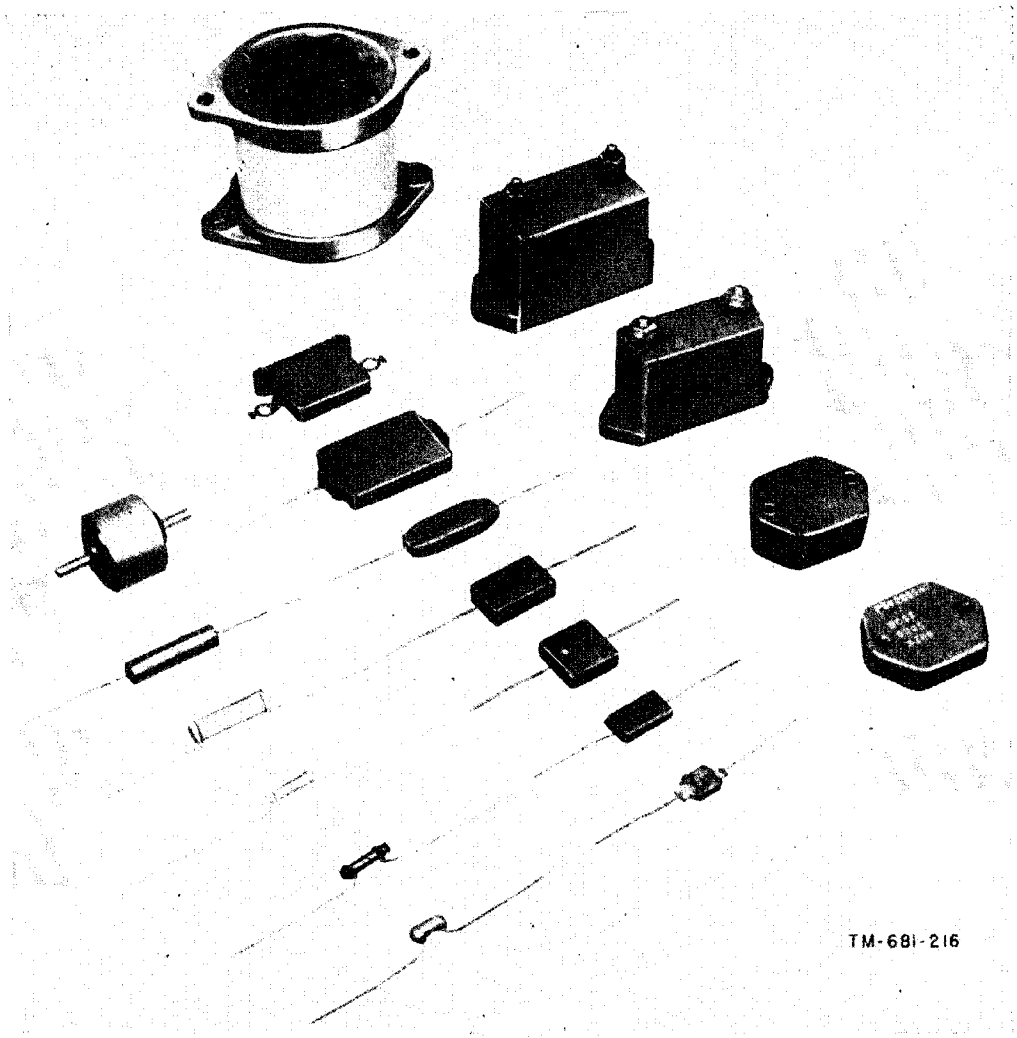


Figure 68. Assortment of mica and ceramic capacitors.

f. ELECTROLYTIC CAPACITORS.

(1) In the study of dry-cell batteries in TM 11–661, it will be remembered that one disadvantage of the dry battery was that the positive terminal became polarized; that is, small bubbles of hydrogen formed on the positive terminal in such a manner as to cut off current flow through the battery by insulating the terminal from the electrolyte. Since this film of gas is essentially an insulating material between two conductors, a totally polarized dry cell may be considered a capacitor of sorts. A similar reaction forms the basis for the study of the so-called electrolytic capacitor. Thus, if two aluminum electrodes are immersed in a suitable electrolyte, such as borax (sodium tetraborate), and connected across a d-c source of power, it is found that a thin film of oxide forms on the positive terminal, and after a few minutes the insulating effect of this oxide film cuts off current flow. Thus, an *electrolytic capacitor* has been formed. In figure 69, a cross section of a basic electrolytic capacitor is shown with the metal container acting as the negative terminal. The positive electrode is one conductor, the film of oxide is the dielectric, and the electrolyte is the other conductor. Hence, the name, electrolytic capacitor, is not strictly applicable, since the electrolyte serves as a conductor, not the dielectric. The second aluminum strip acts simply as a connecting terminal for the electrolyte, which is always the negative terminal. If the dc is re-

versed, current flows until an oxide film has been formed on the second plate. Thus, electrolytic capacitors show polarity and are most effective in pulsating d-c circuits—that is, circuits primarily d-c with an a-c component. Electrolytic capacitors designed for ac use electrodes with previously formed oxide films so that the capacitor presents an insulating dielectric to both polarities.

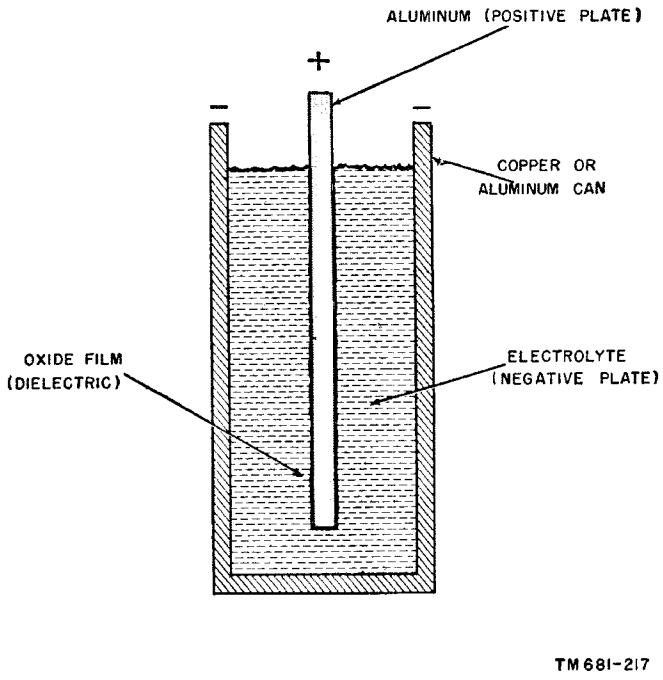


Figure 69. Basic electrolytic capacitor.

(2) The oxide film formed on the positive plate of an electrolytic capacitor is usually less than a hundred-thousandth of an inch (.00001 inch), and may withstand as much as 480 to 500 volts. Since capacity depends upon the distance between conductors as well as on the area of the conductor, it will be seen that an electrolytic capacitor, because of the extraordinary thinness of the dielectric film, is capable of values of capacitance far in excess of values available from other types. In addition, the breakdown voltage of an oxide layer is slightly higher than the usual limits of the low-voltage power supplies used in communications equipment. Thus, electrolytic capacities find wide use in low-voltage (under 600 volts) power-supply filter circuits whose low-frequency pulsating d-c voltages require large amounts of capacitance for effective production of direct current.

(3) In addition, the breakdown voltage of an electrolytic capacitor is determined by the *forming* voltage—that is, the d-c voltage originally applied to the capacitor to form the dielectric oxide film. If this forming voltage is low, the film is very thin and the capacitance very high. For instance, a typical capacitor used in the low-voltage supply of a television receiver is rated at 2,000 microfarads at 20 volts. Higher forming voltages produce thicker dielectric films and less capacitance (for the same plate area). The limit of forming voltage, however, is approximately 500 volts, since no further thickening of the oxide film takes place. The limit of breakdown voltage may be raised to 600 volts, depending on the chemical purity of the electrodes and electrolyte, but these capacitors are more expensive and few will be encountered rated at this voltage.

(4) Since the principal advantage of electrolytic capacitors is their large amount of capacitance and small size, every effort is made to increase the area of the positive plate; the negative plate, as noted above, is the electrolyte. Thus, the positive plate may be a coil of thin aluminum, or it may be accordion-pleated, corrugated, helical, or formed into radial fins to increase surface area. The whole area may then be etched by an electrochemical process, or the plate *fabricated,* that is, sprayed with aluminum. The effective surface area of etched and fabricated plates is considerably increased, since the tiny irregularities provide more contact surface for the same physical size. A perforated celluloid or other insulating material lines the inside of the container to prevent the positive plate

from making contact with the container and short-circuiting the capacitor. A of figure 70 shows the internal construction of such a *wet* electrolytic capacitor and B shows the corrugated positive plate and can container in cross section.

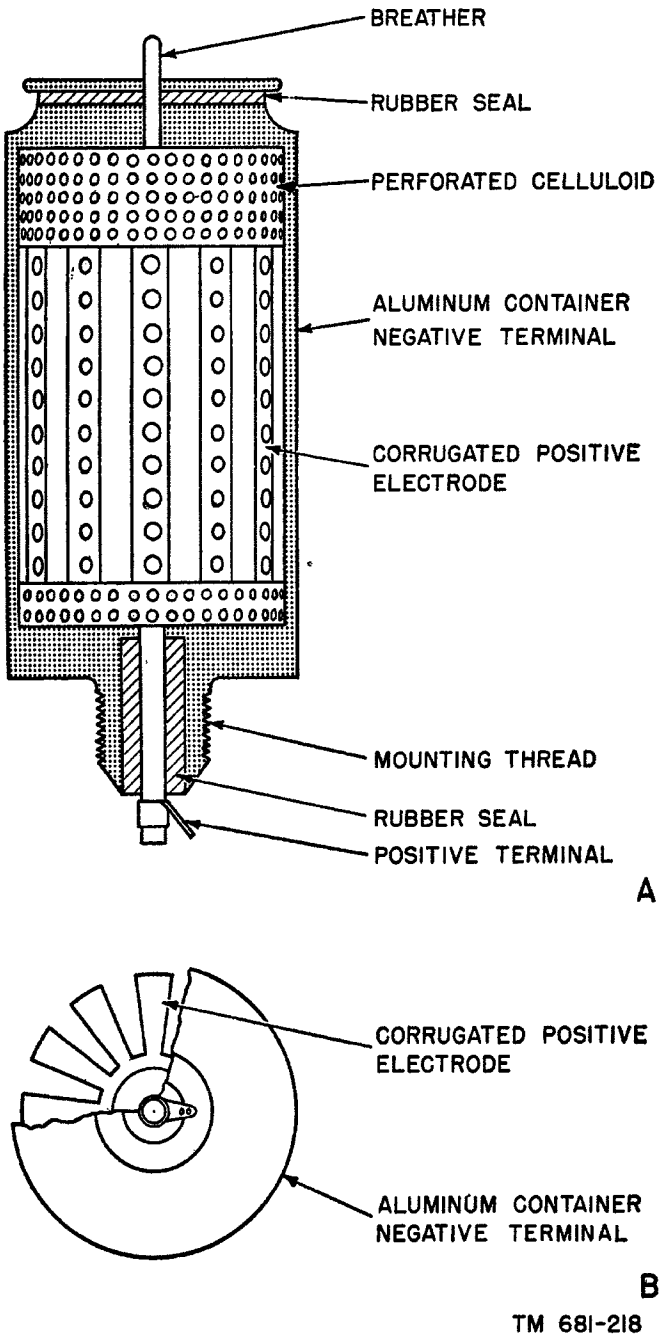


Figure 70. Internal construction of a wet electrolytic capacitor.

(5) The so-called wet electrolytic capacitors described above present certain practical disadvantages. The liquid electrolyte requires a ventilation opening in the top of the can for the escape of gases. Because of this opening, the capacitor may be mounted only in a vertical position, and special care must be taken not to spill the electrolyte. Hence, the so-called *dry electrolytic* capacitor has been developed. In this type of capacitor, the positive plate and the negative connecting terminal are generally flexible sheets of aluminum separated by a plate of jelly-like electrolyte fixed on a separating gauze or paper. The positive plate is *formed* by an electrochemical process which covers it with an extremely thin film of oxide. This dielectric oxide gives a dull gray finish to the positive plate in contrast to the bright foil of the negative terminal. The negative plate, as with wet electrolytics, is the electrolyte. A of figure 71 show this construction. The positive plate of the multisection dry capacitor is cut into sections and separate leads are attached; the electrolyte and negative terminal are common to all sections, as shown in B. In both single-section and multisection units, the sheets are rolled together and encased in a cardboard or metal container in a manner similar to that used for the standard paper capacitor described previously. Since the electrolyte cannot spill or leak out, the capacitor may be mounted in any position and lends itself readily to compact construction. Etched and fabricated positive plates are also used in dry electrolytics and give very high capacities in sizes comparable to the standard .5-μf paper capacitor. Figure 72 is a photograph illustrating the various types and sizes of electrolytic capacitors.

(6) A further disadvantage of electrolytic capacitors, both wet and dry, is their high power loss compared to paper capacitors. Theoretically, a capacitor should show infinite reactance to dc, but since no dielectric is a perfect insulator the d-c resistance is finite and represents a steady loss of power because of the current loss. Hence, a

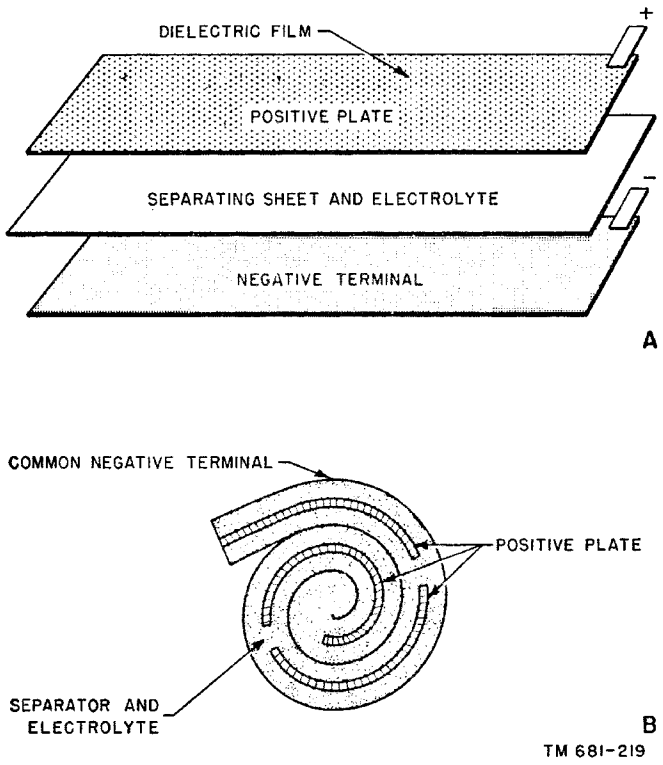


Figure 71. Internal construction of a dry electrolytic capacitor.

good electrolytic capacitor often shows a constant d-c resistance of little more than 1 megohm compared to the minimum 200 megohms of a good paper capacitor of comparable size. Such a resistance means that there is a steady leakage current through the capacitor. Thus, as previously noted, electrolytics are widely used in low-voltage power supplies where such leakage loss is easily sustained.

(7) In filter circuits of low-voltage power supplies the most common type of electrolytic capacitor is the 8-μf or the double-8-μf (marked 8–8) rated at 450 volts. Other typical values are the 12-μf, 16-μf, and 20-μf at 450 volts, the 20-μf and 20–20-μf at 150 volts, and the 50–30-μf at 150 volts, the latter double capacitors both designed for use in the so-called ac/dc receiving set, which operates at line voltage. Numerous very-low-voltage types are also common, such as the 25-μf, or 5θ-μf, at 25 volts, the 200-μf at 25 volts, and 500-μf, 1,000-μf, and 2,000-μf at 10 or 12 volts. Cardboard containers with single, double or triple capacitors generally show color-coded leads. Thus, a typical capacitor may carry the following legend:

20 μf at 450 volts DC	Red
20 μf at 450 volts DC	Yellow
20 μf at 300 volts DC	Blue
50 μf at 25 volts DC	Brown
Common negative	Black

Early types of wet and dry electrolytics in can containers generally used the can as the common negative terminal, which was bolted directly to the

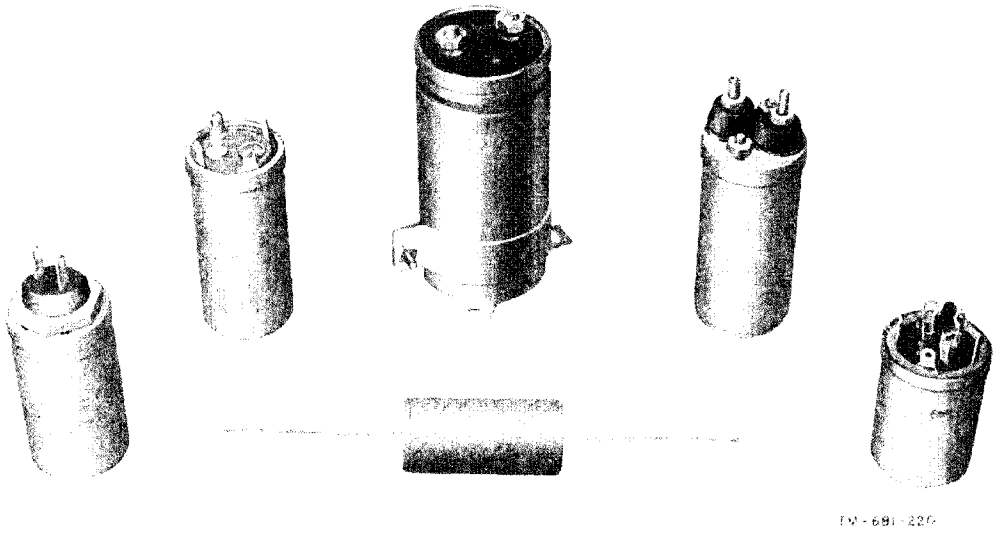


Figure 72. Various types of electrolytic capacitors.

chassis. Since not all equipment uses the metal chassis as ground nor as the negative side for all circuits, such construction proved awkward when a capacitor was used for general replacement. Hence, most dry electrolytics use a separate lead as the common negative, so that the case may be mounted on the chassis regardless of circuit design. Sometimes the leads from cans are flexible, in which case the legend on the case is similar to the one above. If the connecting terminals are rigid, some form of symbol is impressed on or near the terminal for identification. Thus, a typical capacitor will carry the following legend:

8 μf at 450 volts △
8 μf at 450 volts ⌒
25 μf at 25 volts □
Common negative terminal (no symbol)

g. VARIABLE CAPACITORS.

(1) The parallel-plate capacitor lends itself readily to mechanical variation or adjustment of capacity. Since capacity depends both on area of the plates and distance between them, either specification may be varied. Variable capacitors generally vary the effective area of the plates and adjustable capacitors vary the distance between them. A typical variable capacitor consists of two sets of plates which move in and out of mesh to vary capacitance. One set of plates is stationary, called the *stator,* and the other set is movable, called the *rotor.* The rotor plates usually number one less than the stator plates and are mounted on a shaft for rotation into and out of mesh with the stator. Both sets of plates are rigid and made of aluminum to prevent corrosion. The rotor shaft is made a part of the over-all housing which may be bolted to the metal chassis of a piece of communications equipment, and the stator plates are mounted on insulating bars within the housing. Ball bearings, gears, and other mechanical means are used to insure smooth operation of such a capacitor.

(2) Since the thickness of the plates does not affect capacitance they are made as thin as possible while remaining suitably rigid. Air is the dielectric, and care must be taken to prevent the rotor from touching the stator at any point. The distance between any rotor and stator plate varies with the use to which the capacitor is put. Thus, a receiving circuit of very low voltage may use variable capacitors with the plates as close as is mechanically feasible, but transmitting capacitors with high voltages must space plates a considerable distance apart to prevent arcing. The large unit in figure 73 is a high-voltage transmitting capacitor. It should be noted also that if spacing is increased to prevent breakdown, the variable capacitor must be made physically much larger than the low-voltage unit of the same capacity. Generally both the area and the number of plates are increased.

(3) The rotor plates of variable tuning capacitors are often placed off center on the shaft in order to produce a nonlinear change of capacitance with relation to shaft rotation. Thus, as the shaft moves through 180° mechanically the plates mesh very slowly at the minimum capacitance end (high-frequency end) and move rapidly as the full mesh or maximum capacitance (low-frequency end) is reached. This distribution of capacitance change tends to spread out the crowded higher frequencies and bring together the spread of the lower frequencies, thus achieving a more uniform or linear frequency change compared to mechanical movement of the shaft. Other variable capacitors have split-stator arrangements; that is, half of the stator plates are insulated from the other half and both rotors are mounted on a common shaft so that the capacitor becomes in effect two balanced capacitors if connected in series (stator-rotor, rotor-stator), or one capacitor

if connected in parallel. The large transmitting capacitor of figure 73 is a split-stator capacitor. The multisection or *ganged* capacitor is also shown in figure 73. Essentially, this type is two or more variable capacitors mounted in such a manner on a common shaft that all are varied simultaneously. Single-dial tuning of a number of circuits is thus possible.

(4) Low-frequency variable capacitors for broadcast band transmission and reception generally range from 250 $\mu\mu$f to 500 $\mu\mu$f (.00025 μf to .0005 μf) in fully meshed position. For higher-frequency applications, capacitors having ranges between 25 $\mu\mu$f to 150 $\mu\mu$f (.000025 μf to .00015 μf) are used. These smaller variable capacitors are specially designed for use in high-frequency circuits. Other small variable capacitors with a maximum capacity of 5 $\mu\mu$f (.000005 μf), called *micro* capacitors, are also used in certain very-high-frequency applications.

(5) *Adjustable capacitors* are another type of variable capacitor. They are so designed that they can be set to a desired capacitance and then left at that capacitance. Adjustable capacitors are often mica capacitors and consist of a spring-metal plate which is screwed into position against another plate from which it is insulated by the mica. These adjustable capacitors are generally subdivided into *trimmers* and *padders*. It should be noted that there is no electrical difference between the two types: the trimmer is a small adjustable capacitor generally placed in parallel with some larger variable capacitor so that a certain small amount of capacitance is added to the larger unit; the padder is a physically small, high-capacitance capacitor placed in series with a variable capacitor to lessen its capacitance. Trimmer adjustments range from .5 $\mu\mu$f to 10 $\mu\mu$f, others from 10 $\mu\mu$f to 50 $\mu\mu$f. Padder adjustments range from anywhere between 50 $\mu\mu$f to 100 $\mu\mu$f (minimum) and 500 $\mu\mu$f to 1,000 $\mu\mu$f (maximum). These smaller capacitors are also shown in figure 73.

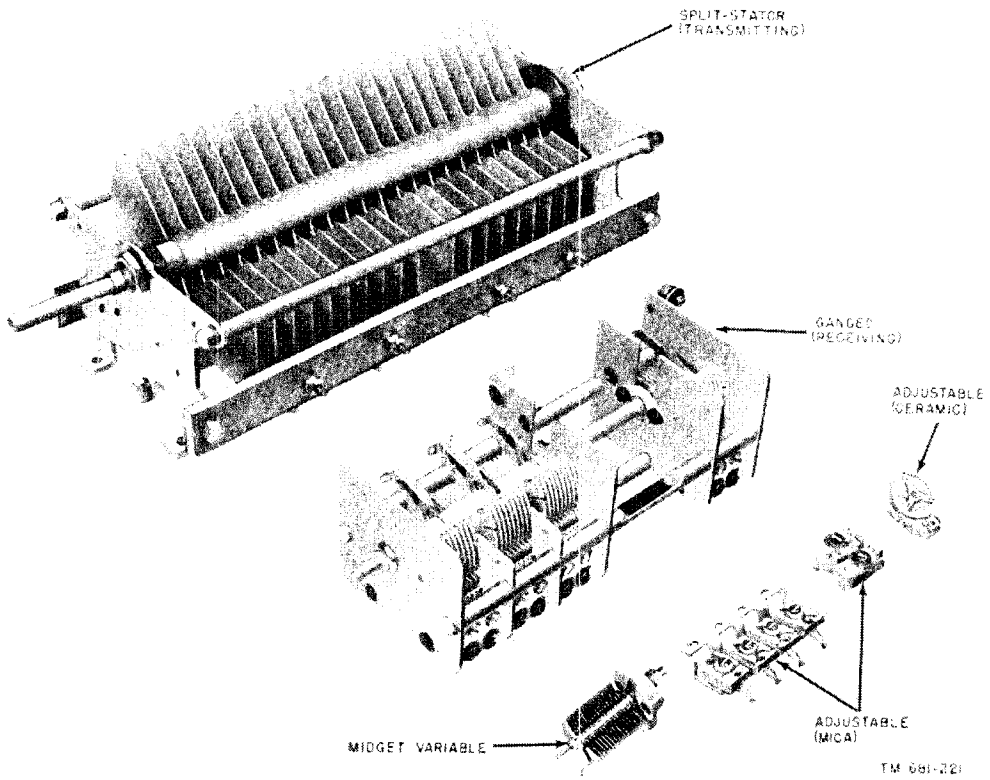


Figure 73. Types of variable capacitors.

47. Summary

a. The inertia of an elastic body is opposite in effect to the inertia of a rigid body and is like the electrical property of capacitance, which produces an effect opposite to that of inductance and complementary to it.

b. A capacitor is any two conductors separated by an insulating material.

c. If voltage is applied to a capacitor, current flows at a maximum almost instantaneously and then falls to a minimum as the voltage across the charged capacitor builds up.

d. A charged capacitor stores energy for an indefinite period of time.

e. If a charged capacitor is short-circuited, current flows at a maximum almost instantaneously in the opposite direction of charging and then gradually falls to a minimum as the voltage across the capacitor falls to zero.

f. For a given capacitor, the ratio between the charge on one plate and the voltage causing it is always a constant.

g. The ratio of the charge Q to the voltage E is the measure of capacitive action, which is called *capacity,* labeled C, and measured in farads. Thus:

$$C = \frac{Q}{E} .$$

h. A farad is the capacity of a capacitor on one plate of which a charge of 1 coulomb is deposited by a difference in potential of 1 volt. A microfarad is one-millionth of a farad (10^{-6} farads). A micromicrofarad is one millionth of a microfarad (10^{-12} farads).

i. Different insulating materials, or dielectrics, show different electrostatic permittivity and different resistances to breakdown voltage.

j. The ratio of dielectric permittivity of a given material compared to the dielectric permittivity of air is K, the dielectric constant of the material.

k. The dielectric strength of an insulating material is the voltage it will withstand per unit thickness, generally .001 inch.

l. The capacity of a parallel-plate capacitor is given by the following formulas:

$$C = \frac{8.84 \times 10^{-8} K A}{d} ,$$

where A and d are in square centimeters and centimeters,

and:

$$C = \frac{22.45 \times 10^{-8} K A}{d} ,$$

where A and d are in square inches and inches.

m. A capacitor prevents the instantaneous rise or fall of *voltage* in a circuit, the delayed counter voltage aiding an alternating current flow.

n. A capacitor is an open circuit to dc and a conducting path to ac.

o. Current through a capacitor is directly proportional to the capacity and the rate of change of the voltage across it:

$$I = C \frac{de}{dt} .$$

p. Capacitance is defined as that property of an electric circuit which tends to oppose a change in voltage:

$$C = I \frac{dt}{de} .$$

q. The unit of capacitance may be redefined: A farad is the capacitance of a circuit in which a voltage change of 1 volt per second causes a current flow of 1 ampere.

r. High-frequency a-c circuits, because of the great speed of change of voltage, use small values of capacitance; low-frequency a-c circuits, because of the slow speed of change of voltage, use large values of capacitance.

s. A capacitor reveals characteristics opposite in effect to those of an inductance: As frequency increases, an inductance generates a greater and greater counter emf, whereas a capacitor under the same circumstances becomes increasingly a conductor.

t. The conductance of a capacitor to ac is equal to the capacitance in farads multiplied by the angular velocity:

$$G_C = 2 \pi f C \text{ (mhos)}.$$

u. The opposition of a capacitor to ac is the reciprocal of its conductance; it is called capacitive reactance, which is assigned the symbol X_C and measured in ohms:

$$X_C = \frac{1}{2 \pi f C} \text{ (ohms)}.$$

The higher the frequency, or the greater the capacitance, the less the capacitive reactance.

v. The current through a capacitor *leads* the applied voltage by 90°.

w. In a series *R-C* circuit, current is everywhere the same, and the voltage across the capacitor is 90° out of phase with the voltage across the resistance. The resultant voltage is:

$$E = \sqrt{E_R^2 + E_L^2} .$$

x. The impedance of a series R-C circuit is:

$$Z = \sqrt{R^2 + X_C^2} .$$

y. The phase angle θ is the angle the *Z*-vector makes with the *R*-vector:

$$\text{Tan } \theta = \frac{X_C}{R}$$

$$\text{Cos } \theta = \frac{R}{Z} .$$

z. The current in a series *R-C* circuit is determined by Ohm's Law:

$$I = \frac{E}{Z} .$$

aa. In a parallel *R-C* circuit, voltage is everywhere the same, and the current through the capacitor is 90° out of phase with the current through the resistance. The resultant current is the line current:

$$I_T = \sqrt{I_R^2 + I_C^2} .$$

ab. The total current is greater than the current in either branch, and the impedance is less than the opposition of either branch:

$$Z = \frac{E}{I_T} .$$

ac. The total current *leads* the applied voltage by the angle the total-current vector makes with the resistance-current vector:

$$\text{Tan } \theta = \frac{R}{X_C}$$

$$\text{Cos } \theta = \frac{Z}{R} .$$

ad. The impedance of the circuit is:

$$Z = \frac{R X_C}{\sqrt{R^2 + X_C^2}}$$

ae. The time required for the rise or fall of voltage in a d-c *R-C* circuit is a function of the capacitance and resistance in the circuit. Voltage rises to 63.2 percent of its maximum value and falls 63.2 percent from its maximum value, in the first unit of time. In each succeeding unit of time, there is an additional 63.2 percent rise or fall of the remainder. The time constant is:

$$t = R \times C$$

where t is the time constant in seconds, R is in ohms, and C is in farads.

af. The total capacitance of a circuit containing more than one capacitor connected in series is equal to the reciprocal of the sum of the reciprocals of the individual capacitances:

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots + \frac{1}{C_n}} .$$

For two capacitances in series this formula is:

$$C_T = \frac{C_1 \times C_2}{C_1 + C_2} .$$

ag. The total capacitance of capacitances in parallel is equal to the sum of the capacitances:

$$C_T = C_1 + C_2 + C_3 \ldots + C_n .$$

ah. Distributed capacitance is the stray or random capacitance that exists between connecting wires, between components located physically near each other, and between different parts of a given component. *Lumped capacitance* is the capacitance of capacitor elements as such.

ai. Fixed capacitors are distinguished, according to the dielectric material used, into paper, oil, mica, ceramic, and electrolytic capacitors.

aj. Variable and adjustable capacitors are distinguished, according to function and size, into tuning, micro, trimmer, padder, and neutralizing capacitors.

ak. Paper capacitors are used for low-frequency work and range in value from .0005 μf to 8 μf at voltages up to 600 working volts.

al. Oil-impregnated paper capacitors and paper capacitors immersed in oil are used for medium high voltages (600 to 2,000 volts) in the same capacitive range as paper.

am. Mica capacitors are used for high-frequency, high-voltage work and range in value from 5 $\mu\mu$f to 50,000 $\mu\mu$f at voltages up to 7,500 volts.

an. Ceramic capacitors are used for very-high-frequency and low-power, very-high-voltage work (up to 30,000 volts).

ao. Electrolytic capacitors show polarity and are largely used in relatively high-power, low-frequency filter circuits at voltages up to 600 volts. Common values of electrolytics are 8, 10, 16, or 20 μf at 450 volts and 100, 200, 500, 1,000, or 2,000 μf at 25 volts.

ap. Variable capacitors are used for low-frequency work in ranges from 250 $\mu\mu$f to 500 $\mu\mu$f, and for high-frequency work in ranges from 25 $\mu\mu$f to 150 $\mu\mu$f. Micro capacitors are variable capacitors used for very-high-frequency work in values up to 5 $\mu\mu$f. Adjustable capacitors are designed for adjustment to a given fixed capacity. Adjustable trimmers are used in parallel with larger variable capacitors and range in values from .5 $\mu\mu$f to 10 $\mu\mu$f. Adjustable padders are used in series with variable capacitors and range in value between 50 $\mu\mu$f to 100 $\mu\mu$f (minimum) and 500 $\mu\mu$f to 1,000 $\mu\mu$f (maximum).

48. Review Questions

a. Describe the effect produced by the electrical property of capacitance in terms of the inertia of elastic bodies.

b. What is a capacitor?

c. Describe the action of current flow in a capacitor when a d-c voltage is applied.

d. Describe the action of current flow when a charged capacitor is short-circuited.

e. What two functions are characteristic of a capacitor?

f. What is capacitance?

g. Give the formula for capacitance in terms of charge and voltage between the plates.

h. Define the farad, the microfarad, the micromicrofarad.

i. What is a dielectric constant?

j. What is dielectric strength?

k. Give the formula for the capacity of a parallel-plate capacitor in square centimeters and centimeters; square inches and inches.

l. How does capacitance vary in respect to the area of the plates, the distance between them, and the dielectric constant?

m. How does the counter emf across a capacitor aid an alternating current flow?

n. What determines the amount of current flow in a capacitor?

o. Define capacitance.

p. Why do high-frequency a-c circuits generally use small values of capacitance, and low-frequency a-c circuits large values of capacitance?

q. How does a capacitor differ from an inductance in terms of reaction to an alternating current?

r. Give the formula for conductance of a capacitor.

s. What is capacitive reactance? Give its formula.

t. How does capacitive reactance vary with frequency? With capacitance?

u. What is the phase shift of current and voltage across a capacitor?

v. In a series *R-C* circuit, what is the phase relationship of the voltage across the resistance to the voltage across the capacitor.

w. Give the formula for the impedance of a series *R-C* circuit.

x. How is the phase angle of a series *R-C* circuit determined?

y. In a parallel *R-C* circuit, what is the phase relationship of the current through the resistance to the current through the capacitance?

z. Give the formula for the impedance of a parallel *R-C* circuit.

aa. How is the phase angle of a parallel *R-C* circuit determined?

ab. What is the time constant of an *R-C* circuit? Give the formula for the time constant.

ac. How many units of time are required for current to reach its practical zero value.

ad. Give the formulas for capacitance in series and in parallel.

ae. What is distributed capacitance? Lumped capacitance?

af. How are classes of fixed capacitors distinguished?

ag. How are variable and adjustable capacitors distinguished?

ah. Give the usual range in capacitance and working voltage for the following capacitors: paper; oil; mica; ceramic; electrolytic; variable.

ai. What is a trimmer capacitor? A padder?

CHAPTER 4

A-C CIRCUITS WITH INDUCTANCE, CAPACITANCE, AND RESISTANCE

49. A-C Circuits

a. In previous chapters, the fundamental properties of inductive, capacitive, and resistive circuits were considered as isolated phenomena and the following facts observed:

(1) The voltage drop across a resistor is in phase with the current through it.

(2) The voltage drop across an inductor *leads* the current through it by 90°.

(3) The voltage drop across a capacitor *lags* the current through it by 90°.

b. The three basic relationships between voltage and current are shown as graphs and as vectors in figure 74. These vectors indicate phase angle by their direction, and their magnitude is dependent on the values chosen for a given circuit. Thus, if I is the effective value of the current in either L, C, or R, then the magnitudes of the effective voltage drops are:

$$E_R = IR$$
$$E_L = IX_L = I\,(2\pi fL)$$
$$E_C = IX_C = I\left(\frac{1}{2\pi fC}\right) .$$

c. Current in an a-c circuit, however, varies with time. The voltage drops across the various elements also vary with time, but the same variation is not always present in each at the same time. Therefore, Kirchhoff's second law (when applied to a-c circuits) must be understood to mean that, *at any instant,* the sum of the voltage drops around a closed circuit equals the sum of the voltage rises, or the total applied voltage. In figure 75, a series L-C-R circuit is shown; the symbols e, e_R, e_L, and e_C denote *instantaneous* voltages. Then:

$$e = e_R + e_L + e_C .$$

This relationship is true for all values of time. But an instantaneous a-c voltage cannot be determined by Ohm's Law, which holds true only for maximum, effective, or average values, as in the formulas of the preceding paragraph. An a-c quantity, however, is fully determined when its *effective* value and phase (in respect to some standard) are known. Therefore, the method of analysis by vectors, which show phase as direction and magnitude as effective value, can be used to add either sine voltages or sine currents.

d. For example, a 60-cycle, 100-volt generator is connected to a circuit having a resistance of 6 ohms in series with an inductive reactance of 8 ohms, as shown in A of figure 76. As was demonstrated in the previous discussion of series L-R circuits, the vector sum of the voltage drops across the resistance and the inductance must equal the applied voltage. Since E_R is in phase with I, and E_L leads I by 90°, their vector sum, 100 volts, is the hypotenuse of a right triangle (B of fig. 76). Then:

$$E^2 = E_R^2 + E_L^2 .$$

Substituting equivalents:

$$(100)^2 = (IR)^2 + (IX_L)^2$$
$$(100)^2 = 36\,I^2 + 64\,I^2$$
$$(100)^2 = 100\,I^2$$
$$I^2 = 100$$
$$I = 10.$$

E_R, the voltage drop across the resistance, is IR, which is 10 times 6 = 60 volts. E_L, the voltage drop across the inductance, is IX_L, which is 10 times 8 = 80 volts. The tangent of the phase angle θ is E_L/E_R or $X_L/R = 8/6 = 1.33$. Then, from the table in appendix II: $\theta = 53.1°$. The current in this circuit lags the applied voltage by 53.1°. C of figure 76 presents these results graphically.

e. In like manner, the current, voltages, and phase angle of a series R-C circuit may be determined. A of figure 77 shows the schematic

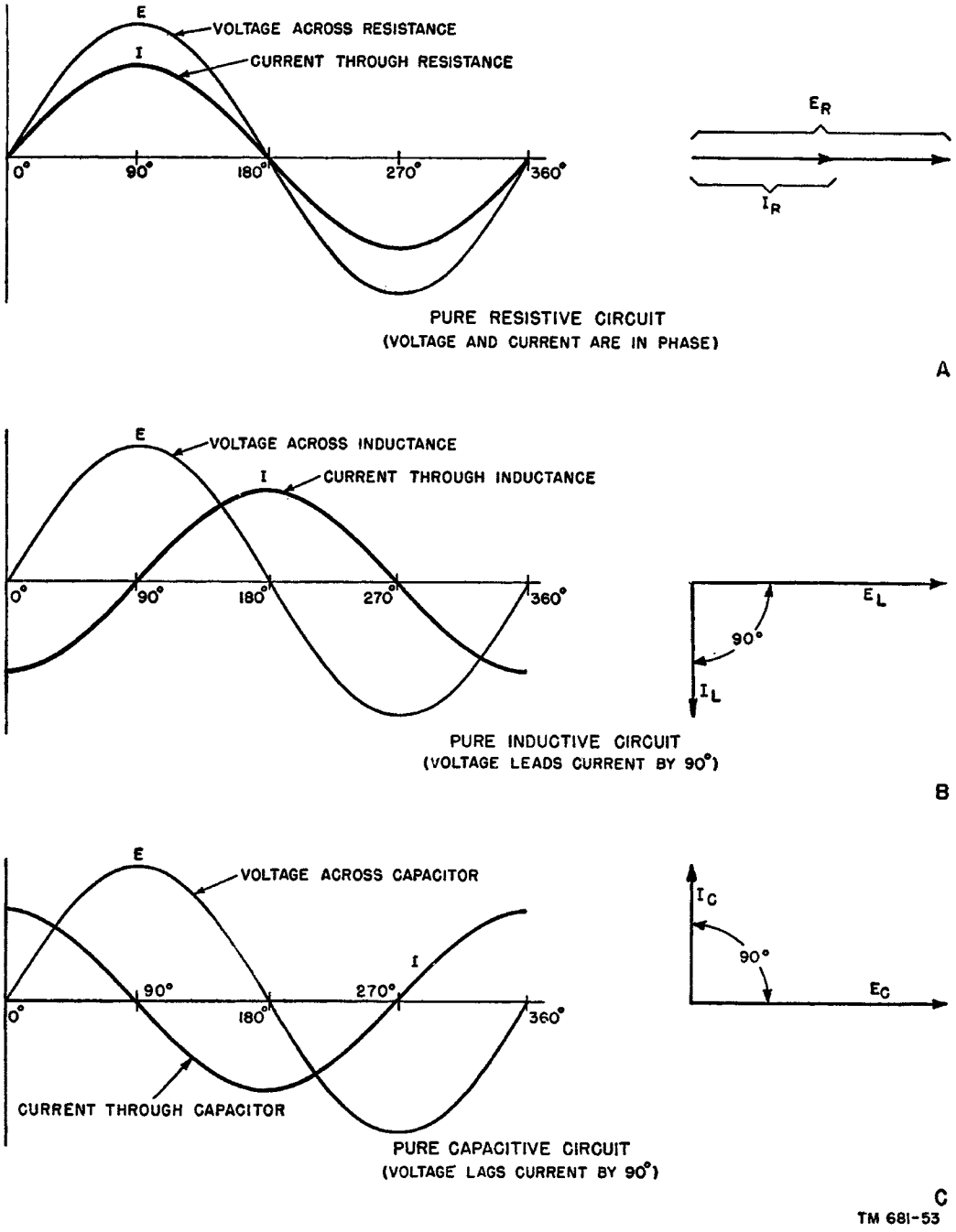


Figure 74. L, C, and R circuits: graphical and vectorial representation.

diagram of a circuit of 3 ohms resistance and 4 ohms capacitive reactance connected to a 60-cycle, 300-volt a-c source. As above, the vector sum of the voltage drops across the resistance and the capacitance equals the applied voltage. Since E_R is in phase with I, and E_C *lags* I by 90°, their vector sum, 300 volts, is the hypotenuse of a right triangle (B of fig. 77). Then:

$$E^2 = E_R^2 + E_C^2.$$

Substituting equivalents:

$$(300)^2 = (3I)^2 + (4I)^2 = 25I^2.$$

The square root of each:

$$300 = 5I$$

$$I = \frac{300}{5} = 60 \text{ amperes.}$$

E_R, the voltage drop across the resistance, is IR, which is 60 times 3 = 180 volts. E_C, the voltage drop across the capacitance, is IX_C, which is 60 times 4 = 240 volts. The tangent of the phase angle θ is $E_C/E_R = X_C/R = 4/3 = 1.33$. From the table in appendix II:

$$\theta = 53.1°.$$

The current in this circuit leads the applied

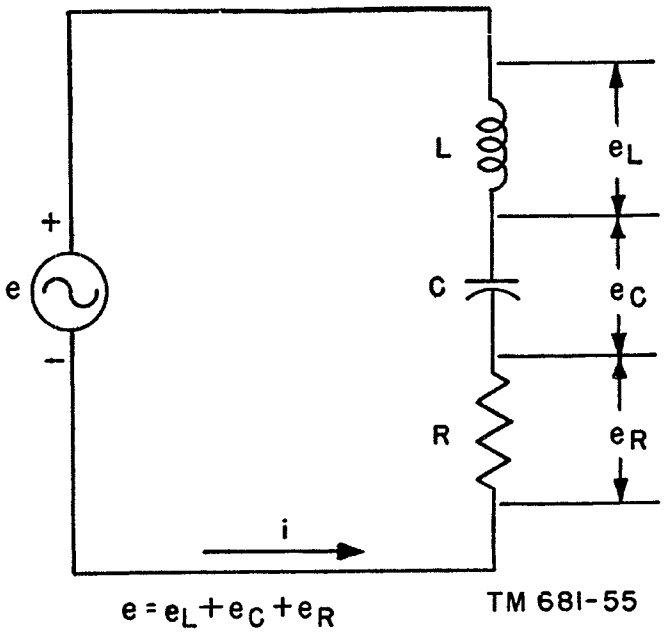
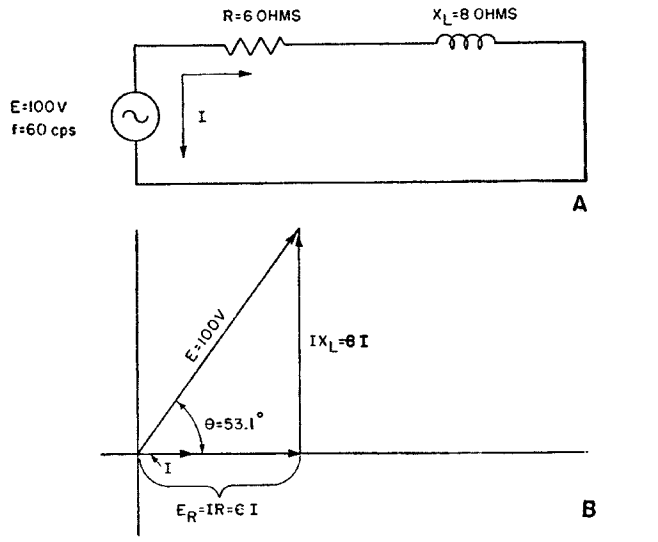


Figure 75. Voltage drops in an L-C-R circuit.



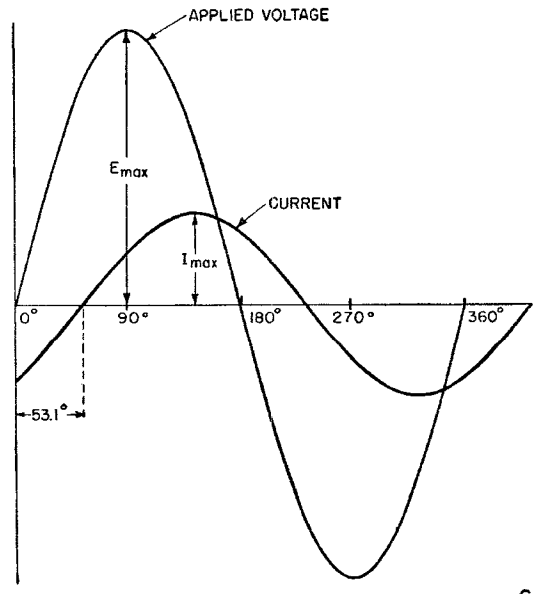


Figure 76. R-L circuit.

voltage by 53.1°. C of figure 77 presents these results graphically.

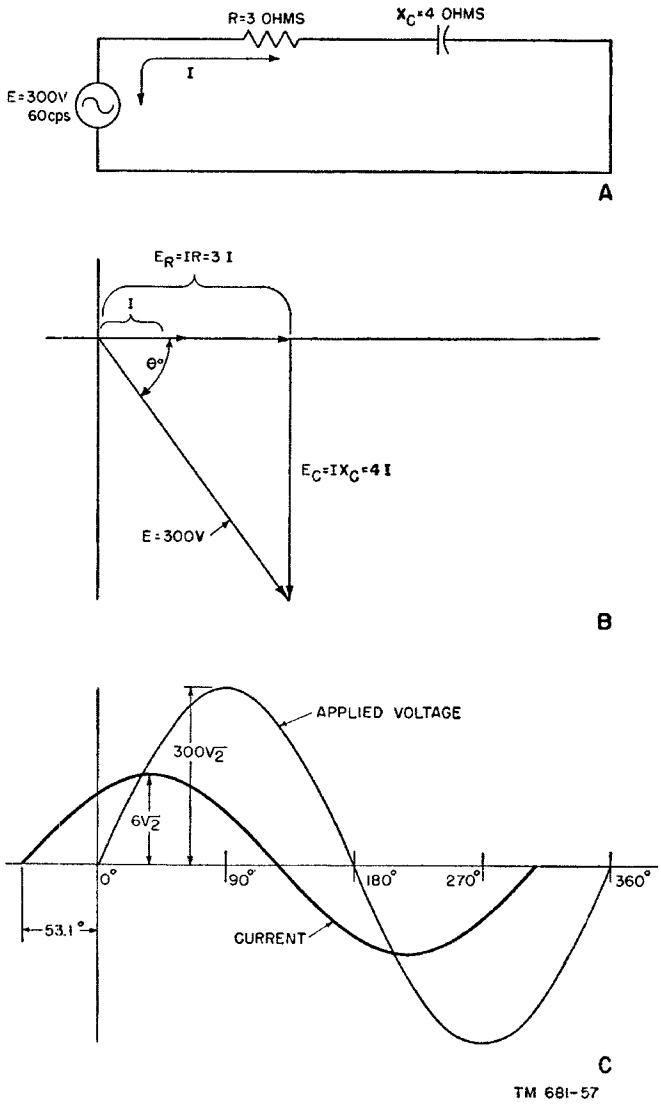


Figure 77. R-C circuit.

50. The Series L-C-R Circuit

a. When the three basic circuit elements of inductance, capacitance, and resistance are brought together in a single circuit, the voltage drops, current, and phase angle may be determined by combining the methods described above. A of figure 78 shows a series L-C-R circuit containing 6 ohms resistance, 8 ohms inductive reactance, and 16 ohms capacitive reactance connected to a 60-cycle, 300-volt source.

b. B and C of figure 78, are the graph and vector diagram, respectively, for this circuit. Since current in a series circuit is everywhere the same, I is the reference vector. Then E_R is

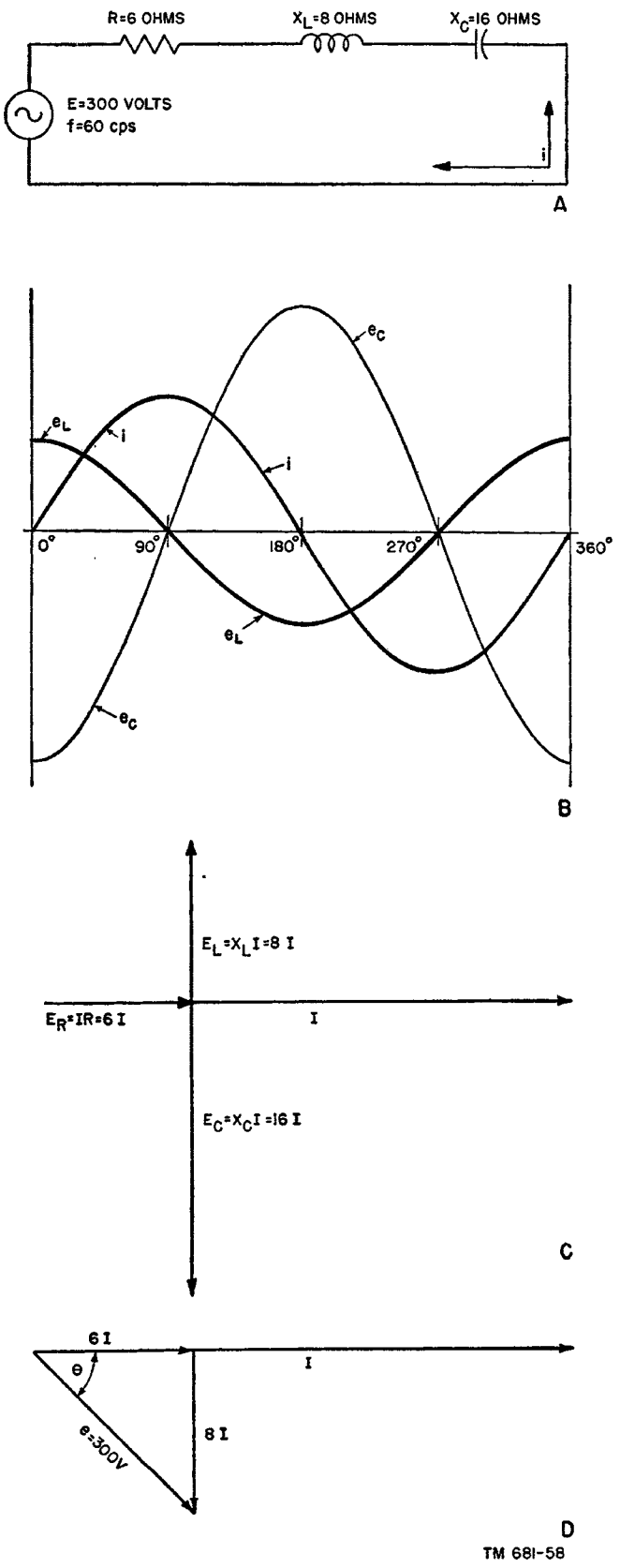


Figure 78. L-C-R circuit.

in phase with I, E_L leads I by 90°, and E_C lags I by 90°. E_C and E_L are 180° out of phase, and so their vector sum is merely the difference between the two. Then, since E_C is larger than E_L, the resultant reactive voltage is $E_C - E_L$, or:

$$16\,I - 8\,I = 8\,I.$$

This resultant reactive voltage lags I by 90° because it has the direction of E_C, the larger vector. D of figure 78 is the resultant vector diagram. From the figure:

$$E^2 = (IR)^2 + (IX)^2$$

$$(300)^2 = 36\,I^2 + 64\,I^2 = 100\,I^2.$$

The square root of each:

$$300 = 10\,I$$

$$I = \frac{300}{10} = 30 \text{ amperes.}$$

The voltage drop across the resistance, $E_R = IR$ = 30 times 6 = 180 volts. The voltage drop across the inductance, $E_L = IX_L$ = 30 times 8 = 240 volts. The voltage drop across the capacitance, $E_C = IX_C$ = 30 times 16 = 480 volts. Because the larger capacitive reactance cancels completely the inductive reactance, the circuit is capacitive, and so the current leads the applied voltage by the phase angle θ. The tangent θ is equal to $\frac{X_C - X_L}{R} = 8/6 = 1.33$. The angle θ is equal to 53.1°.

c. The series L-C-R circuit illustrates the following important points:

(1) The current in a series L-C-R circuit either leads or lags the applied voltage, depending on whether X_C is greater than or less than X_L.

(2) A capacitive voltage drop in a series circuit always subtracts directly from an inductive voltage drop.

(3) The voltage across a single reactive element in a series circuit can have a greater effective value than that of the applied voltage.

d. As noted in (3) above and illustrated in the circuit of figure 78, E_C was found to have an effective value of 480 volts as compared to the applied 300 volts. In d-c circuits, this condition can never exist, since voltage drops are added arithmetically, and accordingly the voltage drop across any one of the component parts is always less than the applied voltage. The contrary condition in a-c circuits was explained in paragraph 49*c* above, in which it was shown that Kirchhoff's Law applies only to the *instantaneous* values of the voltages present.

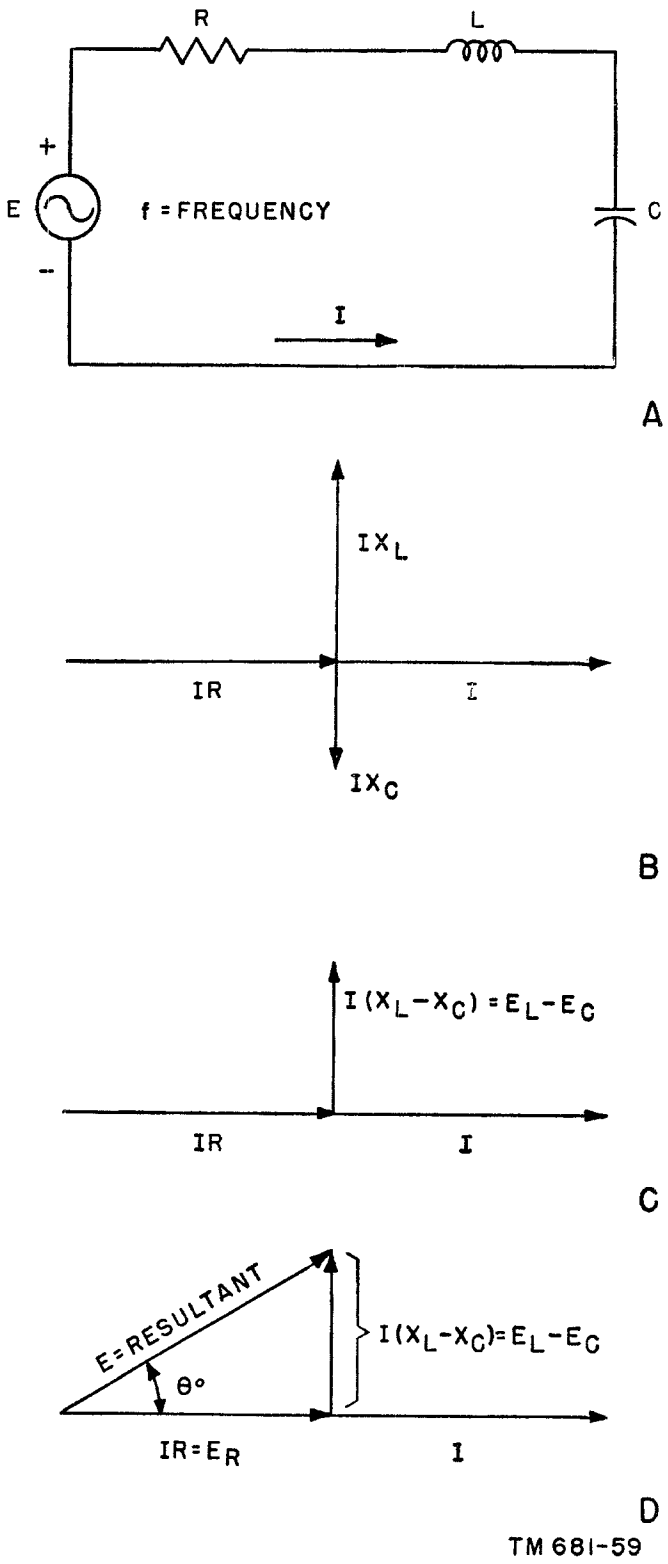


Figure 79. L-C-R circuit: phase relation.

51. Analysis of L-C-R Circuits

a. The various relationships noted above may now be generalized, and the current, impedance, voltage drops, and phase angle of any series L-C-R circuit calculated. In A of figure 79, a schematic diagram of a series circuit containing an inductance L, a capacitance C, and a resistance R is shown. This circuit is connected to an a-c voltage source of magnitude E and frequency f. B of figure 79 shows the initial vector diagram. The voltage drop across the resistance, IR, is drawn in phase with the current I. The voltage drop across the inductance is drawn above the zero axis; the voltage drop across the capacitance is drawn below the axis. C of figure 79 shows the resultant vectors; and the voltage triangle (with base E_R and vertical side $E_L - E_C$) is illustrated in D of figure 79. The resultant voltage E, equal to the applied voltage, is the hypotenuse of the right triangle. Then:

$$E^2 = I^2R^2 + I^2(X_L - X_C)^2$$

$$E^2 = I^2\,(R^2 + (X_L - X_C)^2).$$

The square root:

$$E = I\sqrt{R^2 + (X_L - X_C)^2}\ .$$

And

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}\ .$$

Also, since in any a-c circuit I is equal to E/Z, then Z, the total impedance of an L-C-R circuit, is equal to the square root of the square of the resistance plus the square of the difference of the reactances present. Or:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}\ .$$

From the voltage triangle, it is seen that the current lags the applied voltage by the angle θ. Then, as previously:

$$\text{Tan}\ \theta = \frac{E_L - E_C}{E_R}\ .$$

By substitution and cancellation:

$$\text{Tan}\ \theta = \frac{X_L - X_C}{R}\ .$$

In a like manner, the cosine of θ is equal to E_R/E, or:

$$\text{Cos}\ \theta = \frac{R}{Z}\ .$$

Then:

$$Z = \frac{R}{\cos\theta}\ .$$

These results may be written in another form. Since X_L is equal to $2\pi fL$, and X_C to $\frac{1}{2\pi fC}$, then:

$$Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}\ .$$

In all of the preceding calculations, the values of I and E are the effective or rms values of current and voltage.

b. From the general formulas given above it may be seen that, if the inductive reactance X_L

is greater than the capacitive reactance X_C, the inductive voltage drop is greater than the capacitive voltage drop and their sum is inductive; that is, the voltage leads the current by 90°. If X_C is greater than X_L, the reverse is true; that is, the sum is capacitive and the voltage lags the current by 90°. Finally, if the inductive reactance equals the capacitive reactance, the sum is zero, and a very interesting phenomenon called resonance occurs. In this case, the inductive and capacitive voltage drops are equal in magnitude and 180° out of phase; consequently, their vector sum is zero. Physically, this means that at every instant of time, the voltage drop across the capacitor e_C added to the voltage drop across the inductor e_L gives a resultant voltage of zero. Expressed as an equation:

$$e_C + e_L = 0$$
$$e_C = -e_L \ .$$

c. These results are shown graphically in A of figure 80. Thus, a series L-C-R circuit is said to be resonant when X_L is equal to X_C. Then, since no reactive voltage is present, the impedance of the circuit is equal to the d-c resistance of the circuit, and current is limited by this resistance alone. The phase angle θ has a tangent of zero and a cosine of 1, and θ is 0°, which means that the current in the circuit is in phase with the applied voltage. The vector diagram for this condition is shown in B of figure 80. It should be noted, however, that although the vector sum of the reactive voltages is 0, the voltage across either element will be very high, since the voltage drop across the element is determined by its reactance and the current through it. Current in a resonant circuit is a maximum because it is limited only by R, and therefore the voltage drops across the reactive elements are also maximum and may be far in excess of the applied voltage.

d. A series L-C-R circuit may then appear to its a-c source as any one of the following types:

(1) An R-L circuit ($X_L > X_C$): inductive reactance greater than capacitive reactance.

(2) An R-C circuit ($X_C > X_L$): capacitive reactance greater than inductive reactance.

(3) An R circuit ($X_L = X_C$): inductive reactance equal to capacitive reactance.

e. The resonant frequency of an L-C-R circuit may be determined by reference to the individual reactances, since both are dependent upon frequency. Inductive reactance is equal to $2\pi fL$, and capacitive reactance is equal to $\frac{1}{2\pi fC}$. Then, since at resonance X_L is equal to X_C:

$$2\pi fL = \frac{1}{2\pi fC} \ .$$

Solving for f:

$$f^2 = \frac{1}{4\pi^2 LC}$$
$$f = \frac{1}{2\pi\sqrt{LC}} \ .$$

This is the frequency at which resonance occurs in a series L-C-R circuit. From the formula, it may be seen that regardless of the values of L and C, the circuit will be resonant at *some* frequency. And thus it may be deduced that if the source frequency is fixed, a circuit may be designed to resonate at that frequency.

52. The Parallel L-C Circuit

a. In the discussion of inductance in chapter 2 preceding, and of capacitance in chapter 3 preceding, it was shown that a parallel *L-R* circuit or a parallel *C-R* circuit differed from its respective series circuits in that the *voltage* in the circuit is everywhere the same, the currents 90° out of phase, and the resultant line current either lagging (L-R) or leading (C-R) the applied voltage by some angle less than 90°. In addition, it was shown that the line current is greater than the current in either branch, and that, therefore, the total impedance is less than the impedance of either branch.

b. Unlike the series reactive circuits, an increase in the resistance of a parallel circuit lessens the current through that branch and increases the relative effectiveness of the inductance or capacitance, resulting in a more reactive circuit as the phase angle θ approaches 90°. The limiting condition in this instance would be a resistance of infinite ohms, effectively opening that branch and making current and voltage across the reactive element 90° out of phase. On the other hand, a decrease in the value of the resistance in a parallel circuit causes an increase in current in that branch and decreases the relative effectiveness of the inductance or capacitance, resulting in a more resistive circuit as θ approaches 0°. The limit-

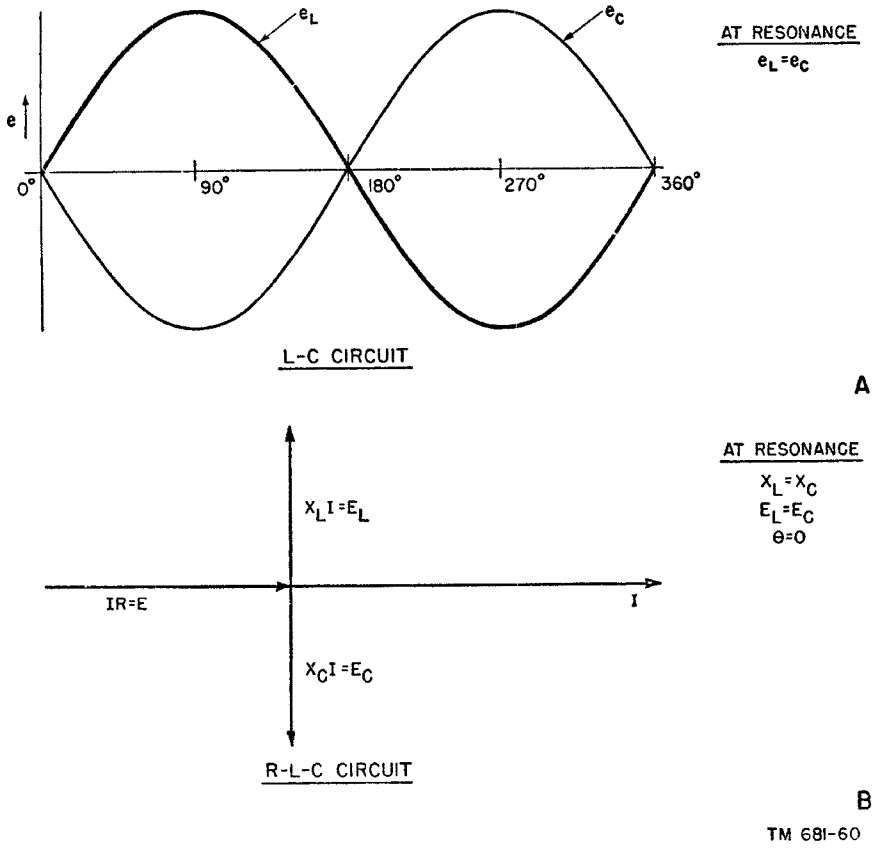


Figure 80. Resonance: graph and vector diagrams.

ing condition in this instance would be a resistance of zero ohms, directly short-circuiting the reactive element so that current and voltage are in phase in this short circuit.

c. In bringing together the parallel *L-R* and *C-R* circuits into a parallel *L-C-R* circuit, new relationships, complicated by the special effect of L and C at resonance, are introduced among the elements. Therefore, by ignoring for the moment the effect of resistance in the circuit, the characteristics of the *L-C* circuit alone may be considered. A of figure 81 shows the schematic diagram of an *L-C* circuit, in which the inductive reactance is 100 ohms, the capacitive reactance 50 ohms, and the applied voltage 300. The graphical representation of the voltages and currents in this circuit is shown in B of figure 81. It will be noted that since the applied voltage is everywhere the same, the voltage may be used as a point of reference. Then, the current through the inductance is plotted as lagging this voltage by 90°, and the current through the capacitance as leading this voltage by 90°. The two currents are then seen to be 180° out of phase.

d. The current through the inductance is I_L:

$$I_L = \frac{E}{X_L} = \frac{300}{100}$$

$$I_L = 3 \text{ amperes.}$$

The current through the capacitance is I_C:

$$I_C = \frac{E}{X_C} = \frac{300}{50}$$

$$I_C = 6 \text{ amperes.}$$

Since these two currents are 180° out of phase, they may be added algebraically to find the total line current:

$$I_T = I_C - I_L$$

$$I_T = 3 \text{ amperes.}$$

The resultant total line current is 3 amperes (dotted sine curve of B of figure 81). This current *leads* the applied voltage by 90°, and the circuit appears to the source as a capacitive circuit. The effect of the inductance is completely cancelled out by the greater effect of the capacitance. C of figure 81 is the vector diagram illustrating this result.

e. The total impedance of this type of parallel circuit is of particular interest. In all previous discussions, it was found that the total impedance of any type of parallel a-c or d-c circuit

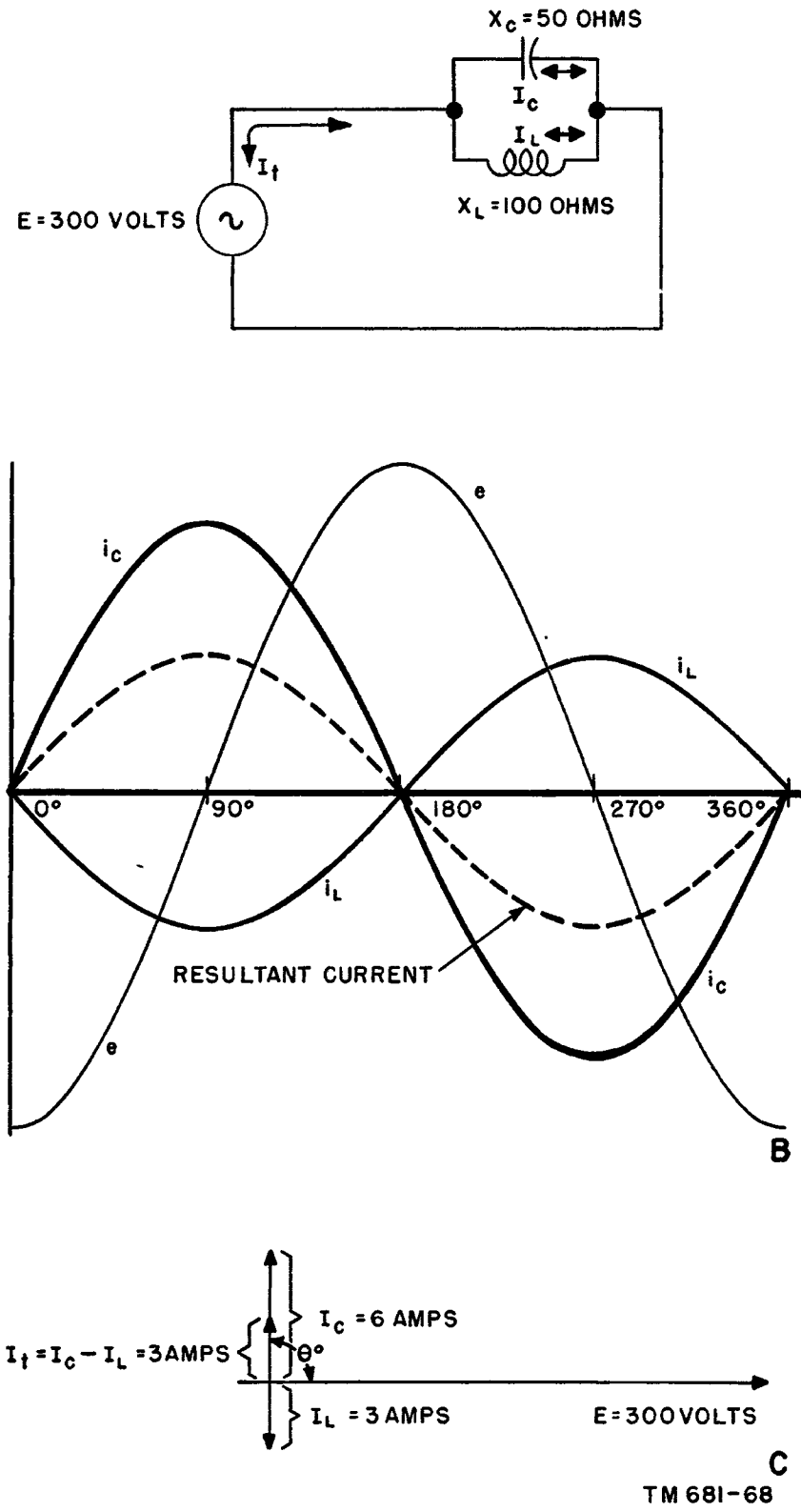


Figure 81. Parallel L-C circuit: graph and vector diagrams.

was always less than the impedance of any branch. But in the circuit above:

$$Z_T = \frac{E}{I_T} = \frac{300}{3}$$

$$Z_T = 100 \text{ ohms.}$$

A total impedance of 100 ohms for this circuit is a value greater than X_C (50 ohms) and, in this case, equal to X_L (100 ohms). This result might have been obtained from the familiar formula for the impedance of a parallel circuit:

$$Z_T = \frac{X_L \times X_C}{X_L + X_C}.$$

Substituting values:

$$Z_T = \frac{100 \times (-50)}{100 + (-50)}.$$

Since X_L and X_C are also 180° out of phase, they may be added algebraically. The minus sign indicates capacitive reactance, conventionally taken as the sign opposite to that of inductive reactance. Then:

$$Z_T = -100 \text{ ohms.}$$

f. From the formula $Z_T = \frac{X_L \times X_C}{X_L + X_C}$ it is interesting and important to realize that at resonance ($X_L = X_C$), the denominator of the equation is equal to zero and the total impedance approaches an infinite amount. Of course, L and C can never offer pure reactance to a circuit, and so it may be said that, at resonance, a parallel L-C circuit offers *maximum* impedance to the applied voltage, line current falling to a *minimum*. On either side of resonance, the total impedance of the circuit falls off rapidly from its maximum (theoretically infinite value), and current rises as the impedance falls. A further examination of the formula will reveal that at the point at which one reactance is twice the value of the other (see example above), the total impedance of the circuit is equal to the larger reactance. As the difference between the values increases, the total impedance falls to a value between X_L and X_C, always greater than the smaller and less than the larger.

53. The Parallel L-C-R Circuit

a. The parallel *L-C-R* circuit may now be seen to be essentially a parallel *L-C* circuit (subject to the interaction of L and C) with an added resistance R in parallel to this circuit. A of figure 82 shows this circuit. At any nonresonant point (X_L not equal to X_C), the currents through L and C will be unequal and 180° out of phase. The resultant current, as shown in the previous example, will be, therefore, either predominantly inductive or capacitive, depending on the branch offering the least opposition to the applied voltage. The generator looking into this circuit, then, would see either a capacitance or an inductance in parallel with the resistance R. The resultant line current is the vector sum of the reactive current and the resistive current, as though the circuit comprised a single reactance and a resistance.

b. For example, the circuit A of figure 82 shows an input voltage of 300, an inductive reactance of 75 ohms, a capacitive reactance of 50 ohms, and a resistance of 100 ohms. B of figure 82 is the graphical representation of the currents in this circuit. It will be seen that the reactive currents are 180° out of phase with each other and 90° out of phase with the current through the resistance, which is in phase with the applied voltage. Then:

$$I_L = \frac{300}{75} = 4 \text{ amperes}$$

$$I_C = \frac{300}{50} = 6 \text{ amperes}$$

$$I_R = \frac{300}{100} = 3 \text{ amperes.}$$

The total reactive current I_X is equal to 2 amperes ($I_C - I_L$). C of figure 82 shows the vector diagram of the currents. Thus, the resultant line current is the vector sum of the reactive current (capacitive) and the resistive current:

$$I_T = \sqrt{I_X^2 + I_R^2}$$

$$I_T = \sqrt{13} = 3.6 \text{ amperes.}$$

Then Z_T, the total impedance of the circuit, is equal to E/I_T:

$$Z_T = \frac{300}{3.6} = 83 \text{ ohms.}$$

The cosine of the phase angle θ that the line current leads the applied voltage is Z/R:

$$\text{Cos } \theta = \frac{83}{100} = .83$$

$$\theta = 34°.$$

c. At the point of resonance, the two reactive currents effectively cancel each other, leaving only the current through the resistance as the total current of the circuit. Therefore:

$$I_C - I_L = 0$$

$$I_T = I_R$$

$$I_T = \frac{E}{R}$$

$$Z_T = R$$

$$\text{Cos } \theta = \frac{Z}{R} = 1$$

$$\theta = 0°.$$

Line current and voltage are effectively in phase across the resistance. Again, it should be noted that a resistance in parallel to an *L-C* circuit tends to destroy the effect of the reactive com-

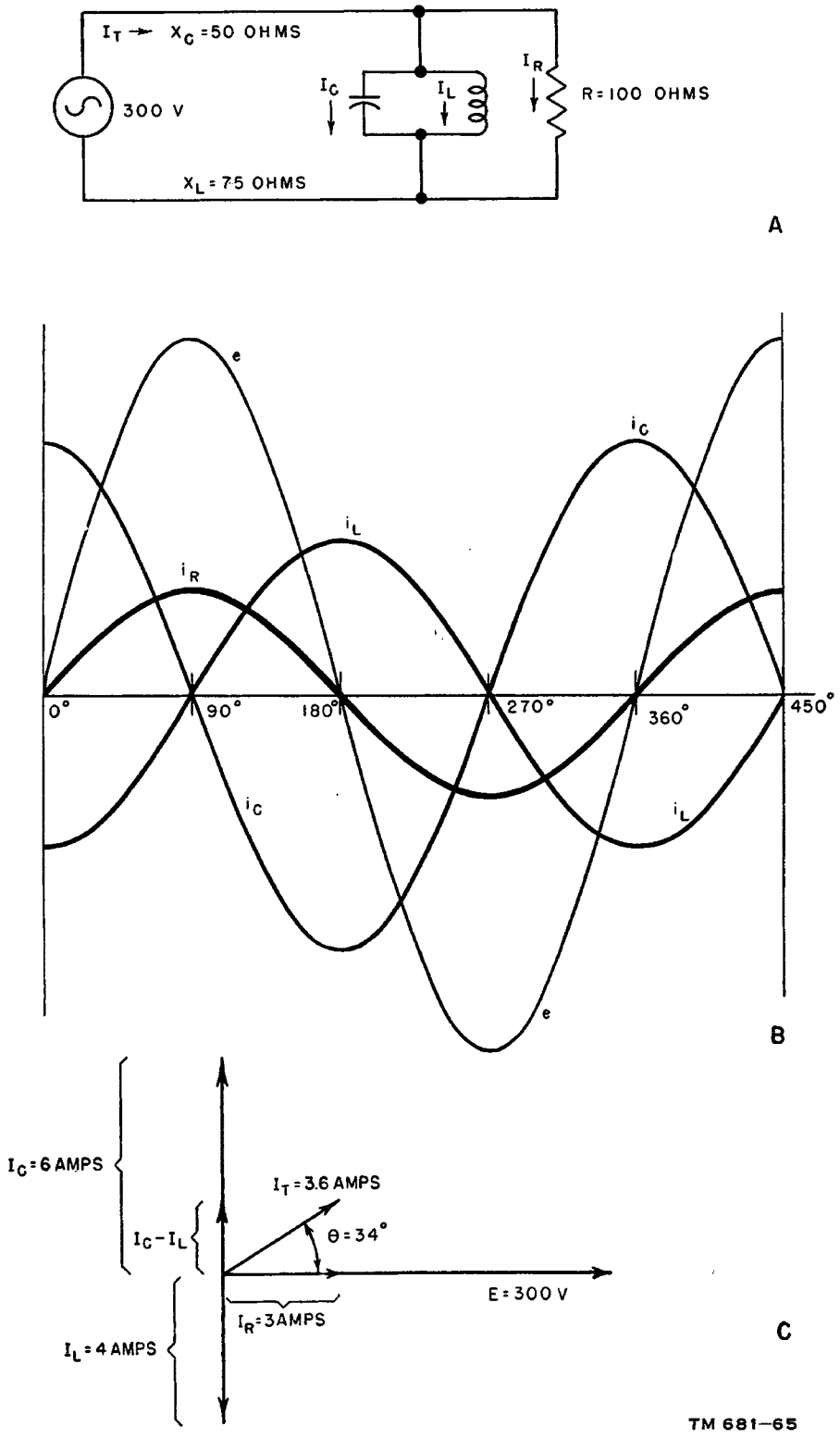


Figure 82. Parallel L-C-R circuit: graph and vector diagram.

ponents, as was shown in the discussion of the parallel *L-R* and *C-R* circuits above. At resonance, the impedance of the circuit is that of the resistance, thus destroying the chief characteristic of a parallel resonant circuit, its maximum impedance to the line.

54. Power in A-c Circuits

a. In d-c circuit analysis, the amount of power absorbed by a resistor or by the resistance of a circuit was determined easily and simply by Joule's Law: $P = I^2R$

where: P = power absorbed in watts,
I = total current in amperes, and
R = resistance of the circuit in ohms.
Since the voltage drop across a resistor R is equal to IR, the formula above may be written:

$$P = IR \times I = EI.$$

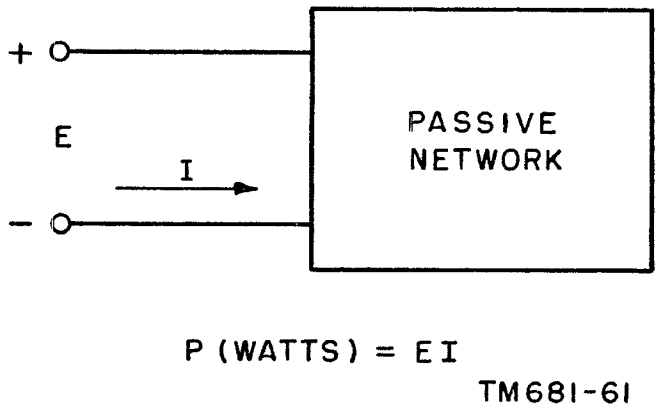


Figure 83. Passive network.

This expression for determining power in d-c circuits is a general one and can be applied to any passive network (one which contains no internal batteries or other source of energy). Figure 83 illustrates such a network. The power absorbed by the network is equal to the voltage applied to it multiplied by the current that flows in it. Thus:

$$P = EI.$$

b. In dealing with a-c circuits, the determination of power is a more complicated process. Since both current and voltage vary with time, the product of e times i (voltage and current at any instant) is also a function of time and is called the instantaneous power p. In general, however, current and voltage in a-c networks are out of phase by some angle θ, as previously shown in the discussions of L-C-R circuits. In figure 84, the conditions in such a circuit with a phase shift of $\theta°$ is illustrated by means of graphs for the current, voltage, and instantaneous power. These graphs reveal several important characteristics of instantaneous power:

(1) The graph of power is of double-frequency variation; that is, the instantaneous power p goes through 2 cycles during one period of the voltage or current.

(2) The power curve has positive loops and negative loops. Thus, for a part of the cycle, p is negative, which must be interpreted to mean that energy is being returned to the generator source during this time. This very important fact indicates that, in an a-c circuit, energy is delivered to the circuit by the source for parts of the cycle and returned to the source by the circuit

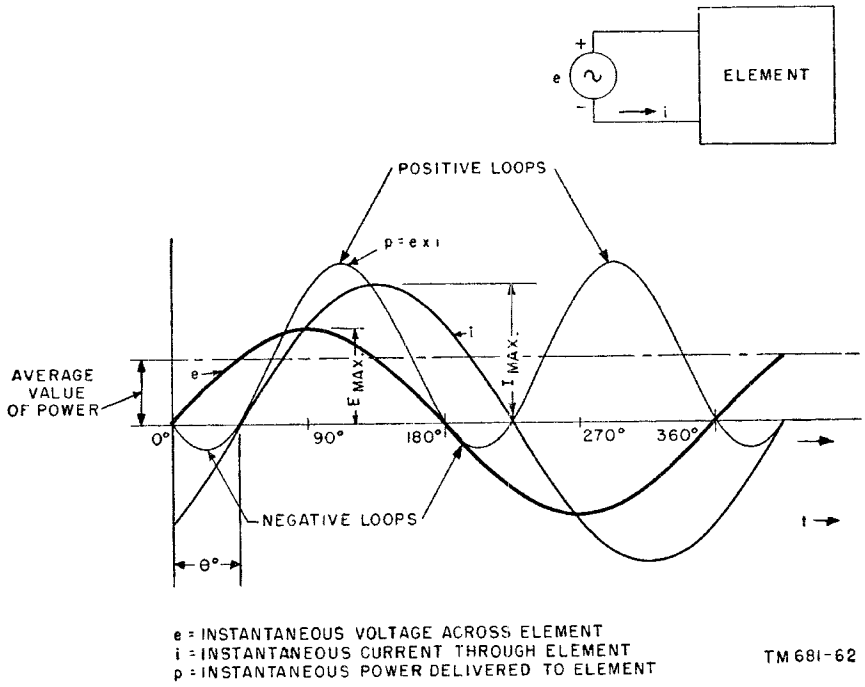


Figure 84. Voltage, current, and power curves.

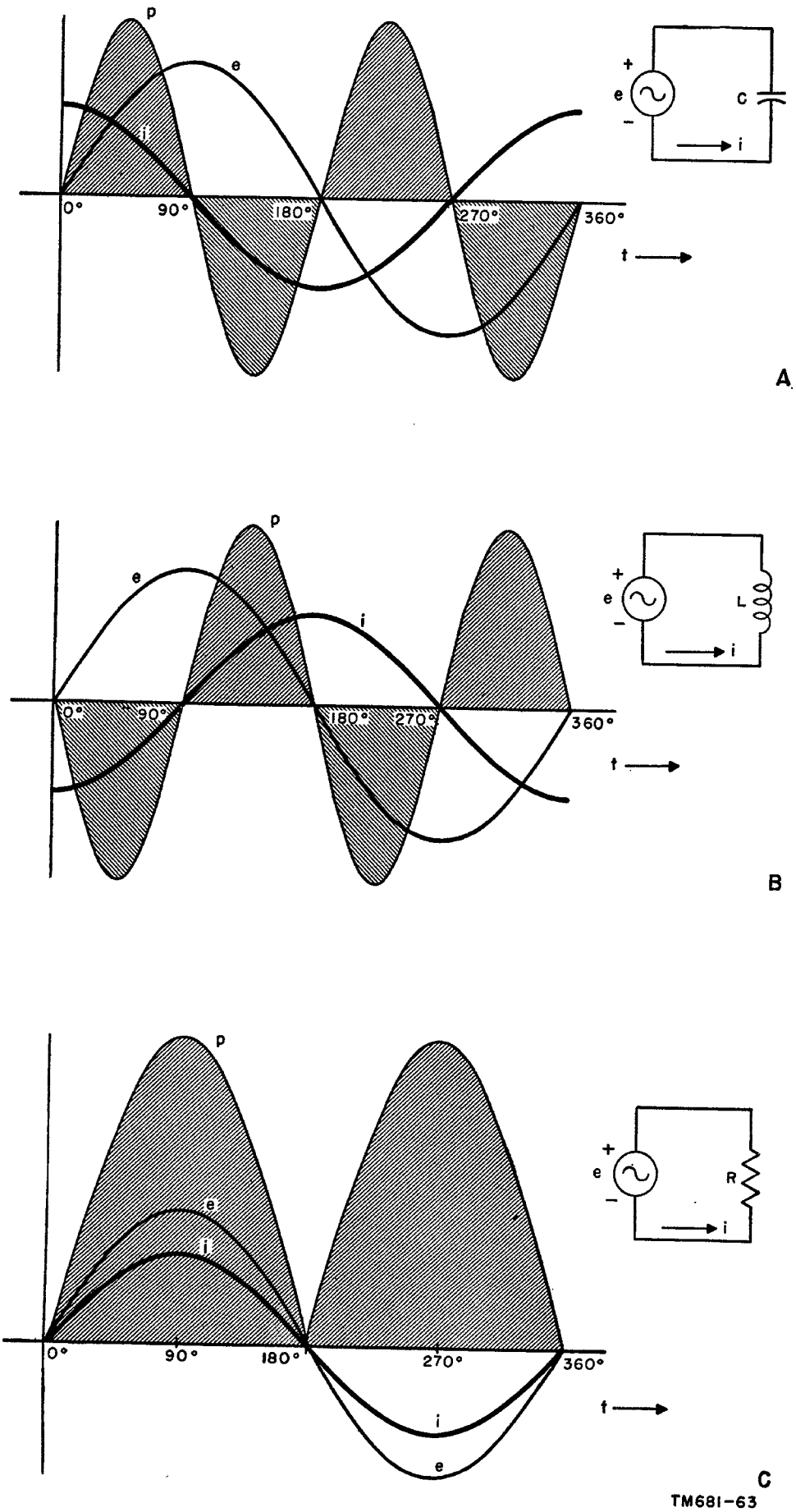


Figure 85. Power curves: capacitive, inductive, and resistive circuits.

for the remainder of the cycle. Accordingly, if, in 1 cycle, the amount of energy returned is equal to the amount delivered by the generator, the net power absorbed by the circuit is zero.

c. The area under the positive loops of the power curve measures the energy delivered to the load, and the area under the negative loops measures the energy returned to the source. In view of these facts, the following generalizations may be made:

(1) If, in 1 cycle (360°), the area under the positive loops of the p curve is greater than the area under the negative loops, the net energy delivered by the generator to the load is positive.

(2) If, in 1 cycle (360°), the area under the positive loops equals the area under the negative loops, the net energy delivered by the generator to the load is 0.

(3) If, in 1 cycle (360°), the area under the positive loops is less than the area under the negative loops, the generator is absorbing a net amount of energy.

d. Therefore, it may be stated that:

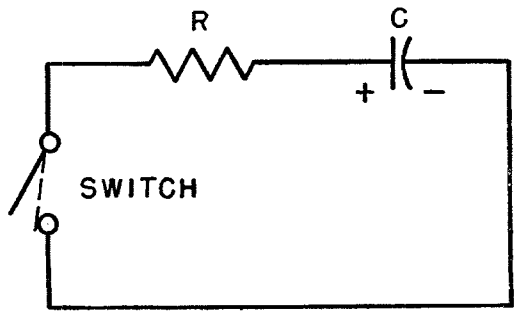
(1) If the net energy delivered by the generator to the load is positive, the circuit contains some resistance, since energy is dissipated in a resistor in the form of heat, which cannot be returned to its source.

(2) If the net energy delivered by the generator to the load is 0, the circuit must contain purely reactive elements, since no energy is used up in the network.

(3) If the net energy delivered by the generator to the load is negative, the generator is absorbing energy and the network contains its own generator, since it is actually delivering power back to the source.

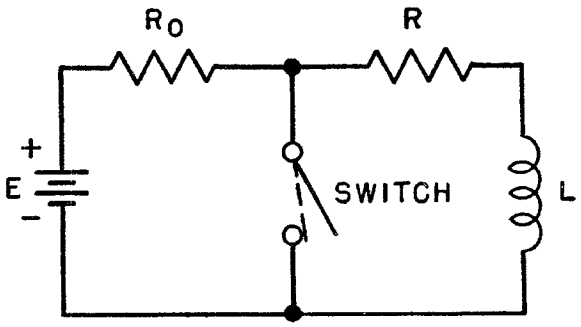
e. Figure 85 illustrates these characteristics with graphs of voltage, current, and power for the three general types of a-c circuits: capacitive, inductive, and resistive. In A and B of figure 85, the area under the positive loops equals the area under the negative loops. The capacitor shown in the schematic of A charges during the first quarter-cycle, and discharges back into the generator during the next quarter-cycle. The same action is repeated during each half-cycle of the generator voltage. B illustrates the same action for a pure inductance. C shows that in the case of a pure resistance the power loops are always positive; that is, the resistor R absorbs energy completely, returning none of it to the source.

f. The behavior of the reactive elements in the circuits above should not be too surprising because, as was pointed out earlier, both capacitors and inductances are elements capable of storing energy without loss. For example, the energy used to charge a capacitor can be completely regained by allowing it to discharge through a resistor; the electrostatic energy of the capacitor is thus converted into heat energy.



C INITIALLY CHARGED. WHEN SWITCH IS CLOSED C DISCHARGES THRU R.

A



BEFORE CLOSING SWITCH, $I_L = \frac{E}{R}$

AFTER CLOSING SWITCH, $I_R = \frac{E_L}{R}$

B

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Figure 86. Recovery of energy in reactive circuits.

Also, the energy used to develop the magnetic field about an inductor can be regained by allowing the current induced in it by the collapsing field to discharge through a resistor. Figure 86 illustrates how the discharging of a reactive element is accomplished. In A, the capacitor C is initially charged. When the switch is closed, C discharges through the resistance R. In B, the initial current through the inductance L stored energy in it by building up a magnetic field; when the switch is closed, the energy stored in the field produces an induced voltage in the inductance and accordingly a current through the resistance R, thus dissipating the energy as heat. The limiting resistor R_o is placed in the circuit to prevent the battery from being short-circuited when the switch is closed.

55. Average Power

a. From these discussions of instantaneous power, it may be seen that in any a-c circuit containing reactive elements, the only power actually dissipated is the power absorbed by the resistance of the circuit. A reactive circuit, however, *appears* to consume large quantities of power, as may be inferred from the area of the positive loops of figure 85. Thus, it is important to note that even though the generator receives back certain amounts of energy *from* the load, it must supply large amounts *to* the load. This power, which the generator must deliver (regardless of the return), is called the *apparent* power; and, as in any a-c circuit, is equal to the product of the effective value of the voltage and the current. Hence:

$$P \text{ (apparent power)} = EI.$$

Or, substituting $\frac{E_{max}}{\sqrt{2}}$ for E, and $\frac{I_{max}}{\sqrt{2}}$ for I:

$$P \text{ (apparent power)} = \frac{E_{max} I_{max}}{2}.$$

b. Apparent power is distinguished from the actual power consumed by the load, which is called the *average* or *true* power and is the energy absorbed by the resistance of the circuit. Average power is defined as the power absorbed by a circuit over a period of 1 cycle of the input voltage. An examination of the power graphs reveals that this concept may be expressed as a formula:

$$P \text{ (average power)} = \frac{\text{(area of positive power loops)} - \text{(area of negative power loops)}}{\text{length of 1 cycle in radians}}.$$

However, the difference between the positive and negative power loops is the difference between the power delivered to the load and the power returned to the source and is the power absorbed by the resistance of the circuit. Therefore, Joule's Law expresses the actual or true power consumed in any circuit, either a-c or d-c. Thus, as previously:

$$P \text{ (average power)} = I^2R.$$

But in a reactive a-c circuit, the current in the circuit is equal to E/Z. Then:

$$P = \frac{E}{Z} \times I \times R.$$

Rewriting:

$$P = EI \times \frac{R}{Z}.$$

However, the ratio R/Z is known from the discussions of L-C-R circuits to be the cosine of the phase angle θ, the angle between current and voltage in the circuit. Therefore, average power is equal to EI multiplied by the cosine of θ.

Or: $P = EI \cos \theta$

where: E = effective value of the voltage across the circuit,
I = effective value of the current in the circuit,
θ = phase angle between voltage and current,
P = average power absorbed by the circuit.

c. The various derived formulas for *apparent* power follow:

$$P = EI$$
$$P = I^2Z$$
$$P = \frac{E^2}{Z}.$$

And those for *average* power:

$$P = EI \cos \theta$$
$$P = I^2R.$$

d. An examination of the formula for average power above reveals that if the phase angle θ equals 90°, its cosine is zero and the actual power absorbed by the circuit is zero. Thus, a phase angle of 90° means that the circuit is purely reactive and returns as much power as it receives. If the phase angle is 0°, its cosine is 1, the circuit is purely resistive, and all the power produced by the source is absorbed by the load.

The cosine θ, then, varies from 0 to 1 as the phase angle varies from 90° to 0°.

e. In the following examples, the effect of the phase angle on power is illustrated by the three circuits of figure 87, to each of which a 300-volt, 60-cycle ac is applied.

(1) The circuit of A of figure 87 comprises an inductive reactance of 100 ohms and a capacitive reactance of 200 ohms. Since these reactances are opposed, the effective reactance is 100 ohms capacitive, and the current in the circuit is 3 amperes. The apparent power in the circuit is:

$$P = EI = 300 \times 3$$
$$P = 900 \text{ watts.}$$

The phase angle of this purely capacitive circuit is 90°, current leading voltage. Then the average power is:

$P = EI \cos \theta = 300 \times 3 \times \cos 90°$
$P = 900 \times 0$
$P = 0$ watt.

Therefore, no power is consumed in this circuit, but the generator must supply 900 watts of power, all of which it receives back.

(2) In B of figure 87, a pure resistance of 100 ohms is connected across the generator. The current in the circuit is again 3 amperes, and since voltage and current are in phase, the phase angle is 0°. The apparent power is:

$P = EI = 300 \times 3 = 900$ watts.

The average power is:

$P = EI \cos \theta = 300 \times 3 \times \cos 0°$
$P = 900 \times 1 = 900$ watts.

Thus, the apparent power and the true power in an a-c resistive circuit are the same.

(3) In C of figure 87, an L-C-R circuit is shown with inductive reactance of 300 ohms, capacitive reactance of 100 ohms, and resistance of 100 ohms. The impedance of this circuit is:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (200)^2}$$
$$Z = \sqrt{50{,}000} = 224 \text{ ohms.}$$

The current in this circuit is:

$$I = \frac{E}{Z} = \frac{300}{224}$$
$$I = 1.34 \text{ amperes.}$$

The phase angle of this effectively inductive circuit may be determined from its cosine:

$$\text{Cos } \theta = \frac{R}{Z} = \frac{100}{224}$$
$$\text{Cos } \theta = .446$$
$$\theta = 63.4°.$$

The average power consumed by the circuit is:

$P = EI \cos \theta = 300 \times 1.34 \times .446$
$P = 179$ watts.

The apparent power in the circuit is:

$$P = EI = 300 \times 1.34$$
$$P = 401 \text{ watts.}$$

Thus, 179 watts of power are consumed in this circuit, but the generator must supply 401 watts, 222 watts of which are returned to the generator by the effective reactive element.

56. Power Factor

a. In reactive a-c circuits, the relative amounts of apparent power and average power are an important consideration from the point of view of efficiency and circuit design. In the circuits above, it may be noted that the average power differs from the apparent power by the factor of the cosine of θ. Thus, the cosine θ determines the percentage of apparent power consumed as true power. The cosine of θ then, which is the phase angle between current and voltage, and by extension a measure of the reactance present in the circuit, is called the *power factor* of the circuit. Then:

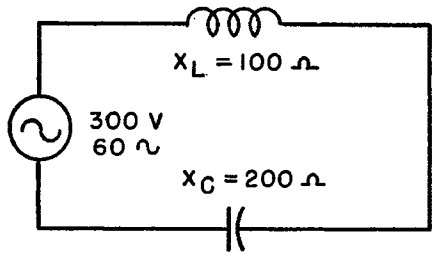
$$P.F. \text{ (power factor)} = \cos \theta.$$

The power factor tells at a glance the relative power dissipated.

b. The concept of power factor may be understood from another point of view. Since the cosine of θ is equal to the ratio R/Z, both numerator and denominator of this expression may be multiplied by I^2. Then:

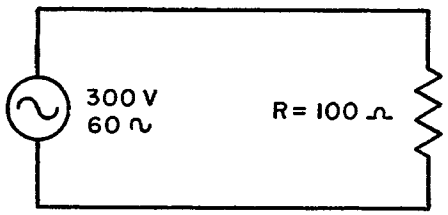
$$\text{Cos } \theta = \frac{I^2R}{I^2Z} = \frac{I^2R}{EI}.$$

However, I^2R is equal to true power, and either I^2Z or EI is equal to apparent power. Therefore, the cosine of θ is seen to be the ratio of true power to apparent power. This ratio is the power factor and is that decimal fraction between 0 and 1 which represents the amount of



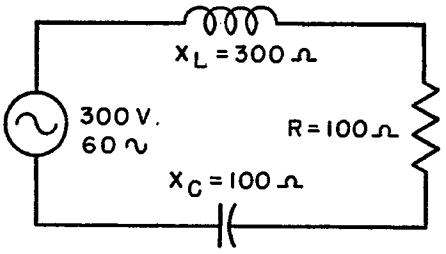
$P_{ap} = E\,I = 300 \times 3 = 900$ WATTS.
$P_{av} = E\,I \cos\theta = 300 \times 3 \times 0 = 0$ WATTS.

A



$P_{ap} = E\,I = 300 \times 3 = 900$ WATTS.
$P_{av} = E\,I \cos\theta = 300 \times 3 \times 1 = 900$ WATTS.

B



$P_{ap} = E\,I = 300 \times 1.34 = 401$ WATTS.
$P_{av} = E\,I \cos\theta = 300 \times 1.34 \times .446 = 179$ WATTS.

C

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Figure 87. Power relationships in a-c circuits.

power actually used when compared to the power flowing in the circuit. Thus:

$$P.F. = \frac{P\ (\text{average})}{P\ (\text{apparent})}.$$

c. It is important to realize that a power factor close to 1 is generally to be desired for all reactive a-c circuits using appreciable power. A low power factor means that there is a large discrepancy between the voltages and currents in the circuit and the small percentage of I and E needed to perform the work desired. The generator source and circuit elements, therefore, would have to be designed to produce and withstand the larger values. As the power factor approaches 1, the generator and circuit elements need be only slightly larger than their useful values. For example, in the circuit C of figure 87 above, the power factor is:

$$P.F. = \frac{P_{av}}{P_{ap}} = \frac{179}{401}$$

$$P.F. = .446 = \cos\theta.$$

The current through all elements is, as shown, 1.34 amperes. The voltage across the inductance E_L is equal to IX_L, and the voltage across the capacitance is equal to IX_C. Then:

$$E_L = 1.34 \times 300$$

$$E_L = 401 \text{ volts.}$$

And: $E_C = 1.34 \times 100$

$$E_C = 134 \text{ volts.}$$

It is interesting to note also that at resonance a special condition in respect to the power factor occurs: Since the reactive elements cancel each other, R is equal to Z, the phase angle is 0°, apparent power is equal to true power, and the power factor is equal to 1. However, because of resonance, the current in the circuit is maximum and accordingly the voltages across the reactive elements may be very high, a condition characteristic of circuit of low-power factor.

57. Summary

a. At any instant, the sum of the voltage drops around a closed circuit equals the applied voltage:

$$e = e_R \times e_L + e_C.$$

b. An a-c value of voltage or current is fully determined when its *effective* value and phase are known.

c. In a series L-R circuit, the voltages are 90° out of phase:

$$E = \sqrt{E_R^2 + E_L^2}$$

$$Z = \sqrt{R^2 + X_L^2}.$$

d. In a series R-C circuit, the voltages are 90° out of phase:

$$E = \sqrt{E_R^2 + E_C^2}$$

$$Z = \sqrt{R^2 + X_C^2}.$$

e. In a series L-C-R circuit, the voltages across the reactive elements are 180° out of phase and may be subtracted directly:

$$E = \sqrt{E_R^2 + (E_L - E_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

f. The voltage drop across a single reactive element in a series L-C-R circuit can have a greater *effective* value than the applied voltage:

$$E_L = I \times X_L$$

$$E_C = I \times X_C.$$

g. If X_L is greater than X_C, the circuit acts inductively and the current *lags* the applied voltage by the phase angle θ:

$$\text{Tan } \theta = \frac{X_L - X_C}{R}.$$

h. If X_C is greater than X_L, the circuit acts capacitively and the current *leads* the applied voltage by the phase angle θ:

$$\text{Tan } \theta = \frac{X_C - X_L}{R}.$$

i. If X_L is equal to X_C, the circuit is resistive and the current is in phase with the applied voltage.

j. If X_L is equal to X_C, the circuit is resonant, and Z is equal to R, E_L is equal to E_C, and current is at a maximum.

k. The voltage drops across the reactive elements in a resonant circuit are maximum and may be far in excess of the applied voltage.

l. The frequency at which an *L-C-R* circuit resonates is found by the formula:

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

m. In a parallel *L-R* circuit, the branch currents are 90° out of phase:

$$I_T = \sqrt{I_R^2 + I_L^2}$$
$$Z = \frac{RX_L}{\sqrt{R^2 + X_L^2}}.$$

n. In a parallel *R-C* circuit, the branch currents are 90° out of phase:

$$I_T = \sqrt{I_R^2 + I_C^2}$$
$$Z = \frac{RX_C}{\sqrt{R^2 + X_C^2}}.$$

o. In a parallel *L-C* circuit, the branch currents are 180° out of phase and may be subtracted directly:

$$I_T = I_L - I_C$$
$$Z = \frac{X_L \times X_C}{X_L - X_C}.$$

p. In a parallel *L-C* circuit, the current in either branch may be greater than the line current:

$$I_L = \frac{E}{X_L}$$
$$I_C = \frac{E}{X_C}.$$

q. In a parallel *L-C* circuit, if X_L is greater than X_C, I_C is greater than I_L, the circuit acts *capacitively*, and line current leads the applied voltage by the phase angle θ; if X_C is greater than X_L, I_L is greater than I_C, the circuit acts *inductively*, and line current lags the applied voltage by the phase angle θ.

r. In a resonant parallel *L-C* circuit, the branch currents are equal and cancel, line current approaches zero, and the impedance of the circuit approaches infinity.

s. A parallel *L-C-R* circuit not at resonance is either a parallel *L-R* or *R-C* circuit, depending on whether I_L is greater or less than I_C.

t. In a d-c circuit, power in watts is determined by Joule's Law:

$$P = I^2R$$
$$P = EI.$$

u. In an a-c circuit, power varies instantaneously with voltage and current. The graph of instantaneous power goes through 2 cycles during 1 cycle of voltage or current.

v. The a-c power curve has positive loops and negative loops:

(1) If the area under the positive loops is greater than the area under the negative loops, the circuit contains some resistance.

(2) If the area under the positive loops equals the area under the negative loops, the circuit contains purely reactive elements.

(3) If the area under the positive loops is less than the area under the negative loops, the external circuit contains a generator and is delivering power to the source.

w. Both inductors and capacitors are capable of storing energy with small loss and return almost as much power to the source as they absorb.

x. In an *L-C-R* circuit, the power which the generator must supply (regardless of the return) is called the *apparent* power:

$$P_{ap} = EI.$$

y. In an *L-C-R* circuit, the only power actually dissipated is the power absorbed by the resistance of the circuit, called the *average* or *true* power:

$$P_{av} = I^2R \qquad P_{av} = EI \cos \theta.$$

z. The cosine of θ is called the *power factor* of an a-c circuit and is the percentage of apparent power actually dissipated, or the ratio of apparent power to true power.

58. Review Questions

a. How does the addition of instantaneous voltage drops around an a-c circuit differ from the addition of the effective rms values of these voltage drops?

b. When is an a-c value of voltage or current fully determined?

c. Give the formula for the impedance of an *L-R* circuit; of an *R-C* circuit.

d. Draw a series *L-C-R* circuit. What is the formula for the impedance of this circuit?

e. What is the phase relationship between the voltages across the reactive elements? Between E_R and E_C?

f. What determines the voltage drop across any single element in an a-c circuit?

g. What condition causes an *L-C-R* circuit to act as an *L-R* circuit? As an *R-C* circuit?

h. How is the phase angle of an *L-C-R* circuit found?

i. What is the phase relationship between current and voltage in an *L-C-R* circuit when X_L is equal to X_C?

j. List the conditions present in a resonant *L-C-R* circuit.

k. Give the formula for determining the resonant frequency.

l. What is the phase relationship between the branch currents of a parallel *L-R* circuit? Of a parallel *R-C* circuit?

m. What is the formula for the impedance of a parallel *L-R* circuit? Of a parallel *R-C* circuit?

n. What is the phase relationship between the branch currents of a parallel *L-C* circuit?

o. Give the formula for the impedance of a parallel *L-C* circuit of negligible resistance.

p. How is the magnitude of the current in either branch of a parallel *L-C* circuit determined?

q. How is the magnitude of the line current of a parallel *L-C* circuit determined?

r. List the conditions present in a parallel *L-C* circuit if X_L is greater than X_C; if X_C is greater than X_L.

s. List the conditions present in a resonant parallel *L-C* circuit.

t. Give the formulas for determining power in a d-c circuit.

u. How does the graph of power in an a-c circuit differ from the graph of either current or voltage?

v. How are the negative loops of a power curve to be understood?

w. How is the net power delivered by a generator to an external circuit determined?

x. Describe in what manner capacitors and inductors return the power they absorb.

y. What is average or true power? Give its formula.

z. What is apparent power? Give its formula.

aa. What is power factor? Why is the cosine of θ considered the power factor?

CHAPTER 5

TUNED CIRCUITS AND RESONANCE

59. Tuned Circuits

a. In the preceding chapter, the special phenomenon of resonance was touched upon in the midst of the more detailed discussion of the fundamental properties of *L-C-R* circuits. In the present chapter, the various effects of resonant circuits will be considered at length, since this type of circuit and the principles controlling it find wide application throughout the practice of radio communications. Resonant circuits, or tuned circuits as they are generally called, are important because their action is such as to set up certain advantageous conditions which may be utilized for purposes of discrimination, or the separation of voltages at one frequency from those at another frequency. The resonant circuit offers to electricity the same sort of directed advantage that the lever and the wheel, for instance, offer to the science of mechanics. When a lever is used to raise some heavy object, the advantage of weight lifting is offset by the added distance through which the lever is moved. Thus, distance is sacrificed to gain power, or, by the use of gears, power may be sacrificed to gain speed. In like manner, tuned circuits may offer high voltages at certain desirable frequencies and low voltages at all other frequencies, or they may be used to short-circuit undesirable frequencies and allow all others to pass at high voltages. Thus, a certain electrical advantage may be gained from tuned circuits.

b. From the discussion in chapter 4 of series and parallel *L-C-R* circuits at resonance, it will be noted that the condition of resonance was characterized by the following facts:

(1) The inductive and capacitive reactances, being equal and opposite in direction, cancel each other. Thus:

$$X_L = X_C.$$

(2) The frequency of the circuit at resonance, f_r, is determined by solving the formula above for f. Thus:

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

(3) In a *series* resonant circuit, impedance is at a *minimum* and current at a *maximum,* since the only effective opposition in the circuit is the resistance R. Thus:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$Z = R.$$

(4) In a *parallel* resonant circuit, impedance *in the line* is at a *maximum* and current *in the line* is at a *minimum,* since the reactances act together in such a manner as to raise impedance. Thus:

$$Z_T = \frac{X_L X_C}{X_L + X_C}.$$

Then, as the sum of X_L and X_C approaches zero, the impedance in the line approaches a maximum.

60. The Series Tuned Circuit

a. The summary above records the conditions in a tuned circuit at the point of resonance; however, in order to understand the tuned circuit more fully it is necessary to consider the conditions existing in the circuit on either side of resonance. A of figure 88 is a representative network which may be solved for voltage, current, and impedance as the frequency of the circuit is varied through the resonant frequency. The a-c generator, whose frequency may be changed at will, delivers 300 volts rms across the 2-millihenry inductance, the 80-$\mu\mu$f-capacitor, and the 100-ohm resistance. The frequency of the generator is varied at intervals from 100 kilocycles to 600 kilocycles in order to determine the behavior of the circuit as it passes

in and out of resonance. As the frequency changes, the reactances of the circuit will change, and consequently total impedance and current will vary. The current in the circuit at the various positions of the generator may be determined by use of the now familiar formulas. Assuming the first generator frequency to be 100 kcs, then:

$$X_L = 2\pi fL = 6.28 \times 10^5 \times 2 \times 10^{-3}$$
$$X_L = 6.28 \times 2 \times 10^{-2}$$
$$X_L = 1{,}256 \text{ ohms.}$$

And:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 10^5 \times 8 \times 10^{-11}}$$
$$X_C = 19{,}890 \text{ ohms.}$$

Then X, the effective reactance, is $X_C - X_L = 19{,}890 - 1{,}256 = 18{,}634$ ohms. The impedance of the circuit may be found:

$$Z = \sqrt{R^2 + X^2}\ .$$

But since X is more than 10 times R, the total impedance may be taken (for practical purposes) to be that of the reactance value. Then:

$$Z = 18{,}634 \text{ ohms.}$$

And the current $I = \frac{E}{Z} = \frac{300}{18{,}634}$

$$I = .0161 \text{ ampere} = 16.1 \text{ ma.}$$

b. In like manner, the conditions in this circuit may be calculated for each frequency of the generator as it is varied up to 600 kc (kilocycles) in steps of 100 kc each. The table below lists the values of the separate reactances, the difference between them, the impedance, and the current in the circuit for each step. B of figure 88 is a graph of the variation in current

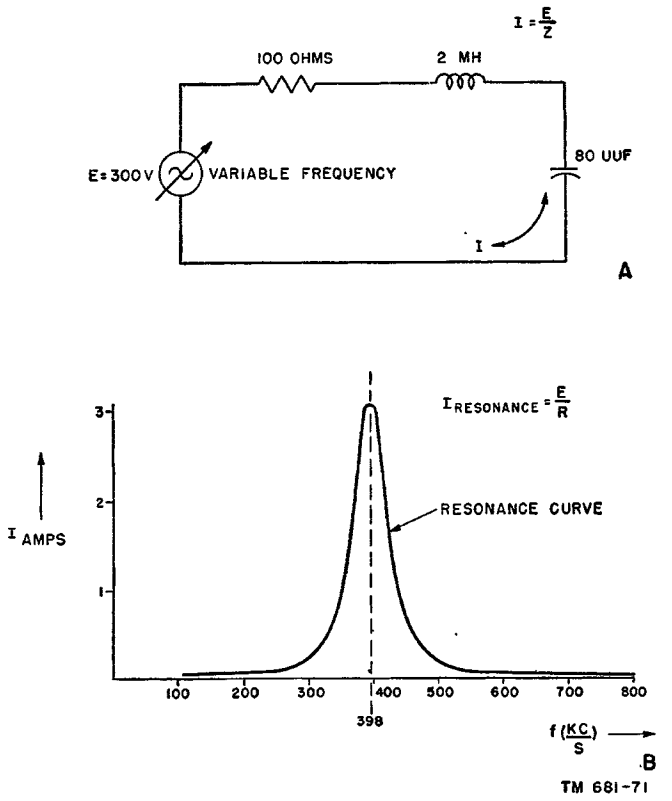


Figure 88. L-C-R circuit and resonance curve.

I plotted against frequency f. The following chart and the graph together clearly show that at the resonant frequency of 398,000 cps, impedance is a minimum and equal to R, the current is a maximum, and the reactances are equal. The current at resonance in the circuit illustrated above is 3 amperes, or 3,000 milliamperes, determined by Ohm's Law:

$$I = \frac{E}{R} = \frac{300}{100} = 3 \text{ amperes.}$$

Frequency kc/sec	X_L ohms	X_C ohms	X $(X_L - X_C)$	R ohms	Z ohms	E volts	I milliamperes
0	0	∞	∞	100	∞	300	0
100	1,256	19,890	18,634	100	18,634	300	16.1
200	2,512	9,945	7,433	100	7,433	300	40.0
300	3,768	6,630	2,862	100	2,862	300	104.8
350	4,400	5,670	1,270	100	1,270	300	236
*398	5,000	5,000	0	100	100	300	3,000
400	5,030	4,960	70	100	122	300	2,460
450	5,620	4,410	1,240	100	1,240	300	242
500	6,280	3,978	2,302	100	2,302	300	130
600	7,536	3,315	4,221	100	4,221	300	71

* Resonant frequency.

c. Thus, a resonant *L-C-R* circuit acts as a simple resistance circuit and has a current flow limited only by this resistance. However, it is important to note that the voltage across the reactive elements, although equal and opposite, may be very high and is determined by the current in the circuit multiplied by the reactance of the element. Thus:

$$E_L = IX_L = 3 \times 5{,}000$$
$$E_L = 15{,}000 \text{ volts}$$
$$E_C = 3 \times 5{,}000$$
$$E_C = 15{,}000 \text{ volts.}$$

Figure 89 shows the current and voltage readings to be observed on meters placed in the circuit of figure 88. It is apparent that the voltage across either L or C is 50 times the applied voltage, and that this reactive voltage depends on the current in the circuit, which in turn depends on resistance. Thus, a resonant circuit of very little resistance is capable of producing enormous voltages across the reactances. Certain communications circuits require such a high-voltage gain over a low-input voltage, and this type of resonant circuit offers this electrical advantage.

61. Frequency at Resonance

a. The condition of resonance in any circuit is determined initially by the equal and opposite reactances in the circuit. Since both inductive reactance and capacitive reactance depend directly upon the frequency of the applied voltage, the condition of resonance may be stated as:

$$2\pi fL = \frac{1}{2\pi fC}.$$

Then: $4\pi^2 f^2 LC = 1.$

And: $f^2 = \frac{1}{4\pi^2 LC}.$

Taking the square root of each side:

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

Since f is the frequency when X_L is equal to X_C, it is the frequency at resonance and may be written f_r. Then:

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

where: L = the inductance of the circuit in henrys,
C = the capacitance of the circuit in farads,
f_r — the resonant frequency of the circuit.

Thus, in the previous example, the resonant frequency might have been determined from the values of L and C alone, without reference to the graph of B of figure 88. As given, L is equal to 2 *mh* and C is equal to 80 $\mu\mu$f. Then:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28 \times \sqrt{2 \times 10^{-3} \times 8 \times 10^{-12}}}$$
$$= \frac{1}{6.28 \quad \frac{16}{10^{14}}} = \frac{10^7}{25.12}$$
$$= 398{,}000 \text{ cps.}$$

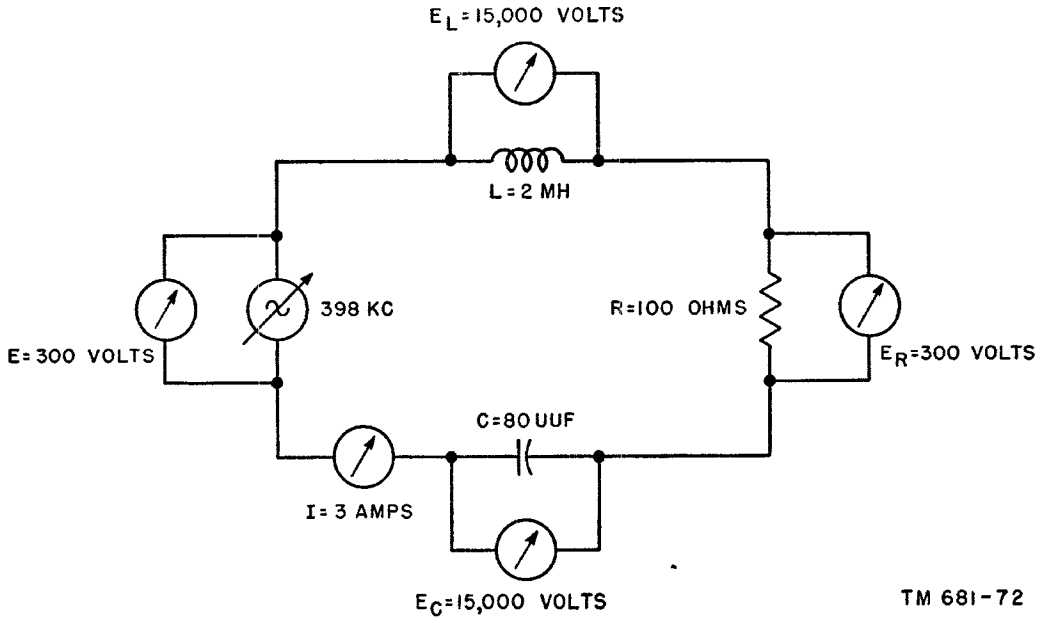


Figure 89. L-C-R circuit: voltage and current readings at resonance.

b. A further examination of the formula for frequency at resonance reveals that for any value of L and C there is only one definite resonant frequency. This frequency depends on the product of L and C alone, since all other factors are constant. Thus, various combinations of values of L and C may be used in a circuit to achieve resonance at a given frequency as long as the product of the two values is the same. A large value of L and a small value of C may resonate at the same frequency as a large value of C and a small value of L. For instance, in the example above, an inductance of .5 *mh* and a capacitance of 320 $\mu\mu$f resonates at the same frequency (398,000 cps) as the 2-*mh* L and the 80-$\mu\mu$f C used originally. The chart below illustrates this important fact by listing a series of values of L and C which when connected across a household circuit would resonate at the standard 60-cycle frequency.

Frequency cycles/sec	L henrys	C microfarads	$L \times C$ (any constant)
60	1	7	7
60	2	3.5	7
60	3	2.33	7
60	4	1.75	7
60	5	1.4	7
60	6	1.16	7
60	7	1	7
60	8	.875	7
60	9	.77	7
60	10	.70	7

c. It will have been noticed also that the values of L and C for resonance at 398,000 cps were relatively small compared with those which resonated at 60 cps. This fact is indicated by the formula, since as either L or C increases, the resonant frequency *decreases*, and as L or C decreases, the frequency *increases*. Thus, high-frequency tuned circuits generally use very small components, and low-frequency tuned circuits use relatively large coils and capacitors. Furthermore, if the value of either L or C is known, the value of the other component required for resonance may be computed from the formula.

Since:
$$f^2 = \frac{1}{4\pi^2 LC}$$
then:
$$L = \frac{1}{4\pi^2 f^2 C}$$
and:
$$C = \frac{1}{4\pi^2 f^2 L}.$$

62. Resonance Curves

a. In the calculations for frequency in the examples above, it will have been noted that the determination of frequency at resonance is independent of the resistance in the circuit. A circuit having 10,000,000 ohms resistance will have the same resonant frequency as a circuit having 1-ohm resistance, if the product of L and C is the same in both cases. The current in the circuit, however, is not independent of resistance. As noticed previously, the smaller the resistance of the circuit, the greater will be the value of the maximum current. In fact, if it were possible to design a circuit with zero resistance, the current at resonance would be infinitely large. In practice, the resistance is never zero, but it may be very small, and thus current may be high at resonance but always well within a limiting value.

b. The graph which shows how current varies in an *L-C-R* circuit as the frequency changes (voltage and resistance remaining fixed) is called a resonance curve. The characteristic shape of a typical resonance curve was shown in B of figure 88. The steepness of the slope of the curve as it approaches and leaves resonance, as well as the height of the maximum point above the reference line, is dependant on the resistance in the circuit. Figure 90 shows some typical resonance curves for a circuit having the same L and C values but different values of resistance. The difference in the peak values for each of the curves is due to the different values of resistance in the circuit. Notice, too, that as the resistance R increases, the response curves become flatter and broader about the resonant frequency. If the resistance in a resonant circuit is too large, the circuit loses its utility as a frequency selector; this means that little discrimination in amount of current flow (hence voltage) is made between frequencies at resonance and those not at resonance. Thus, the

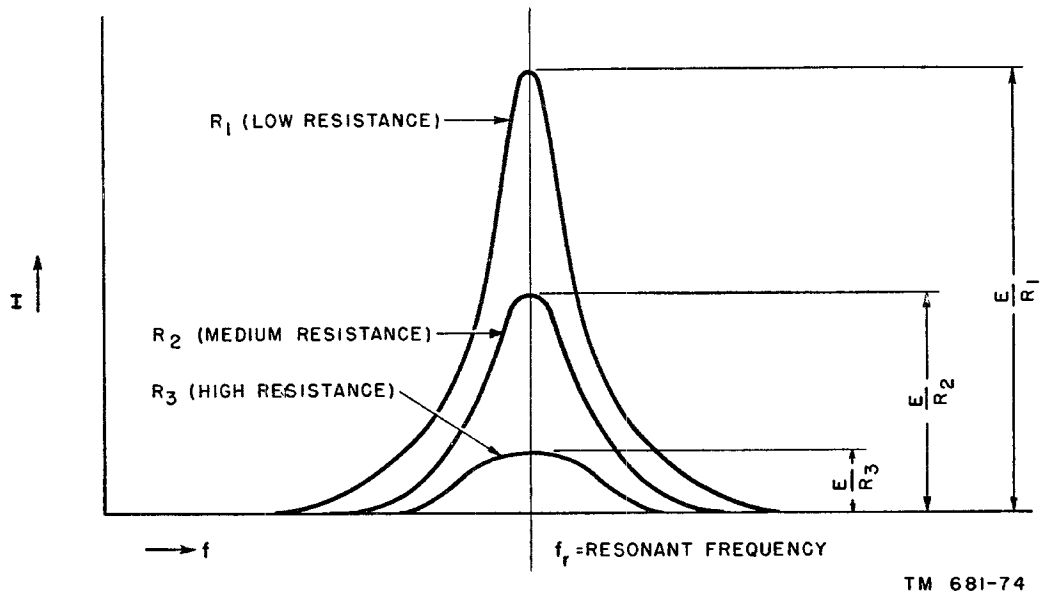


Figure 90. Typical resonance curves.

initial selective frequency advantage of a resonant circuit is lost.

63. Q of a Series Tuned Circuit

a. In the circuit of figure 89, the voltages appearing across the reactances at resonance are 15,000 volts as compared with the input voltage of 300, which is also the voltage appearing across the resistance. This high voltage is directly dependent on the current in the circuit, which is in turn dependent on the applied voltage and the resistance. A comparison, then, of the voltage across one of the reactances to the applied voltage gives an immediate indication of the *quality* of a resonant circuit. Thus, a large difference in these voltages would indicate little resistance in the circuit, high current, and a steep, narrow resonance curve. This ratio of the voltages is termed the Q of a circuit and tells at a glance to what extent the special effects which are characteristic of resonance are present. The Q of a series tuned circuit, then, is defined as the ratio, at resonance, of the voltage across the inductance E_L to the applied voltage E, or of the voltage across the capacitance E_C to the applied voltage E. Since E_L is equal to E_C at resonance, either voltage may be used to compute Q. Thus:

$$Q = \frac{E_L}{E} = \frac{IX_L}{IR}$$

$$Q = \frac{X_L}{R} = \frac{2\pi fL}{R}.$$

Referring to the circuit of figure 88 and its table of values:

$$Q = \frac{15{,}000}{300} = 50$$

or:

$$Q = \frac{5{,}000}{100} = 50.$$

b. The preceding expressions for Q indicate that Q varies inversely with the resistance in the circuit; the lower the resistance, the higher the Q will be. The curves of figure 90 indicate that the smaller the resistance in the circuit, the greater will be its frequency discrimination. Thus, it will be seen that Q is a measure of the ability of a resonant circuit to select or reject a band of frequencies. The higher the Q of a series resonant circuit, the greater will be its value as a frequency selector—that is, the narrower will be the band of frequencies showing a voltage gain at resonance.

64. Bandwidth

a. The bandwidth of a circuit is the total number of cycles above and below the resonant frequency which realize practically the same voltage gain as the resonant frequency itself. The width of this band is also called the band pass of the circuit. The effective limits of the band pass are taken to be the points on the resonance curve corresponding to .707 of the peak voltage or current, whichever is plotted. In figure 91, the shaded area represents the

band of frequencies for which the current is greater than .707 of the peak current. Note that one half of the band lies above the resonant frequency (f_r to f_2) and the other half of the band lies below the resonant frequency (f_r to f_1).

b. The width of the band pass may always be found from the resonance curve. However, since the Q of a circuit determines the over-all width of the resonance curve, the band pass may also be found in terms of the resonant frequency and the Q of the circuit. Thus:

$$\text{Band pass (cps)} = \frac{f_r}{Q}.$$

From this formula, it can be seen that the higher the Q, the smaller the band pass, and, conversely, the smaller the Q, the greater the band pass. This result agrees with the previous conclusions regarding low resistance in series resonant circuits and its effect on the voltage gain of frequencies at resonance over those near to, but not at, resonance.

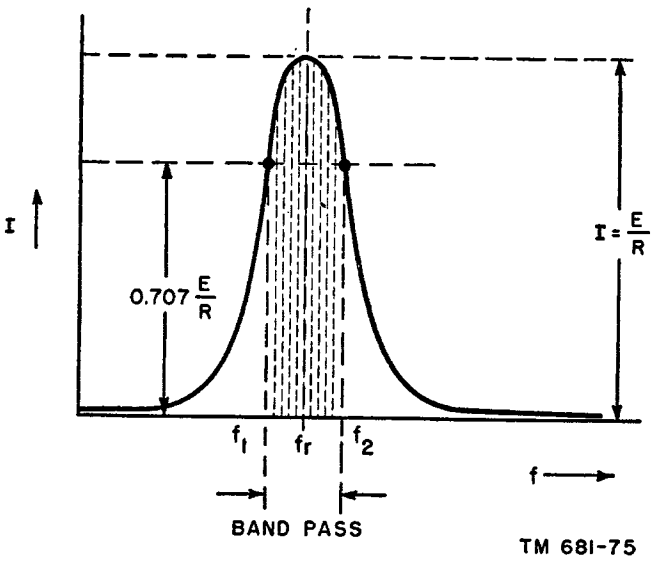


Figure 91. Bandwidth of a series resonant circuit.

65. Analysis of Series Tuned Circuits

a. A of Figure 92 illustrates the schematic of a series L-C-R circuit of 4-millihenry inductance L, 250-micromicrofarad capacitance C, and 50-ohm resistance R. A 30-volt rms a-c generator of variable frequency is applied to the network. The resonant frequency of this circuit is:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{4\times10^{-3}\times25\times10^{-11}}}$$
$$= \frac{1}{6.28\sqrt{\frac{100}{10^{14}}}} = \frac{10^6}{6.28}$$
$$= 15{,}000 \text{ cps.}$$

The current at resonance is:

$$I = \frac{E}{R} = \frac{30}{50} = .6 \text{ ampere.}$$

The Q of the circuit is:

$$Q = \frac{X_L}{R} = \frac{2\pi fL}{R} = 6.28\times159\times10^3\times4\times10^{-3}$$
$$Q = \frac{4{,}000}{50} = 80.$$

The band pass of the circuit is:

$$\text{Band pass} = \frac{f_r}{Q} = \frac{159{,}000}{80}$$
$$= 1{,}990 \text{ cycles.}$$

The resonance curve is shown in B of figure 92, illustrating the band-pass limits and characteristic shape determined by the Q of the circuit. Thus, if greater bandwidth should be desired, the Q could be lowered, and the curve broadened simply by adding resistance to the circuit.

b. In the circuit above, a 5-millihenry inductance is substituted for L. Then, in order to maintain resonance at 159,000 cps, the capacitor C must change. The new value of C may be determined from the formula for frequency at resonance. Thus:

$$C = \frac{1}{4\pi^2 f^2 L}$$
$$= \frac{1}{4\times9.86\times159\times10^3\times159\times10^3\times5\times10^{-3}}$$
$$C = 200\ \mu\mu\text{f.}$$

In changing the values of L and C, however, the Q of the circuit has been changed, since Q depends on the ratio of reactance to resistance. The reactances are equal and opposite, as expected, but of different value. Then:

$$Q = \frac{2\pi fL}{R} = \frac{5{,}000}{50}$$
$$Q = 100.$$

In raising the Q of this circuit, the bandwidth narrowed:

$$\text{Band pass} = \frac{f_r}{Q} = \frac{159{,}000}{100} = 1{,}590.$$

Conversely, if L is lowered and C increased, then both X_L and X_C decrease, the Q of the circuit falls, and bandwidth is increased. In general, it may be said then that a high L/C ratio is desirable if a wide bandwidth is not the primary

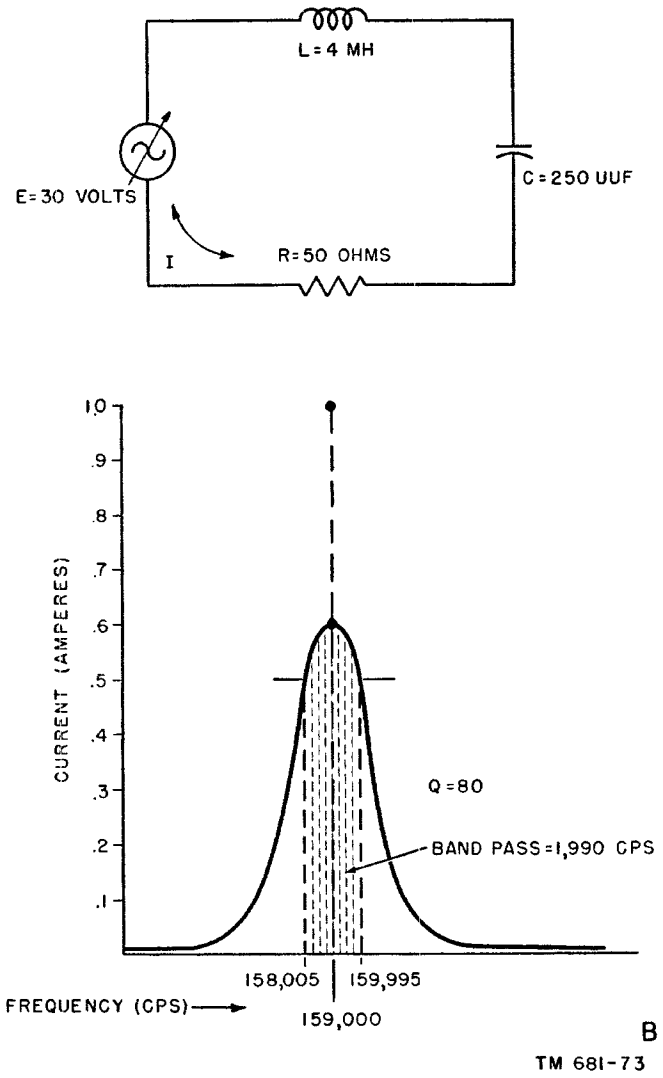


Figure 92. Series tuned circuit and resonance curve.

consideration, since Q and hence the gain of the circuit are increased.

c. In figure 93 is shown a series tuned circuit as it appears in an intermediate stage of a radio receiver. This circuit is designed to resonate at 455 kilocycles with a bandwidth of 10 kc. The resistance of the wire in the inductance is shown as the separate resistance R (30 ohms). Since the input voltage of the circuit is induced in the coil, it may be thought of as an a-c generator in the circuit. The resonant high-voltage gain, which appears as a voltage across the capacitor, is delivered to the vacuum tube for further amplification. The essential determining factors in this circuit are its resonant frequency, band pass, and resistance. Therefore, the Q of the circuit is fixed:

$$Q = \frac{f_r}{\text{band pass}} = \frac{455}{10}$$

$$Q = 45.5.$$

Since R is fixed, and since Q is equal to $\frac{X_L}{R}$, then X_L is fixed:

$$X_L = QR = 45.5 \times 30$$

$$X_L = 1{,}365 \text{ ohms.}$$

The value of the inductance L is determined by X_L. Thus:

$$L = \frac{X_L}{2\pi f} = \frac{1{,}365 \times 10^{-3}}{6.28 \times 455 \times 10^3}$$

$$L = .478 \text{ millihenry.}$$

Since X_C is equal to X_L, then $X_C = 1{,}365$ ohms

$$C = \frac{1}{2\pi f X_C} = \frac{1}{6.28 \times 455 \times 10^3 \times 1365}$$

$$C = 256\ \mu\mu\text{f.}$$

These values for L and C may be checked to determine their resonant frequency. Thus:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{\frac{.478 \times 25.6}{10^{14}}}}$$

$$f_r = \frac{107}{22} = 455{,}000 \text{ cps.}$$

Thus, a series tuned circuit consisting of an inductance of .478 millihenry, a capacitance of 256 micromicrofarads, and a resistance of 30 ohms has a resonant frequency of 455 kc and a bandwidth of 10 kc. B of figure 93 shows the resonance curve. A larger resistance will not change the resonant frequency, but it will increase the bandwidth and decrease the Q of the circuit.

66. Some Applications of Series Tuned Circuits

a. The principles of series resonant circuits considered above offer many interesting applications in communications work. In A of figure 94, the output of the L-C-R network is the voltage appearing across the resistance R. The input is a variable frequency generator. At resonance, the impedance of the circuit is a minimum, current in the resistance is a maximum, and the output voltage a maximum. For frequencies off resonance, the current falls off from maximum, as shown by the current versus frequency curve. The output voltage accordingly falls off in the same manner. Such a circuit may be used to pass a narrow band of frequencies and to discriminate against all others. In B of figure 94, the same series resonant circuit is rearranged to suppress a narrow band of frequencies and to pass all others. The resistance

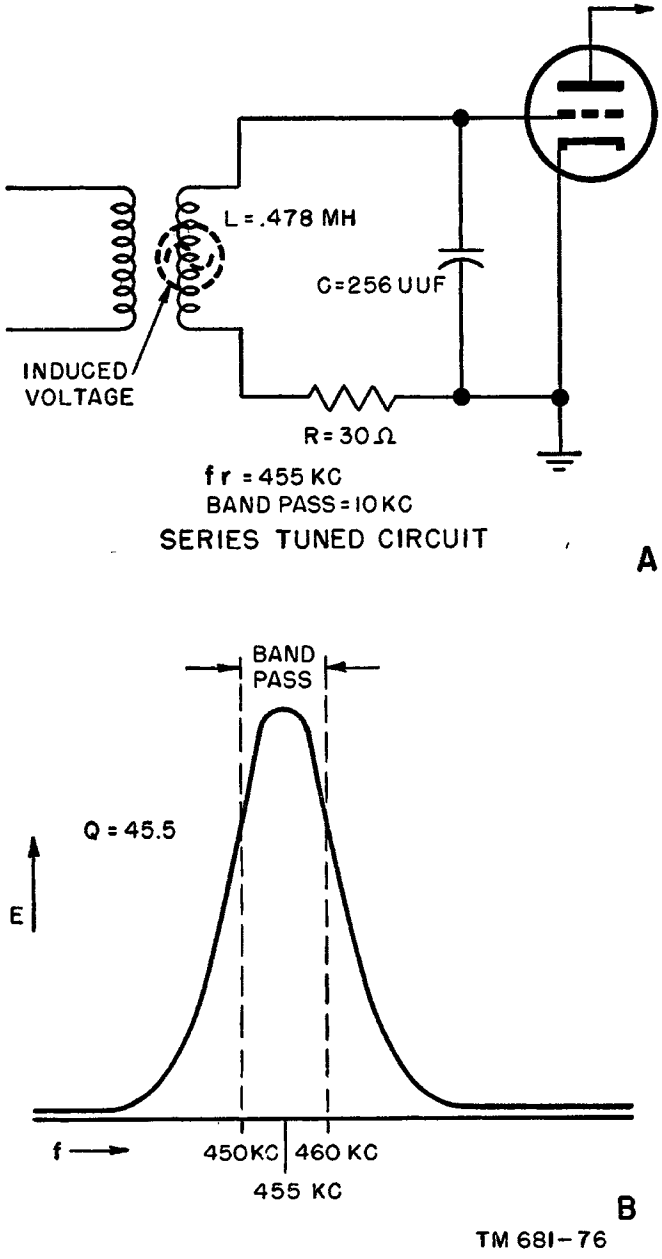


Figure 93. Series tuned circuit and band pass.

R prevents current in the circuit from becoming too high. At resonance, the voltages appearing individually across L and C are opposite in direction and cancel each other across the output resistance R_o. At all other frequencies, the voltages across L and C do not cancel completely and the difference voltage appears across the output resistance. Such a circuit may be used in band-rejection networks. In C of figure 94, another application of a series resonant circuit is shown. In this case, use is made of the fact that at resonance the voltage across the inductor or the capacitor is very large compared to the applied voltage. The voltage across the output resistance R_o varies as the voltage across the capacitor, thus passing a band of frequencies at and near resonance and suppressing all others.

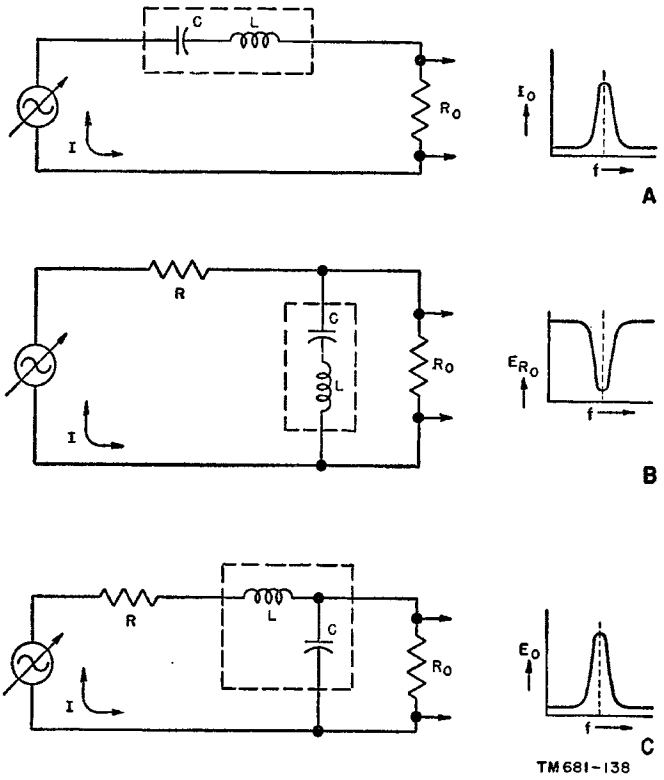


Figure 94. Applications of series resonant circuits.

b. Another excellent example of the use of a tuned series circuit is the first radio-frequency stage of a radio receiver. Figure 95 is a representative schematic of such a circuit. The antenna of a radio receiver is cut by the electromagnetic waves from a radio transmitting station and thus has an emf induced in it. The frequency of the induced voltage is the same as that radiated by the broadcasting station. This induced voltage produces an ac in the primary of the transformer which couples the antenna to the first stage of the receiver. This current creates a changing magnetic field, which links the turns of the inductance in the tuned circuit. Thus, an emf is induced in the inductance of the tuned circuit, and this emf can be replaced by an equivalent a-c generator of effective value E. If the capacitance in the tuned circuit is varied until the resonant frequency is the same as the frequency of this emf, then the current I will be a maximum. Since the voltage across the capacitor is equal to $Q \times E$ (applied), and a properly designed circuit has a high Q, the voltage across

the capacitor terminals c-d in figure 95 is considerably greater than the voltage induced in the antenna. The normal signal voltage picked up by the antenna is of the order of microvolts, and, if a usable signal is to be obtained, this small voltage must be amplified many times by vacuum tubes and a number of resonant circuits.

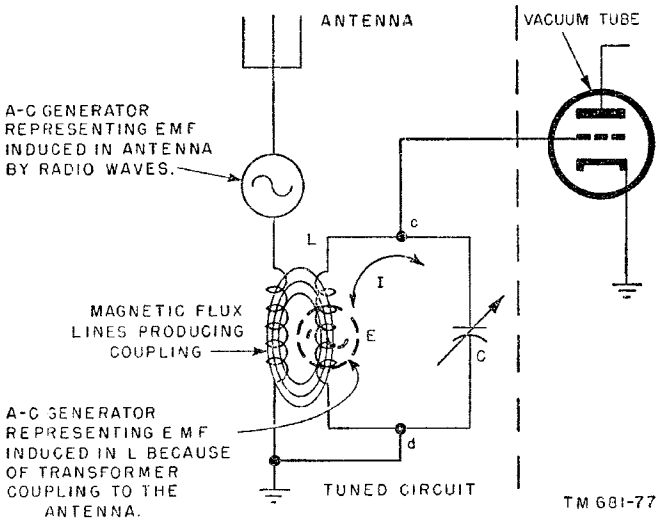


Figure 95. Series tuned radio-frequency circuit.

67. The Parallel Tuned Circuit

The phenomenon of resonance, which was described above in terms of series circuits, also lends itself to analysis in terms of parallel circuits. However, it is important to note that the so-called parallel resonant circuit is essentially the same circuit that has been analyzed above, but its application is such as to reveal a different set of operating conditions. Therefore, in order to avoid confusion and to keep the distinction between series and parallel circuits quite clear, the *series* resonant circuit is defined as one in which the signal *orginates within* the resonant circuit (A of fig. 96) ; and the *parallel* resonant circuit is defined as one in which the signal *originates outside* of the resonant circuit (B of fig. 96). The term *tank circuit* is also used commonly in connection with resonant circuits, both series and parallel. The name *tank* is derived from the ability of an L-C combination to store energy, as will become evident in later discussions. In general, any resonant L-C combination is called a tank, especially when the reactances are shown connected without the signal or the resistance in the circuit being shown (C of fig. 96). Thus, in figure 95, the tuned circuit is a series resonant circuit because its input voltage is induced in the coil, and may be thought of as an a-c generator in series with L and C.

68. Parallel Resonance: the Ideal Tank Circuit

a. A of figure 97 is the schematic diagram of a parallel-resonant circuit. The resistances R_L and R_C are shown because the inductor and the capacitor always have some resistance associated with them in practical circuits. Ordi-

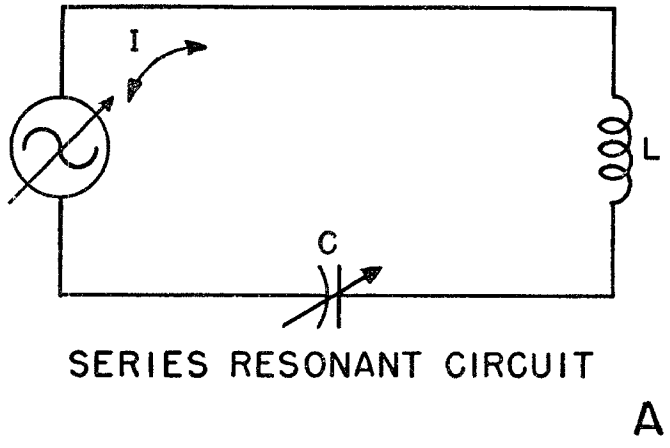
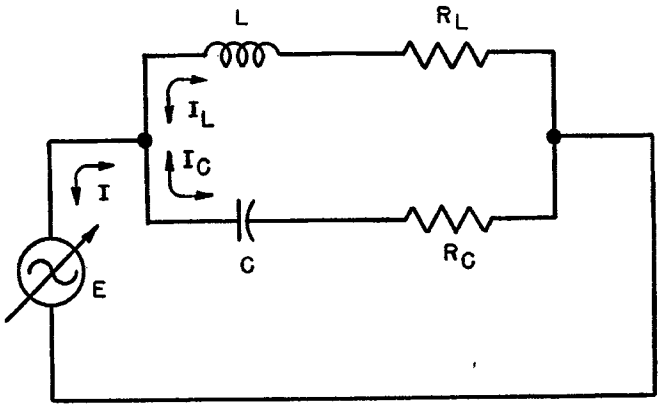


Figure 96. Series and parallel tuned circuits.

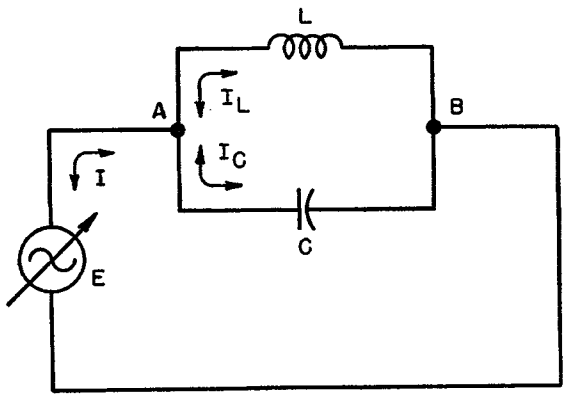
narily R_C is much smaller than R_L. In the ideal tank circuit (a theoretical instance only), R_L and R_C are assumed to be zero. B of figure 97 is the schematic of an ideal tank circuit. An a-c voltage of rms value E and frequency f is applied across a combination of L and C. C of figure 97 is the vector diagram of this circuit. The applied voltage E appears across both the inductance and the capacitance, and is accordingly taken as the reference vector.



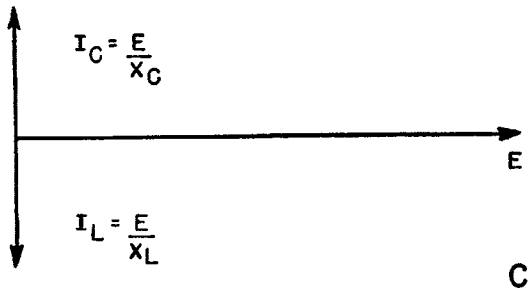
R_L = RESISTANCE IN INDUCTIVE BRANCH

R_C = RESISTANCE IN CAPACITIVE BRANCH

A



B



C

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Figure 97. Ideal tank-circuit and vector diagram.

b. By Kirchhoff's Law, the current flowing into the junction A (B of fig. 97) must equal the sum of the currents leaving A; therefore, the current I delivered by the generator is equal to $I_L + I_C$. Furthermore, the current through the inductance has the value E/X_L and lags E by 90°; the current through the capacitor has the value E/X_C and leads E by 90°. Thus, these two currents are 180° out of phase in respect to the applied voltage, and partial or complete cancellation will take place in the line, depending on the amount of each of the branch currents. If the capacitive reactance X_C is less than the inductive reactance X_L, the circuit acts capacitively, since the current I_C will be greater than the current I_L. If the inductive reactance X_L is less than the capacitive reactance X_C, the inductive current I_L will be greater than the capacitive current I_C, and the circuit will act inductively. However, when X_L is equal to X_C—the same condition which produced resonance in the series circuit—the inductive and capacitive currents are equal, the branch currents cancel in the line, and the line current in the ideal circuit is zero. Thus, an ideal parallel resonant circuit presents infinite impedance to the line and acts as an open circuit. A of figure 98, is the vector diagram illustrating the capacitive circuit (I_C greater than I_L); B the vector diagram for the inductive circuit (I_L greater than I_C); and C the vector diagram for resonance (I_L equal to I_C).

c. The third case above, in which the parallel tuned circuit acts as an open circuit, is the one which is the most interesting. Despite the fact that a non-zero voltage has been applied by the generator to the tank, the current delivered by the generator is zero. This can be interpreted to mean that the ideal parallel resonant circuit presents infinite opposition or impedance to the source when the condition $X_L = X_C$ is satisfied. Previously, it was found that this equation yields the frequency at resonance:

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

Thus, the resonant frequency of an L-C circuit is the same whether it be connected in series or in parallel. Sometimes, however, the term *anti-resonant frequency* is applied to a parallel circuit, resonant frequency being restricted solely to the series circuit. In any case, a given L-C

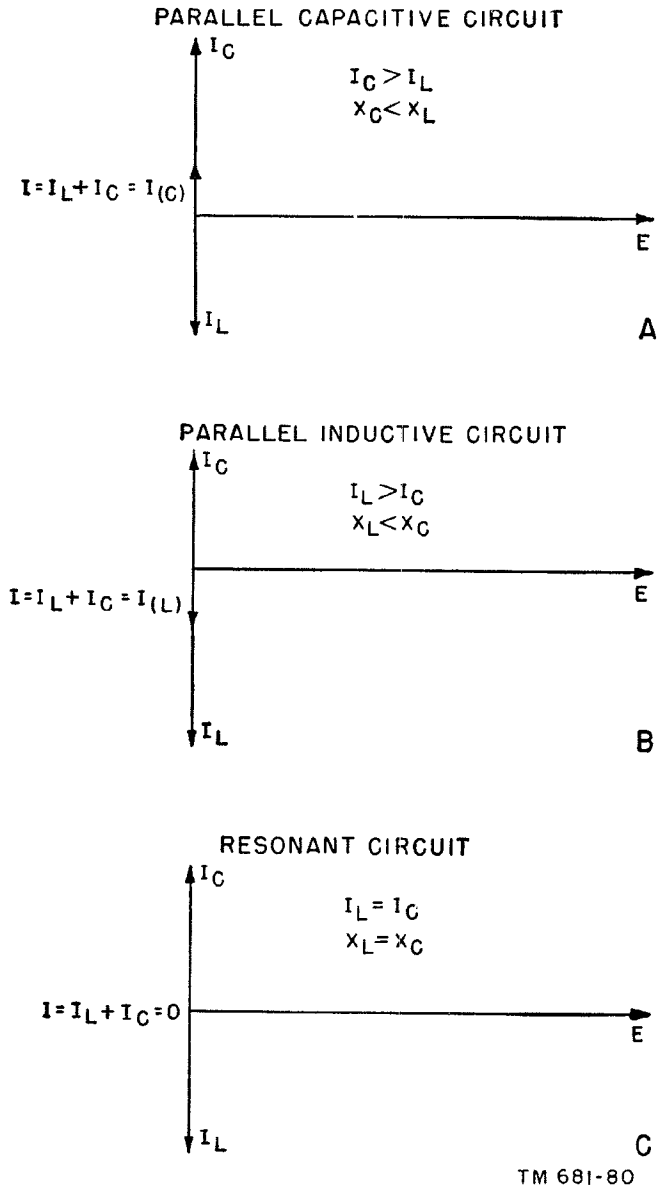


Figure 98. Vector diagrams of parallel L-C circuits.

combination has *one* resonant frequency, whatever name is applied to it.

69. Current in the Ideal Tank Circuit

a. An analysis of current present in the ideal parallel tank circuit is instructive, although the circuit is not practicable. Such a device proves useful in setting up limiting conditions for comparison with practical circuits. A of figure 99 is the schematic of an ideal tank circuit in which three meters have been placed to measure the currents in the inductive branch, in the capacitive branch, and in the line. The frequency of the applied voltage is variable. At frequencies much lower than the resonant frequency, X_L is much less than X_C, and the meter in the inductive branch will show a value almost equal to the current in the line; whereas, at frequencies much higher than the resonant frequency, X_C is much less than X_L and the meter in the capacitive branch will show a value almost equal to the current in the line. At the resonant frequency, X_L is equal to X_C, the meter reading in the inductive branch is the same as that in the capacitive branch, and the meter in the line reads zero. This last fact is illustrated by the graphs of B of figure 99, which show that at any and every instant the branch currents are equal in magnitude and that their resultant current is zero.

b. The following generalizations may now be made about an *ideal* parallel tank circuit:

(1) At zero frequency, the inductance behaves as a short circuit (X_L is zero), the capacitor behaves as an open circuit (X_C is infinite), and the impedance offered to the line is zero.

(2) At infinite frequency, the capacitor behaves as a short circuit (X_C is zero), the inductance behaves as an open circuit (X_L is infinite), and the impedance offered to the line is zero.

(3) At resonance, the inductance and capacitor individually present finite reactance to the line (X_L and X_C neither zero nor infinite), the reactive currents are 180° out of phase, and the impedance offered to the line is infinite.

c. In the discussions above, attention has been directed to the currents *in the line* and in the individual branches. It is important to realize, however, that although line current is zero at resonance, the current in the capacitor, and the current in the inductance are the *same current;* that is, the reactive currents are *in phase* in the tank itself and circulate from capacitor to inductance. This reciprocating action is like the harmonic motion of a pendulum. Once set in motion, a pendulum stores energy as it moves to one side and this energy is given back as it returns. Thus, the capacitor (once charged) delivers energy to the inductance, which stores the energy and returns it to the capacitor. In an ideal tank circuit, this reciprocity of energy would continue indefinitely. The current arrows of C of figure 99 help one to visualize this action. It will be seen that the currents set up by the applied voltage in *L* and *C* at any given instant

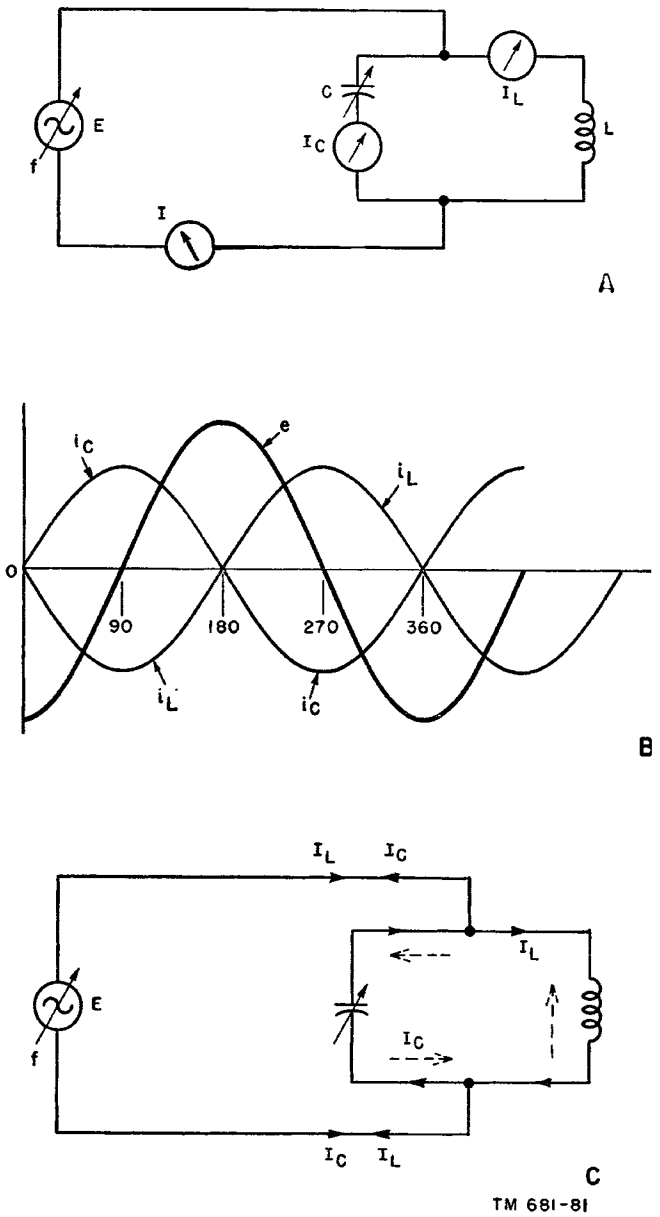


Figure 99. Ideal parallel resonant tank circuit.

are opposite in direction in respect to the line and cancel each other. Within the tank itself, which may be considered a *series* circuit once the capacitor has been charged, the currents flow together first in one direction and then in the other. The dotted arrows show the current in the tank in alternate half-cycles. Thus, within an ideal tank circuit, current at resonance oscillates perpetually; off resonance, the difference between the branch currents is the current in the line.

d. As frequency varies through resonance, the current in the line varies through zero. This variation may be shown by a resonance curve which is similar in shape to the series resonant curve but inverted as to position. Figure 100 shows series and parallel resonance curves superimposed one upon the other. The circuits are considered to be ideal; that is, the resistance is zero, resulting in infinite current at series resonance instead of a finite amount, and resulting in zero current at parallel resonance.

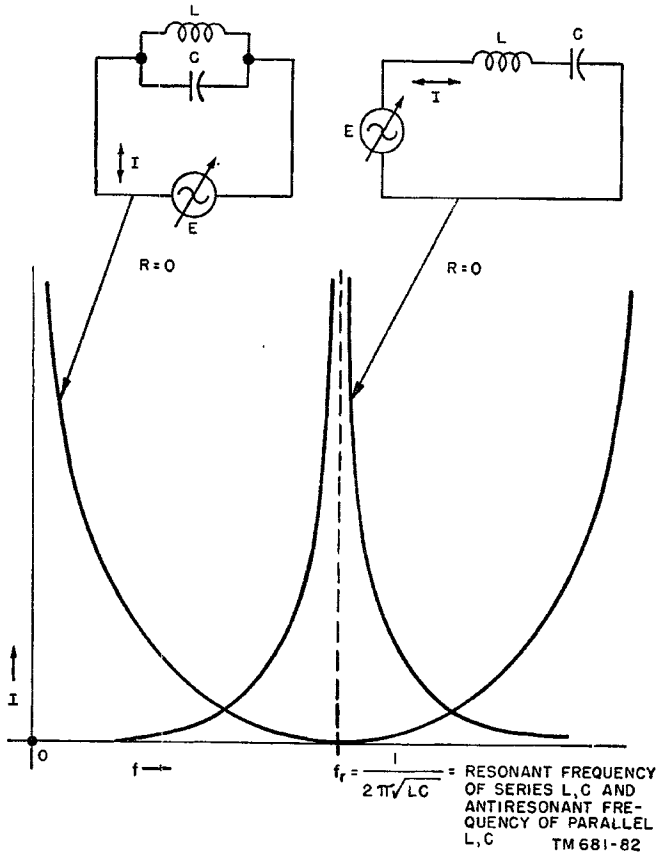


Figure 100. Resonance curves for ideal parallel and series circuits.

70. Impedance and Reactance in the Ideal Parallel Tank Circuit

a. In the series L-C circuit previously discussed, current is everywhere the same and voltages across any element may be determined simply by Ohm's Law for a-c circuits. In the parallel-resonant circuit, the voltage is everywhere the same, and currents in the separate branches and in the line are determined by the impedances present in the branches (taken separately) and by the total impedance to the line. Thus, current in the inductive or capacitive branch at any moment is found:

$$I_L = \frac{E}{X_L}$$

and:

$$I_C = \frac{E}{X_C}.$$

The total line current I is found by Ohm's Law:

$$I = \frac{E}{Z}.$$

Moreover, as noted above, the total line current is equal to the sum of the branch currents (added vectorially):

$$I = I_L + I_C.$$

Since these currents are 180° out of phase and X_C is conventionally taken as negative, then:

$$I = I_L - I_C.$$

Then:

$$\frac{E}{Z} = I_L - I_C.$$

And:

$$Z = \frac{E}{I_C - I_C} = \frac{E}{\frac{E}{X_L} - \frac{E}{X_C}} = \frac{E}{\frac{E(X_C - X_L)}{X_L X_C}}$$

$$Z = \frac{X_L X_C}{X_C - X_L}.$$

b. The impedance of a parallel *L-C* circuit differs from that of a series *L-C* circuit in one important respect, the change of sign affecting X_C and X_L. A greater inductive reactance in a series circuit causes the circuit to act *inductively;* whereas a greater inductive reactance in a parallel circuit causes the circuit to act *capacitively,* since more current flows in the capacitive branch. Thus, an examination of this formula for impedance reveals the following facts:

(1) If X_C is greater than X_L, the denominator of the expression is positive; thus, Z is positive, and the circuit behaves inductively.

(2) If X_C is less than X_L, the denominator of the expression is negative; thus, Z is negative, and the circuit behaves capacitively.

(3) If either X_L or X_C is equal to zero, then Z is zero.

(4) Because both X_L and X_C depend on frequency, the impedance of the circuit will also vary with frequency.

c. A graph of the variation of impedance of the ideal parallel tank circuit as frequency changes and it passes through resonance is shown by the reactance curve of figure 101. This curve illustrates clearly the following points:

(1) At zero frequency, the impedance is zero.

(2) At resonance, the impedance is infinite.

(3) As frequency increases beyond the resonant frequency, the impedance approaches zero.

(4) The circuit behaves like an inductance for frequencies below the resonant frequency, and behaves like a capacitance for frequencies above the resonant frequency.

The preceding points indicate that the tank circuit is very versatile. It can be used to simulate the properties of either a capacitor or an inductor.

d. For example, figure 102 is the schematic of a parallel *L-C* circuit of 4-millihenry inductance and 40-micromicrofarad capacitance; resistance is present in this circuit, but the amount is negligible. The variable frequency generator delivers 300 volts rms. The frequency at resonance is determined:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{4 \times 10^{-3} \times 4 \times 10^{-11}}}$$

$$f_r = \frac{1}{6.28\sqrt{\frac{16}{10^{14}}}} = \frac{10^7}{25.12}$$

$$f_r = 398{,}000 \text{ cps.}$$

The current in either branch is determined by the reactance of that branch. Since I_L is equal to I_C, either reactance may be used:

$$X_L = 2\pi fL = 6.28 \times 398 \times 10^3 \times 4 \times 10^{-3}$$

$$X_L = 10{,}000 \text{ ohms.}$$

Then:

$$I_L = \frac{E}{X_L} = \frac{300}{10{,}000}$$

$$I_L = .03 \text{ ampere.}$$

Thus, a current of .03 ampere circulates in the tank, but the current in the line is practically zero, since, as has been shown, the resonant tank offers a maximum impedance to the line.

e. If the frequency of the generator is changed to 200,000 cps, the currents in the branches differ:

$$X_L = 2\pi f L = 6.28 \times 2 \times 10^5 \times 4 \times 10^{-3}$$

$$X_L = 5{,}024 \text{ ohms.}$$

Then:

$$I_L = \frac{E}{X_L} = \frac{300}{5{,}024}$$

$$I_L = .059 \text{ ampere.}$$

And:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 2 \times 10^5 \times 4 \times 10^{-11}} = \frac{10^6}{50.24}$$

$$X_C = 19{,}900 \text{ ohms.}$$

Then:

$$I_C = \frac{E}{X_C} = \frac{300}{19{,}900}$$

$$I_C = .015 \text{ ampere.}$$

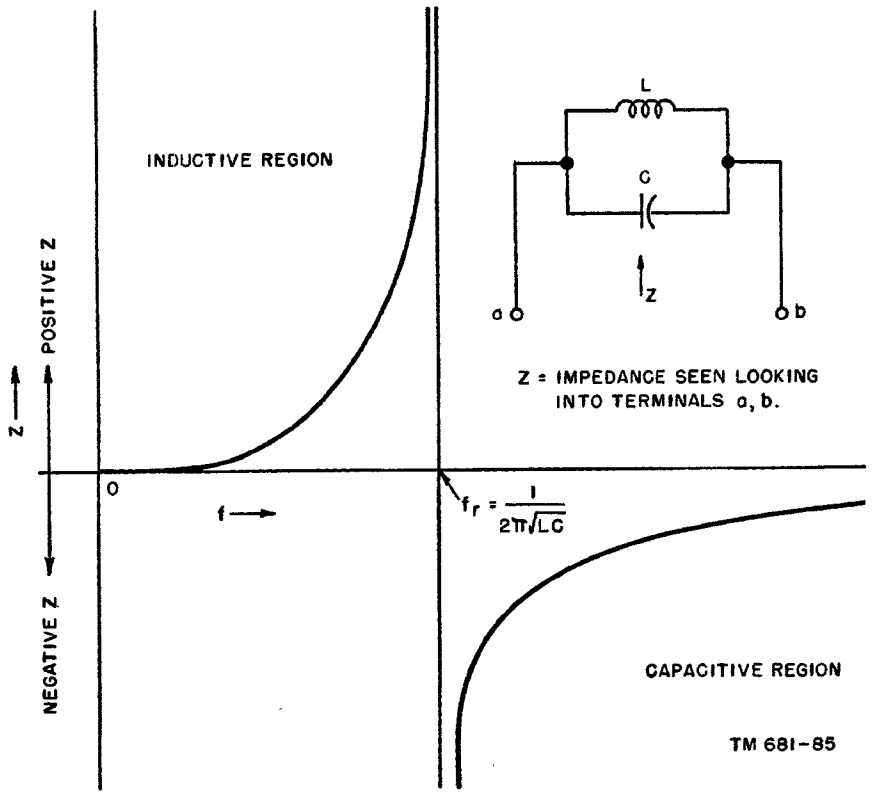


Figure 101. Reactance chart for ideal parallel tank circuit.

Since the inductive current is greater than the capacitive current, the circuit behaves as an inductance. The line current is:

$$I = I_L - I_C = .059 - .015 = .044 \text{ ampere.}$$

Thus, the current in the line is 44 milliamperes, lagging the applied voltage by 90°. B of figure 102 is the vector diagram of this circuit at a frequency of 200,000 cps (resistance negligible). Using the formula for impedance:

$$Z = \frac{X_L X_C}{X_C - X_L} = \frac{5{,}024 \times 19{,}900}{19{,}900 - 5{,}024} = \frac{10^8}{14{,}876}$$

$$Z = 6{,}720 \text{ ohms.}$$

Since the sign of Z is positive, the circuit behaves inductively. Then:

$$I = \frac{E}{Z} = \frac{300}{6{,}720} = .044 \text{ ampere.}$$

These results check with the findings above: The line current is 44 milliamperes, lagging the applied voltage by 90°.

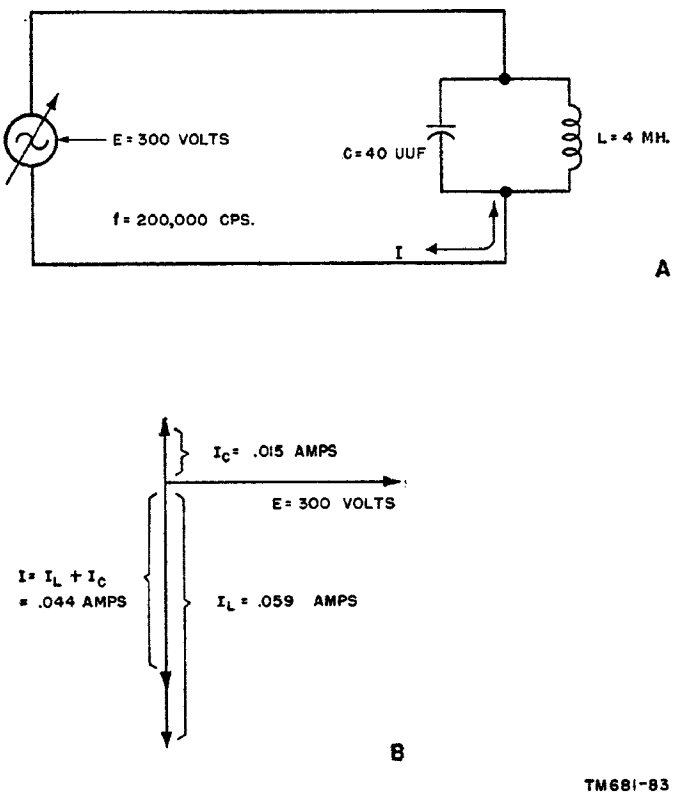


Figure 102. Parallel L-C circuit and vector diagram.

71. Q of a Parallel Resonant Circuit

a. The conclusions reached in the discussion of the ideal tank circuit, and the results of the circuit analysis above, were based on the assumption of zero resistance or negligible resistance in the parallel branches. A of figure 103 shows an equivalent schematic diagram of a practical circuit. R_L and R_C are the resistances

in the inductive and capacitive branches, respectively. The magnitude of these resistances determines the *quality* or Q of the resonant circuit, as was shown to be true of the series circuit. The presence of resistance in either or both branches of the parallel circuit means that the branch currents are not exactly 180° out of phase at resonance. The resistance changes the phase angle of each branch, as illustrated by the vector diagram B of figure 103. Since the current in either branch of a practical circuit is determined by the *impedance* of the branch and not by the reactance alone, the branch currents do not cancel completely, and the line current results. Thus, at resonance the *reactive components* of the currents are equal and opposite, and the resultant current is *always* along the horizontal vector, which represents the applied voltage E. As indicated in the diagram, the resultant line current is in phase with the applied voltage and is proportional to the arithmetical sum of the resistive components of the branch circuits ($R_L + R_C$). Thus, the greater the sum of R_L and R_C, the smaller the respective phase angles and the greater the line current. The magnitude of the line current at resonance, then, is indicative of the amount of resistance present in the circuit, and hence of the Q of the circuit.

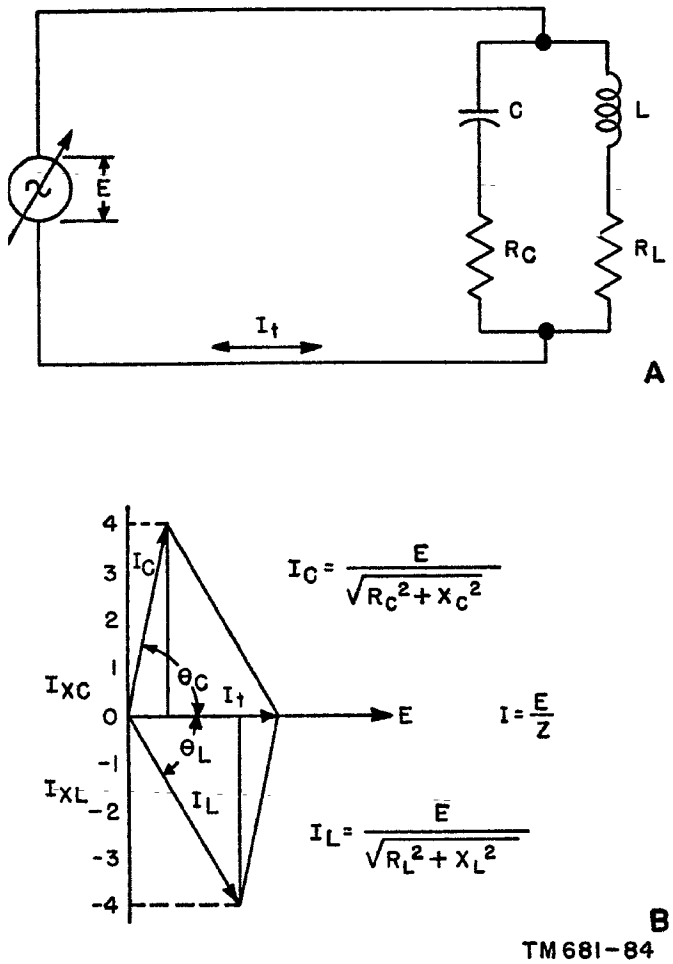


Figure 103. Parallel resonant L-C-R circuit and vector diagram.

b. In the *series* resonant circuit, the quality or Q of the circuit is determined by the ratio of the voltage across either reactance as compared to the applied voltage. Since voltage is everywhere the same in the parallel resonant circuit, the Q of the circuit is determined by the ratio of the current in the tank as compared to the line current. Then:

$$Q = \frac{I_{tank}}{I_{line}}.$$

Since the capacitive branch generally has negligible resistance, the capacitive current may be taken to be the resonant tank current, and:

$$Q = \frac{I_C}{I} = \frac{\frac{E}{X_C}}{\frac{E}{Z}} = \frac{Z}{X_C}.$$

Since $Z = \frac{X_L X_C}{X_C - X_L}$ and the sum of the reactances in the denominator of this expression is the impedance *within* the tank, then $X_C - X_L$ may be replaced by $\sqrt{R^2 + (X_L - X_C)^2}$, that is, by R. Therefore:

$$Q = \frac{\frac{X_L X_C}{R}}{\frac{X_C}{}} = \frac{X_L}{R}.$$

Thus, the Q of a parallel resonant circuit is found to be the same as that of a series resonant circuit: the ratio of the inductive (or capacitive) reactance to the resistance of the circuit. If the Q of a circuit is known, the total impedance and the current in the tank may be determined from the formulas above.

Since:

$$Q = \frac{Z}{X_C}$$

then: $Z = QX_C$ (or QX_L).

And since: $Q = \frac{I_C}{I}$

then: $I_C = QI.$

c. From this last equation it is evident that the important fact about parallel resonance is *not* the magnitude of the current in the tank (equal to E/X_L or to E/X_C), but the *ratio* of

this current to the line current, that is, the Q of the circuit. As with series resonance, the greater the resistance in the circuit, the lower the Q and accordingly the flatter and broader the resonance curve of either line current or circuit impedance. The bandwidth of the parallel resonant circuit thus follows the specifications for bandwidth at series resonance. The bandwidth, then, of the parallel resonant circuit is defined as the number of frequencies between the points .707 down from maximum on the impedance curve. Thus:

$$\text{Band pass} = \frac{f_r}{Q}.$$

Also, it can be shown mathematically that when the Q of a tank circuit is 10 or greater, all the conclusions reached for the ideal tank circuit (zero resistance) are valid for the tank circuit with resistance.

d. A comparison of two resonant circuits, one series and one parallel, with applied voltage E and with L, C, and R of the same values, reveals the following interesting relationships between the two. Since circuit values are the same, the Q of both circuits is the same. Also, as shown above, the current within the tank of the parallel circuit is equal to Q times the line current:

$$I_{tank} = Q \times I_{line}.$$

And:
$$I_{tank} = \frac{E}{X_L}.$$

Also:
$$I_{series} = \frac{E}{R}.$$

Comparing the current in the series circuit to that in the parallel tank:

$$\frac{I_{series}}{I_{tank}} = \frac{\frac{E}{R}}{\frac{E}{X_L}} = \frac{X_L}{R} = Q.$$

Then:
$$I_{series} = Q \times I_{tank}.$$

The current in a series resonant circuit then is Q times that in the tank of a parallel resonant circuit; and this latter is Q times greater than the parallel line current. Therefore, substituting $Q \times I_{line}$ for I_{tank}:

$$I_{series} = Q \times QI_{line} = Q^2 I_{line}.$$

The current in the series resonant circuit then is Q^2 times greater than the current in the line of the parallel resonant circuit. In like manner, a comparison of voltages reveals that the voltage across either reactance in the series resonant circuit is Q^2 times the voltage across the resistance R in the parallel tank circuit.

72. Universal Curves for Parallel Resonance

a. The universal curves for a parallel resonant circuit are shown in figure 104. The impedance curve is the characteristic resonance curve; the reactive component curve will be recognized as a composite of the curves of figure 102. The Q of the circuit is relatively high, since the impedance curve is steep and narrow. At resonance, the reactive components cancel at zero, and the impedance curve maximum corresponds to the maximum of the resistive component curve. This means that, at resonance, the source or generator *sees* a maximum impedance with zero reactance; therefore, the impedance is by definition a pure resistance. It should be noted, however, that this resistance is not R, the physical resistance of the tank circuit. By the resistive component is meant the opposition to the circuit source over and above the resultant reactances. For example, at some point near resonance, the impedance of the circuit is far greater than either reactance and far greater than the difference between the reactances. This difference between the reactances is the only reactive component in the circuit, and the balance of the opposition to the source must be, by definition, a pure resistance. The resistive component curve has the same characteristic shape as a resonance curve but with steeper sides. Of particular interest is the fact that at the limits of the bandwidth (points on the impedance curve .707 down from the maximum impedance), the resistive component drops to 50 percent of the resonant impedance, and that these points correspond to the maxima and minima on the reactive component curve, each also 50 percent of the resonant impedance.

b. The preceding analyses of parallel resonance may be summed up as follows:

(1) The *impedance* presented to the source is a *maximum* and a *pure resistance* at the resonant frequency.

(2) The line current is a *minimum* and in phase with the applied voltage.

(3) For frequencies above and below resonance, the tank circuit impedance is no longer purely resistive, containing a reactive component.

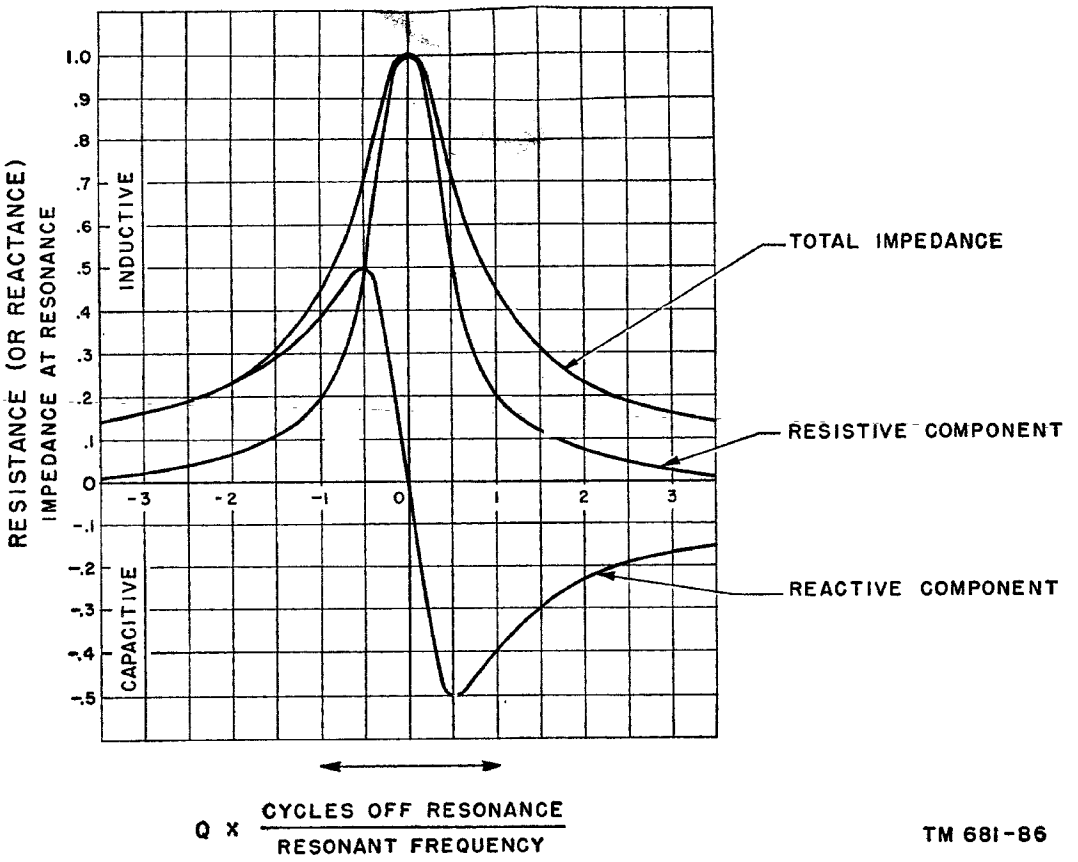


Figure 104. Universal curves of a parallel resonant circuit.

(4) At the resonant frequency, the tank circuit has zero reactance.

(5) Above resonance, the circuit behaves capacitively; below resonance, the circuit behaves inductively.

c. For example, a parallel *L-C-R* circuit resonant at 100,000 cps has an inductive reactance of 10,000 ohms, capacitive reactance of 10,000 ohms, a resistance R of 200 ohms (the combined R_L and R_C), and an applied voltage of 300 volts. The Q of the circuit is determined:

$$Q = \frac{X_L}{R} = \frac{10{,}000}{200} = 50.$$

The impedance Z to the source is:

$$Z = Q \times X_L = 50 \times 10{,}000 = 500{,}000 \text{ ohms.}$$

The line current I may be determined:

$$I = \frac{E}{Z} = \frac{300}{500{,}000} = .0006 \text{ ampere}$$

$$I = .6 \text{ milliampere.}$$

The current in the tank is:

$$I_{tank} = Q \times I_{line} = 50 \times .0006 = 30 \text{ milliamperes.}$$

The bandwidth of the circuit is:

$$\text{Band pass} = \frac{f_r}{Q} = \frac{100{,}000}{50} = 5{,}000 \text{ cycles.}$$

On either side of resonance at the limits of the bandwidth, the impedance of the circuit is .707 of the maximum impedance of 500,000 ohms, or 353,000 ohms. The resistive component of this impedance is 50 percent of maximum, or 250,000 ohms; the reactive component at the lower frequency is also 50 percent of maximum, or 250,000 ohms positive, and at the upper frequency 250,000 negative. The power consumed by the circuit at resonance is equal to $E \times I$, since the impedance is a pure resistance. Thus:

$$P = E \times I = 300 \times .0006 = .18 \text{ watt.}$$

To prove that this power is actually consumed by the resistance R, that is, by $R_L + R_C$, it is necessary only to calculate the power consumed by that resistance because of the current through it. Thus:

$$P = I^2R = .03^2 \times 200 = .18 \text{ watt.}$$

At the limits of the bandwidth the current in the line is doubled (since Z is halved) and the power consumed by the resistive component is the same as the foregoing (since the resistive component is halved):

$$P = I^2R = \frac{E^2}{Z^2} \times R = \frac{300 \times 300 \times 250{,}000}{353{,}000 \times 353{,}000}$$

$$P = \frac{9 \times 10^5 \times 2.5 \times 10^5}{1.25 \times 10^{12}} = .18 \text{ watt.}$$

The resistive component of the impedance of a parallel *L-C-R* circuit, then, is *not* imaginary; it consumes power and the concept is a way of expressing the power dissipated by the physical resistance ($R_L + R_C$). Also of interest is the fact (revealed by the foregoing) that the average power consumed by a parallel *L-C-R* circuit is the same in or out of resonance (if the load $R_L + R_C$ remains constant); whereas, the average power consumed by a *series L-C-R* circuit is greatest at resonance.

73. Some Applications of Parallel Tuned Circuits

a. Like the series tuned circuit, the parallel tuned circuit may be used in band-pass or band-rejection networks. A of figure 105, the resistance R_o is placed in series with a parallel *L-C-R* circuit and a variable-frequency generator. At resonance, the impedance of the tuned circuit is a maximum and line current a minimum; therefore, the current through R_o is a minimum. Off resonance to either side, the impedance of the tuned circuit falls and current in the line rises, and accordingly current through R_o rises. Since current rises and falls through R_o, the voltage across it follows the same pattern, as shown by the graph. Thus, if the terminals across R_o are used as the output of this circuit there will be good voltage output at all frequencies except at those making up the bandwidth of the resonant frequency of the tuned circuit. This, then, is a band-rejection circuit.

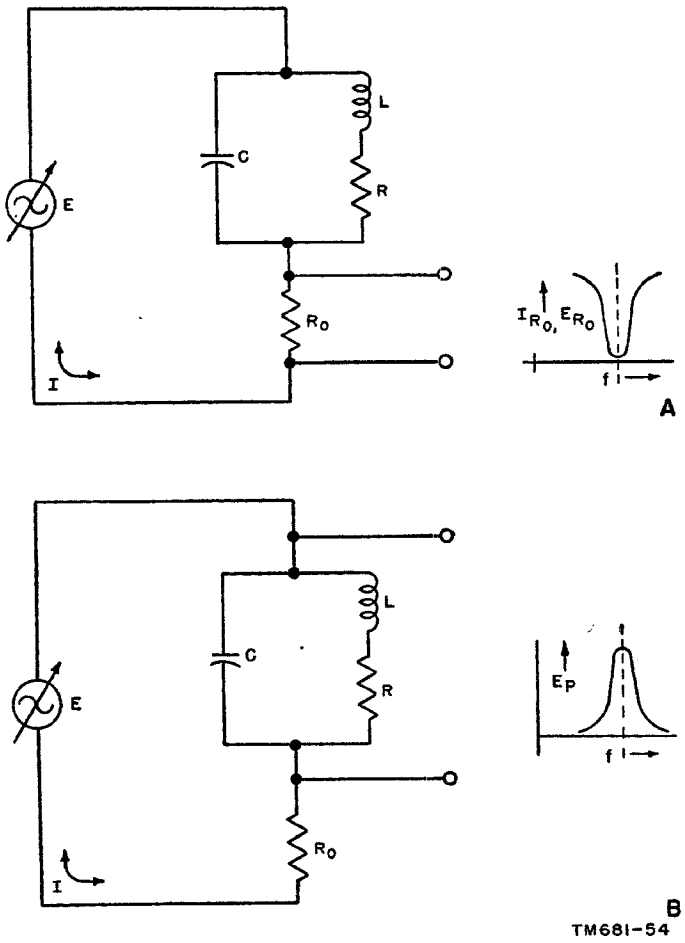


Figure 105. Applications of parallel resonant circuits.

b. In B of figure 105, the same circuit is shown except that the output terminals are across the tuned circuit. At resonance, the current in the line is a minimum, and the impedance of the tuned circuit is high. Since the voltages E_R and E_P divide proportionately to their respective ohmic resistance (their sum equal to the applied voltage), E_P is greatest and E_R least at resonance. Off resonance, the impedance across the tuned circuit falls, and accordingly the voltage across it falls. This rise of E_P at resonance is shown by the graph. Thus, this circuit will show little voltage output to any frequency except to those of the bandwidth at resonance. This, then, is a band-pass circuit.

74. Filter Circuits and Wavetraps

a. The principles of the variation of inductive and capacitive reactance with frequency, and the principles of the variation of impedance in series and parallel tuned circuits, are often used in filter circuits and wavetraps either to pass all frequencies below or above a certain frequency, or to pass or reject certain bands of frequencies. A and B of figure 106 illustrate the fundamental filter circuits. In A, high frequencies at the input meet a relatively high inductive reactance in *L* and a low capacitive reactance in *C*. Thus, high frequencies are stopped by *L* and effectively short-circuited by *C*. Low frequencies meet little opposition in *L* and high opposition in *C*. Therefore, low frequencies pass from input to output, and this arrangement is called a *low-pass filter*. In B, low frequencies meet a relatively high capacitive reactance in *C* and a low inductive reac-

tance in L. Thus, low frequencies are stopped by C and effectively short-circuited by L. High frequencies meet little opposition in C and high opposition in L. Therefore, high frequencies pass from input to output, and this arrangement is called a *high-pass filter.*

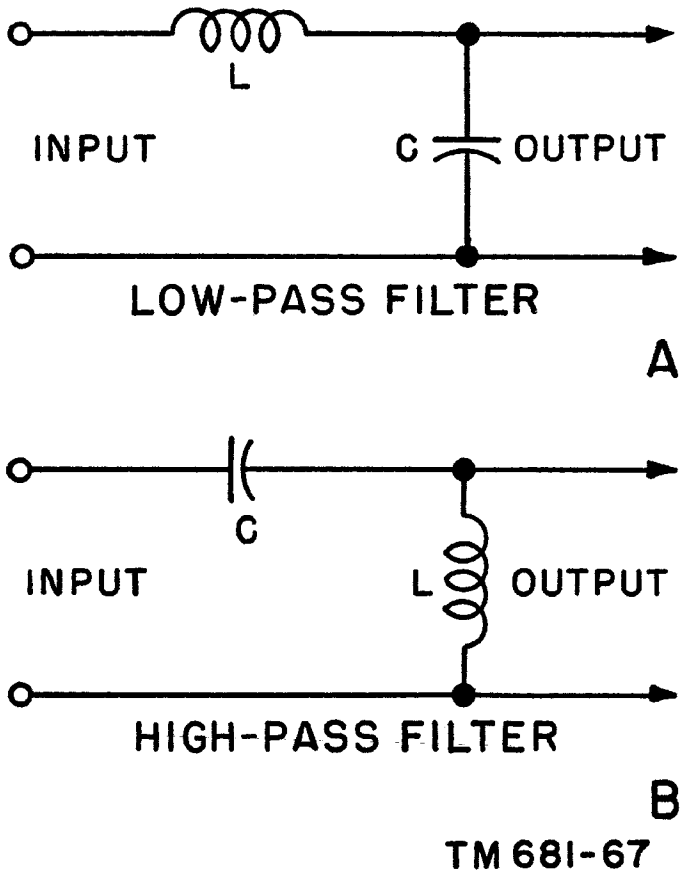


Figure 106. Fundamental filter circuits.

b. A and B of figure 107 illustrate more elaborate low-pass and high-pass filter circuits known as π-type filters, so-named from the resemblance of the schematic to the symbol. The element nearest the input characterizes the filter. Thus, A shows a capacitor input π-type low-pass filter, and B an inductor input π-type high-pass filter. C and D of figure 107 show inductor input low-pass and capacitor input high-pass filters, respectively.

c. A and B of figure 108 illustrate band-pass and band-rejection filters. In A, the resonant band frequencies find the series resonant circuits S and S' low-impedance paths and the parallel resonant circuit P a high-impedance path. Thus, the resonant band of frequencies is rejected or suppressed. All other frequencies

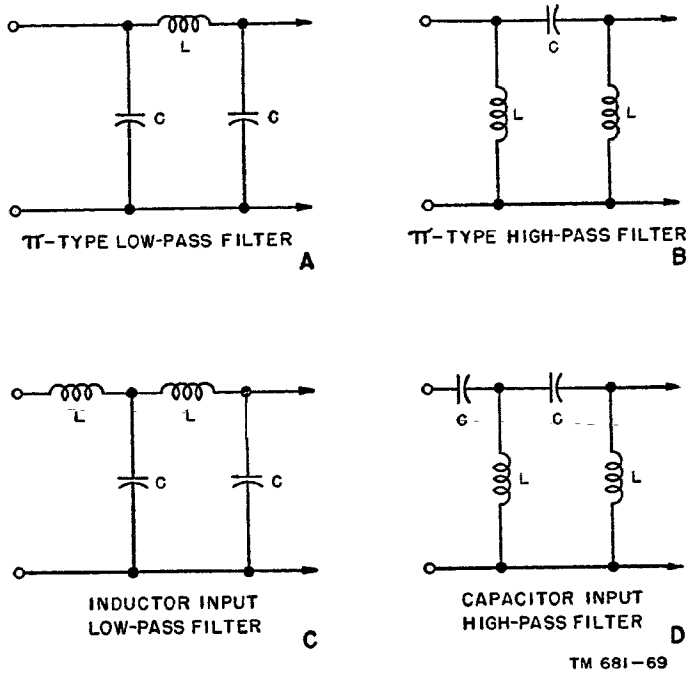


Figure 107. Characteristic filter circuits.

find S and S' high-impedance paths and P a low-impedance path, and accordingly they pass from input to output with little opposition. In B, the resonant band frequencies find the parallel resonant circuit P and P' high-impedance paths, and the series resonant circuit S a low-impedance path. Thus, the resonant band is passed from input to output with little opposi-

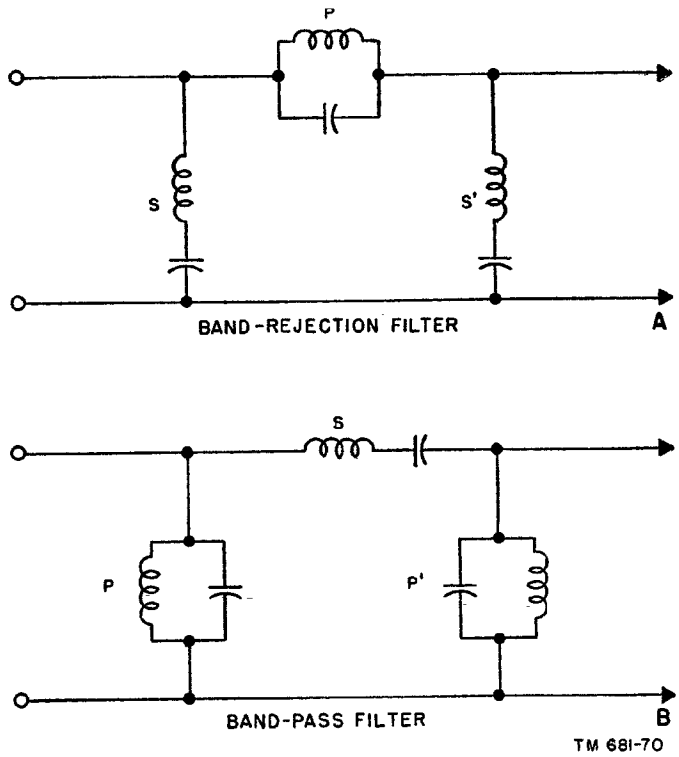


Figure 108. Band-filter circuits.

tion. All other frequencies find P and P' low-impedance paths and S a high-impedance path, and accordingly these frequencies are rejected.

d. The band-pass filter considered above finds wide application in radio communication circuits. By means of variable capacitors, the various resonant circuits may be adjusted at will to pass one band of frequencies and then readjusted to pass some other band, thus selecting first one broadcast station and then another to the exclusion of all others. A of figure 109 shows a typical band-pass filter as used in the intermediate stages of a radio receiver. The heavy lines emphasize the filter; the functioning of the other elements shown need not be taken up at this point. B of figure 109 is this same circuit drawn out as an equivalent filter circuit in terms of previous illustrations. Parallel resonant circuit P offers high opposition and the series resonant circuit S little opposition; after passing through the vacuum tube, circuits P' and S' offer impedances similar to P and S. Thus, this circuit is a band-pass filter.

e. The principles of band rejection are also used in television circuits to eliminate some unwanted signal. These circuits are known as wavetraps. Figure 110 shows a dipole antenna connected to a television receiver by lead-in wires. An undesired signal may be eliminated by placing a series circuit (resonant to that frequency) to ground from each lead wire. The undesired signal finds minimum impedance and is short-circuited to ground; all other signals find maximum impedance and pass on into the receiver. Thus, the undesired signal is said to have been trapped out.

75. Summary

a. A resonant circuit offers a directed electrical advantage similar to the directed advantage of the lever in mechanical systems.

b. A series resonant or series tuned circuit is defined as one in which the signal originates *within* the resonant circuit.

c. A series circuit is resonant if:

(1) $X_L = X_C$.
(2) $E_L = E_C$.
(3) $E = E_R$.
(4) $Z = R$.

d. At resonance, the reactances cancel, current is a maximum, impedance is a minimum, and the phase angle is 0°.

e. On either side of resonance, current is low and impedance is high.

f. At resonance, current is a maximum and the voltages across the reactances are maximum.

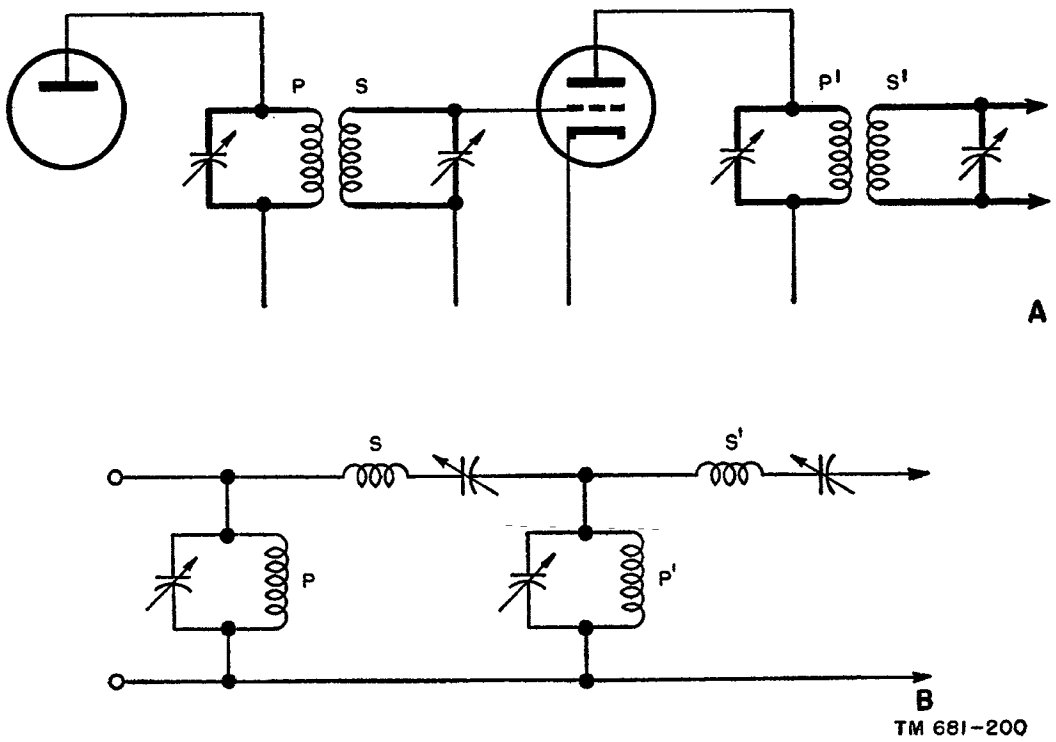


Figure 109. Band-pass filter in communications receiver.

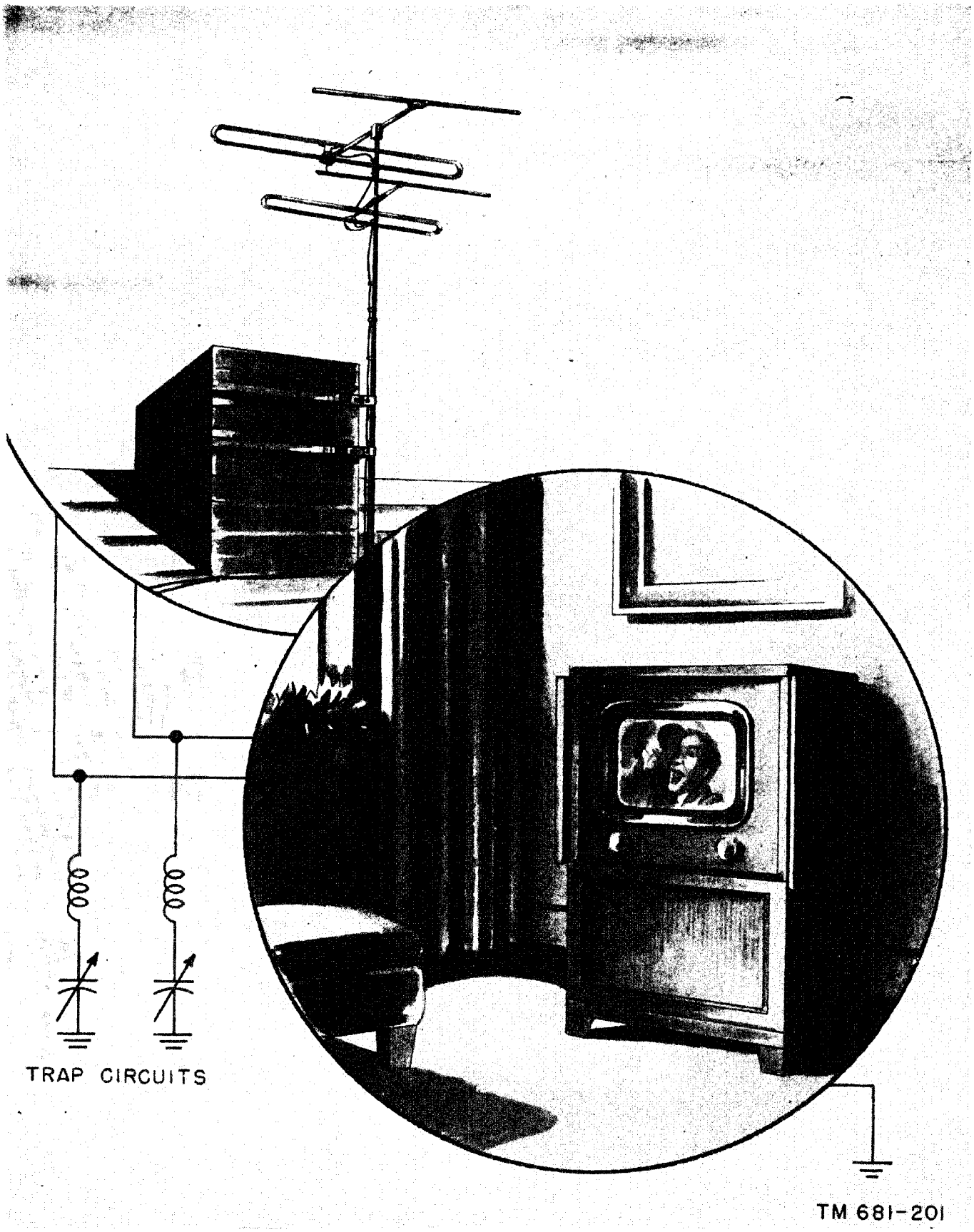


Figure 110. Wavetrap for television receiver.

g. The frequency at resonance is:

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

h. For any given frequency, the product of L and C is a constant: At low frequencies, L and C are large, and at high frequencies, L and C are small.

i. The frequency at resonance is independent of the resistance of the circuit.

j. Resistance in a resonant circuit determines the maximum current and the steepness of the slope of the resonance curve.

k. The Q of a series resonant circuit is the ratio of E_L (or E_C) to E, the applied voltage:

$$Q = \frac{E_L}{E} = \frac{X_L}{R}.$$

l. Q is a measure of the selectivity of a circuit and varies inversely with the resistance in the circuit: the lower the resistance, the higher the Q and the greater the difference between the voltages available at the resonant frequency and the voltages at frequencies off resonance.

m. The bandwidth of a circuit is the total number of cycles above and below the resonant frequency which realize practically the same voltage gain as the resonant frequency.

n. The effective limits of the band pass are points on the resonance curve .707 down from the maximum.

$$\text{Band pass} = \frac{f_r}{Q}.$$

o. A series resonant circuit may be used as a band-pass or band-rejection network, depending on the position of the output terminals.

p. A parallel resonant circuit is defined as one in which the signal originates *outside* of the resonant circuit.

q. A tank circuit is any resonant circuit, either series or parallel.

r. A parallel circuit is resonant if:

(1) $X_L = X_C$.
(2) $I_L = I_C$.

s. The frequency of resonance of a parallel circuit (sometimes called the anti-resonant frequency) is:

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

t. In a parallel resonant circuit, the branch currents cancel, the line current is a minimum, impedance is a maximum, and the phase angle is 0°.

u. On either side of resonance, line current is high (the difference between the branch currents) and impedance is low.

v. At resonance, the current in the inductance and the current in the capacitor are the same current (the tank current) circulating from inductance to capacitor and back again indefinitely:

$$I_L = \frac{E}{X_L}.$$
$$I_C = \frac{E}{X_C}$$
$$I_{tank} = \frac{E}{X_C} \text{ or } \frac{E}{X_L}.$$
$$I_{line} = \frac{E}{Z}.$$

w. At frequencies above resonance, X_L is greater than X_C, I_C is greater than I_L, and the circuit acts capacitively; at frequencies below resonance, X_C is greater than X_L, I_L is greater than I_C, and the circuit acts inductively.

x. A tank circuit can be used to simulate the properties of either a capacitor or an inductor.

y. The Q of a parallel resonant circuit is the ratio of the current in the tank (I_L or I_C) to the current in the line:

$$Q = \frac{I_C}{I} = \frac{X_L}{R} = \frac{Z}{X_C}.$$

z. The frequency at resonance, the Q (X_L/R), and the bandwidth of both series and parallel resonant circuits are the same.

aa. The resistance of the tank R ($R_L + R_C$) determines the impedance of the line at resonance:

$$Z = \frac{X_L \times X_C}{R} = \frac{X_L^2}{R} \text{ or } \frac{X_C^2}{R}.$$
$$Z = QX_L \text{ or } QX_C.$$

ab. At resonance, the impedance of the tank is purely resistive.

ac. The parallel resonant circuit may be used as a band-pass or band-rejection circuit.

ad. A filter circuit is one which discriminates between signals of different frequencies.

(1) A low-pass filter passes low frequencies at a higher voltage than high frequencies of the same input voltage.
(2) A high-pass filter passes high frequencies at a higher voltage than low frequencies of the same input voltage.
(3) A band-pass circuit passes a given band of frequencies at a higher voltage than all other frequencies.

(4) A band-rejection circuit passes all frequencies at a higher voltage than those of a given band.

ae. A wavetrap is a band-pass or band-rejection filter so arranged that an undesired signal is eliminated.

76. Review Questions

a. Define a series resonant circuit.

b. List the conditions present in a series resonant circuit.

c. What is the relationship between current and impedance at resonance? Off resonance?

d. What is the relationship between the voltages across the reactances at resonance? Off resonance?

e. Give the formula for frequency at resonance.

f. Show that the product of L and C is a constant for any given frequency.

g. What effect does the resistance in a circuit have upon the frequency of resonance?

h. What is the Q of a resonant circuit? How is it determined?

i. What is the relationship between the voltage across either reactance and the applied voltage in a series resonant circuit?

j. Why is the Q of a circuit the measure of its selectivity?

k. What is bandwidth? Give its formula.

l. Draw the schematics of series resonant circuits showing output terminals for band pass or band rejection.

m. Define a parallel resonant circuit.

n. What is a tank circuit? Why is it so named?

o. List the conditions present in a parallel resonant circuit.

p. What is the anti-resonant frequency of a circuit?

q. What is the relationship between branch currents, and between line current and impedance, of a parallel resonant circuit?

r. What is the phase angle of a parallel resonant circuit?

s. Describe the circulating current in a parallel tank circuit.

t. How is the magnitude of the current in a resonant tank determined?

u. What is the relationship between current in the tank and current in the line of a parallel resonant circuit?

v. Explain the fact that a parallel L-C circuit may be used to simulate the properties of either a capacitor or an inductance.

w. Give the formulas for the Q of a parallel resonant circuit.

x. What is the formula for the impedance of a parallel resonant circuit?

y. Explain what is meant by the fact that the impedance of a resonant tank is said to be purely resistive.

z. Draw the schematics of:

(1) a low-pass filter,
(2) a high-pass filter,
(3) a band-pass filter,
(4) a band-rejection filter,
(5) a choke-input low-pass filter.

aa. Show how a series or parallel resonant circuit may be used as a wavetrap.

CHAPTER 6
TRANSFORMERS

77. The Transformer

a. The efficient use of electrical energy, like the efficient use of mechanical energy, often requires the introduction of some means of converting the form of the energy at the source to a form that can be used at the load. A gear train, for instance, introduced between a gasoline engine and certain power saws may be designed in such a way that the gear can be adjusted to turn the saw slowly with great force, or with great speed and less force, depending on the quality and hardness of the material milled. In the same manner, it is necessary often to adjust electrical circuits so that the power available may appear at the load as one of various combinations of voltage and current—that is, as high voltage with low current, as high current at low voltage, or as any convenient combination of the two. The electrical device which corresponds to a gear train in mechanics is the *transformer,* which converts the electrical power available from one voltage-current level to another voltage-current level. It should be noted, however, that neither the gear train nor the transformer changes the *amount of power* available. The amount of power used by the saw, at any instant, is the power delivered by the gasoline engine (less friction losses) ; and the amount of power dissipated by the load in an electrical circuit is the same as the power delivered by the source (less internal losses) ; but in either case, the power to be used can be adapted to the particular work to be done.

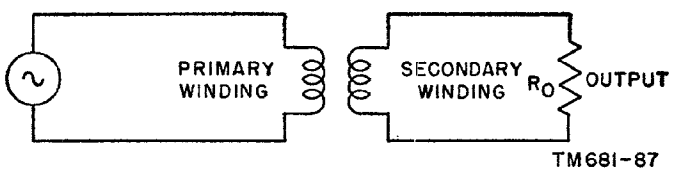


Figure 111. Basic transformer.

b. The operation of a transformer depends on the principle of electromagnetic induction, which was treated at length in chapter 2. Basically, a transformer consists of any two inductors (in separate circuits) so placed physically that the changing electromagnetic field set up by an alternating current in one induces an alternating emf in the other. Thus, mutual inductance exists between the coils, and the two circuits are said to be inductively coupled. Figure 111 shows such a basic transformer connected between an a-c generator and a resistance load. The coil connected to the source of power is called the *primary* winding, and the coil connected to the load is called the *secondary* winding. The power delivered by the generator passes through the transformer and is delivered to the load, although no direct connection exists between the primary and the secondary winding, or between generator and load. The connection that does exist is the flux linkage between the coils, and power is effectively transferred by induction. Thus, the power consumed by the primary is equal to the power delivered by the secondary, and: $P_P = P_S$. If the coils of a transformer were completely shielded from each other, no power transfer could take place and the transformer would be useless.

c. For a maximum transfer of power from the primary to the secondary of the transformer the flux linkage must be complete; that is, all of the lines of force set up by the primary winding must link the secondary winding. For this reason, the secondary is often wound directly on the primary with only protective insulation separating the two coils. Then, since the reluctance of air is very great and its permeability small (as noted in the study of electromagnetism in TM 11–661), the introduction of a soft steel core of high permeability in the transformer increases the flux linkage between the coils and makes possible a high percentage of power transfer. Even with the use of high

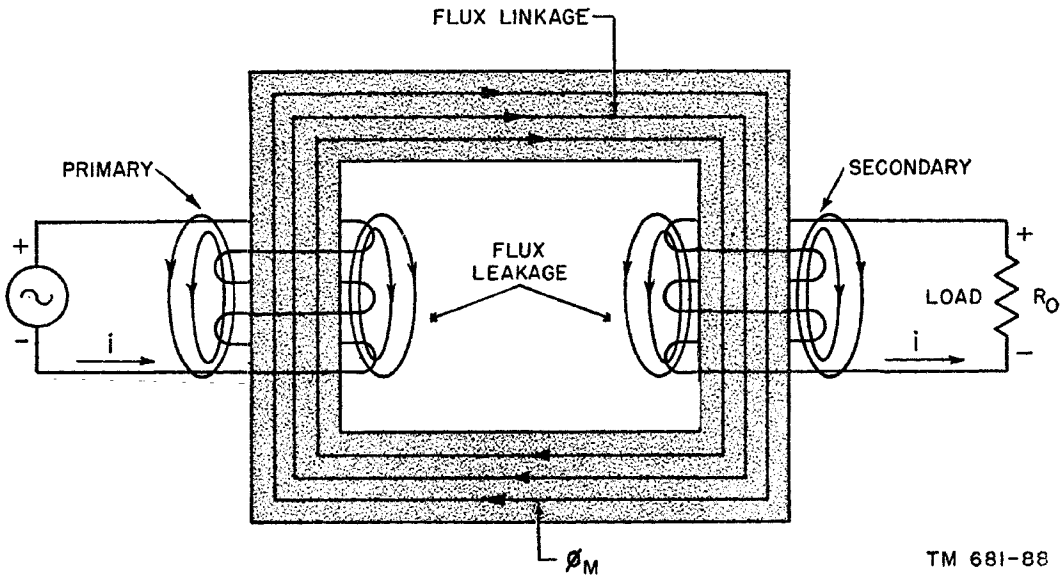


Figure 112. Iron-core transformer.

permeability cores, a few of the flux lines fail to link the secondary winding and are effectively lost, constituting a flux leakage which prevents the transformer from being a perfect conductor of power from the generator to the load. However, a well designed iron-core transformer may effect a 98-percent flux linkage, which means that K, the coefficient of coupling between the coils, is .98. Figure 112 shows a typical iron-core transformer with the flux lines set up by the primary linking the secondary by means of the low reluctance path of the core. The small flux leakages are also shown. A of figure 113, shows the shell type of transformer core, which is the most efficient core type, and B is a cross section of the windings as they usually appear. Each layer of wire is separated from the other by sheets of waxed paper, and the primary winding is separated from the secondary winding by thicker varnished paper or cardboard.

78. Theory of Transformer Operation

a. The ability of a transformer to transfer energy from its primary to its secondary through the agency of flux linkage is a function of inductive coupling or high mutual inductance. This means that the inductance of each winding should be as great as possible. If the transformer were ideal (the windings showing infinite inductance), the inductive reactance of the primary would also be infinite for any a-c frequency. At the usual 60-cycle power-line frequency, however, the inductance of the primary must be large in order to generate an appreciable reactance. Thus, if the iron core of a 60-cycle transformer were removed, the inductive reactance would fall and the primary circuit would show a high current even with no load on the secondary. This initial *magnetizing* current in the primary of a transformer under no load should be kept as low as possible since it represents a loss. The greater the inductance, the greater the reactance, and the less magnetizing current needed to set up the flux linkage.

b. The importance of the reactance of the primary, and the effect of the secondary winding on the primary, may be understood by a series of step-by-step analyses. In A of figure 114, a simple transformer with primary and secondary windings on a soft iron core is shown. The primary is connected to an a-c source, and the secondary is left open so that it has no appreciable effect on the first analysis. The primary winding is then, in effect, nothing but a simple iron-core inductance. As such, the primary offers inductive reactance to the source, and current which is dependent on the magnitude of the reactance flows in the primary circuit, causing a changing magnetic field to be set up in the core. The current through the inductor and the voltage across it are 90° out of phase, the current lagging, as demonstrated in chapter 2. At any given instant of time, the current is in the direction shown. The direction of the flux lines is determined by the *left-hand rule*: if the fingers of the

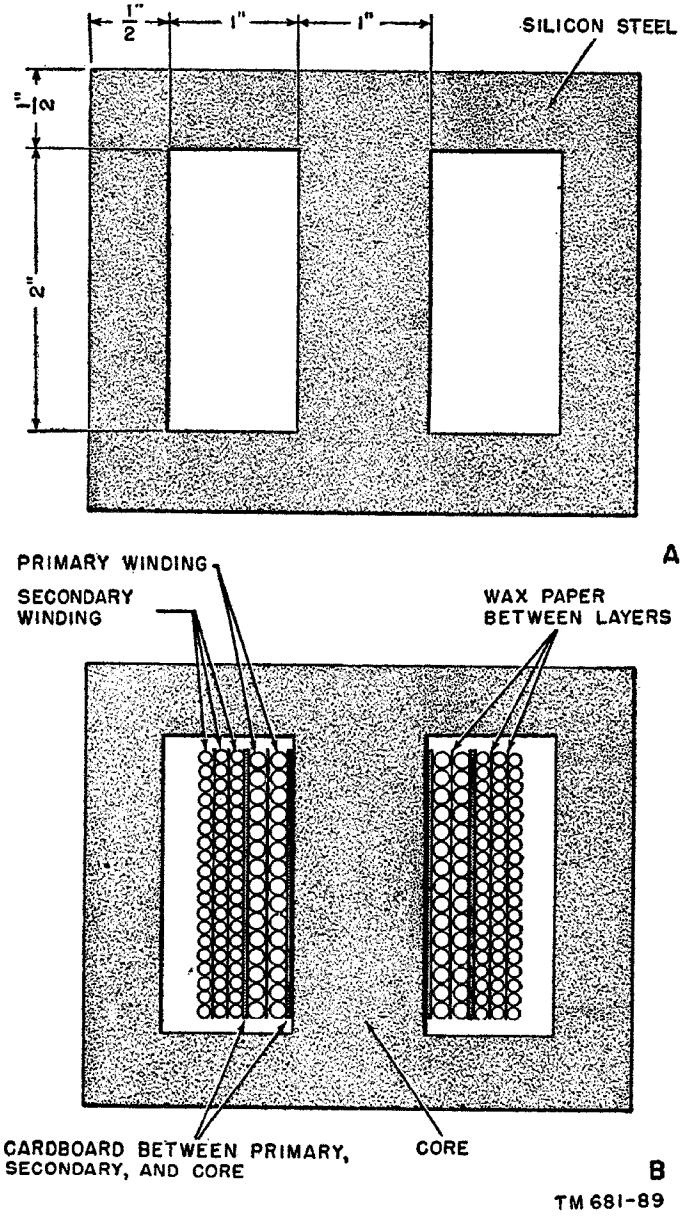


Figure 113. Cross section of iron-core transformer.

left hand point in the direction of electron flow in the coil winding, the thumb points in the direction of the magnetic field. At this same instant, a counter emf E_p' is set up in the primary 180° out of phase with the applied voltage E_p and 90° out of phase with the current in the primary I_p, the primary current leading, as shown by the vector diagram B of figure 114.

c. In A of figure 115, the same transformer is shown with the effect of this primary action on the secondary indicated (for ease of explanation, the primary action is not shown). Thus, it will be seen that the secondary alone constitutes also a simple inductance in which a voltage has been induced by the changing flux lines set up by the primary. This induced voltage has the same direction as the counter emf of the primary. The secondary circuit is shown closed by a resistance so that a small current may flow in the circuit. The induced voltage is conceived as a generator in series with the secondary considered as an inductance. Then, the current through the secondary and the voltage across it are 90° out of phase, the current lagging, as shown in the vector diagram of B of figure 115. Note that the initial direction of the voltage is the same as that of the primary counter emf of B of figure 114.

d. At the same instant in time taken for the primary analysis, the secondary current and the flux lines are in the direction shown in A of figure 115 (the left-hand rule). The flux lines set up by the secondary current are opposite in direction to those set up by the primary current. Therefore, the secondary current I_s effectively decreases the impedance of the primary circuit by opposing the flux lines set up by the primary current. The primary current must accordingly increase to maintain the flux linkage between the two coils. If the load on the secondary causes more current to flow in the secondary, more current flows in the primary from the source. This opposition in flux lines in a transformer is in accordance with Lenz' Law, which states that an induced voltage (and the current resulting from it) is always in such a direction as to oppose the force setting it up. If the action of the secondary aided the lines of flux set up by the primary, an increase in secondary current would effect an increase in flux lines, an increase in the inductance and inductive reactance of the primary, and a corresponding decrease in primary current—an impossible condition, since the secondary would then be delivering more power to the load than the primary receives from the source.

e. The effect of the secondary upon the primary of a transformer may be considered from another point of view. The secondary current sets up an emf of self induction or counter emf E_s' in the secondary 180° out of phase with the secondary voltage, and at the same time an induced voltage in the primary in phase with this counter emf. Since the voltage induced in the primary is 180° out of phase with E_s, the voltage of the secondary, it is in phase with E_p, the voltage applied to the primary, and 180°

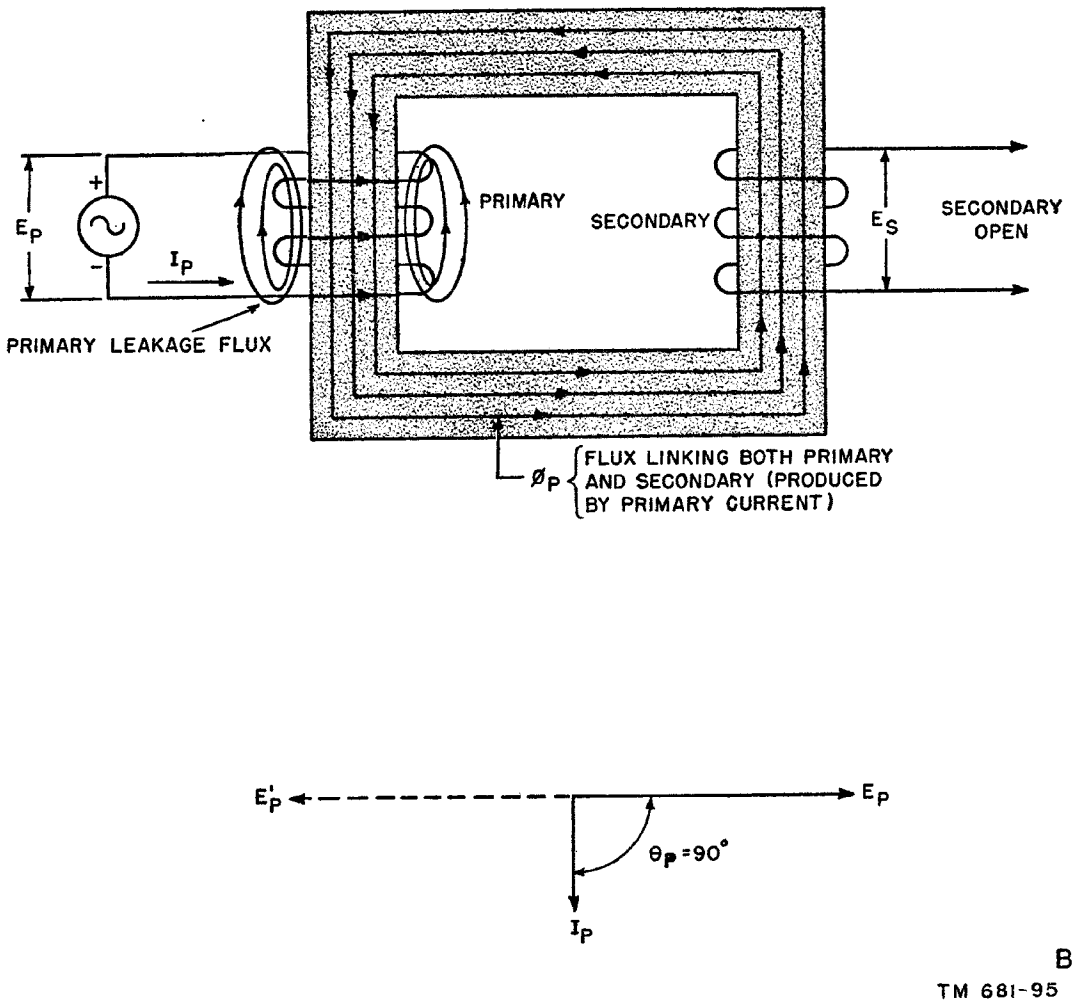


Figure 114. Direction of flux lines produced by primary current.

out of phase with E_p', the counter emf of the primary. Therefore, the effect of current in the secondary is to cancel to some extent the counter emf of the primary, or to increase the applied voltage, thereby allowing more current to flow in the primary. These induced voltages are shown as dotted line vectors in B of figures 114 and 115.

f. The total action of the transformer may now be understood in terms of the vector diagrams of figure 116. With a token current flowing in the secondary of the circuit shown in A, the vector diagram of voltages and currents is as shown in B. Voltages across the transformer are 180° out of phase, and currents are 180° out of phase. The phase angle between voltage and current in the primary is 90°, current lagging, and between voltage and current in the secondary also 90°, current lagging. As R_o, the load resistance, is decreased, the load is said to increase, and current in the secondary increases. Since R_o is shunted directly across the secondary, the resistive current increases and the circuit becomes more and more resistive; that is, voltage and current in the secondary circuit approach an in-phase condition and the phase angle θ_s approaches 0°. Since primary current increases with an increase in secondary current, as explained above, the inductive reactance of the primary has been in effect reduced and therefore the circuit appears more resistive. Hence, voltage and current in the whole primary circuit approach an in-phase condition and θ_p approaches 0°. The vector diagram of C of figure 116 shows these phase relationships for the transformer of A operating under the load for which it was designed. An ideal transformer would show the current and voltage vectors superimposed (θ_s and θ_p equal to 0°)—that is, voltage and current in each side in phase, and the secondary voltage and current 180° out of phase. Such a trans-

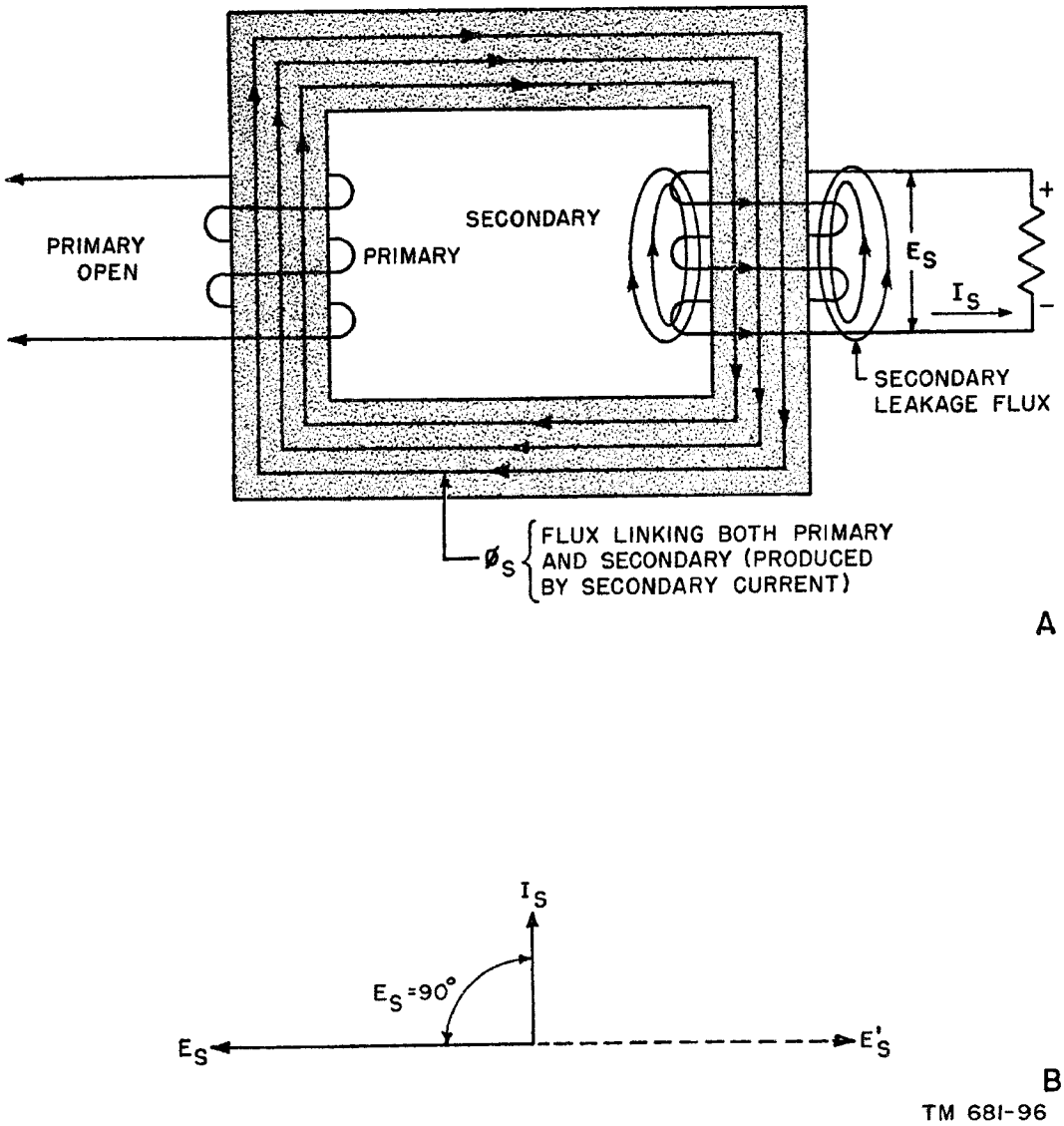


Figure 115. Direction of flux lines produced by secondary current.

former effects a perfect transfer of energy, and the source generator *sees* the load as an equivalent pure resistance (as though the transformer were not in the circuit) but with voltage and current shifted 180°.

79. Turns Ratio: Voltage

a. In the study of inductance in chapter 2, the total magnitude of any emf induced in a coil was shown to be dependent on the number of turns and the rate of change of the flux linking the circuit, or:

$$e = -N\frac{d\phi}{d_t}.$$

Since the rate of change of the flux is fixed by the frequency of the current supplied by the generator, the magnitude of the voltage induced in either side of a transformer depends directly upon the number of turns. Thus, the back emf induced by the changing current in the primary is not equal to the emf induced in the secondary unless the number of turns in the primary equals the number in the secondary. Furthermore, since the back emf in the primary is equal to the applied voltage, a ratio may be set up to determine the emf induced in the secondary in terms of the applied voltage and the *turns ratio* of the two coils.

Thus:
$$\frac{E_p}{N_p} = \frac{E_s}{N_s}$$

where: E_p = the voltage applied to the primary,

N_p = the number of turns in the primary,

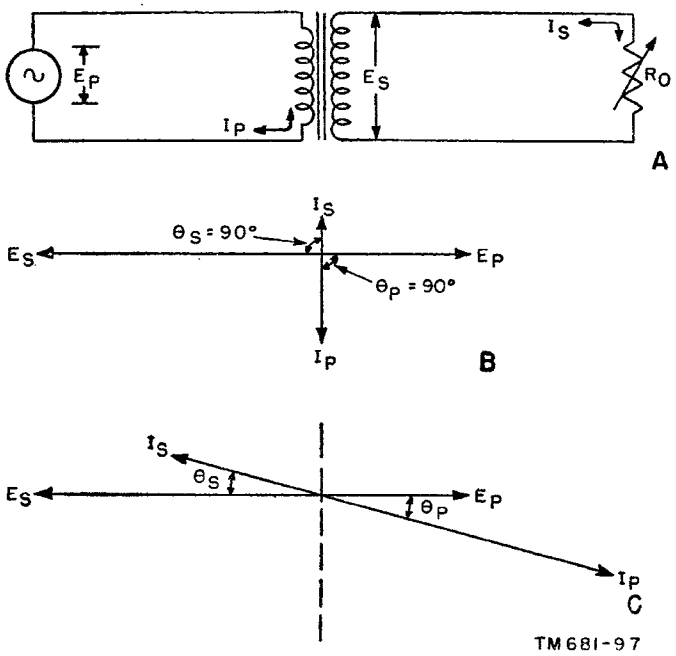


Figure 116. Phase relationships of voltage and current in a transformer.

E_s — the voltage induced in the secondary,

N_s = the number of turns in the secondary.

This equation may be rewritten:

$$E_p N_s = E_s N_p$$

or:

$$E_s = \frac{E_p N_s}{N_p}.$$

b. The expression N_s/N_p or N_p/N_s is called the turns ratio and may be expressed as a single factor. Figure 117 shows the schematic of a transformer with 1,000 turns in the secondary and 250 turns in the primary. The turns ratio is 4:1 or 4. If 110 volts ac is applied to the primary of this transformer, the voltage induced in the secondary is:

$$E_s = \frac{E_p N_s}{N_p} = \frac{110 \times 1000}{250}$$

$$E_s = 110 \times 4 = 440 \text{ volts.}$$

Such a transformer is called a *step-up* transformer. If the ratio N_s/N_p is less than 1, the secondary winding has fewer turns than the primary, and the secondary voltage is less than the applied voltage. Such a transformer is called a *step-down* transformer. It should be noted, however, that the terms *step-up* and *step-down* as applied to transformers always refer to voltage level, not to current level, nor are they ever applied to power, which in any carefully designed transformer is taken to be practically the same in both primary and secondary.

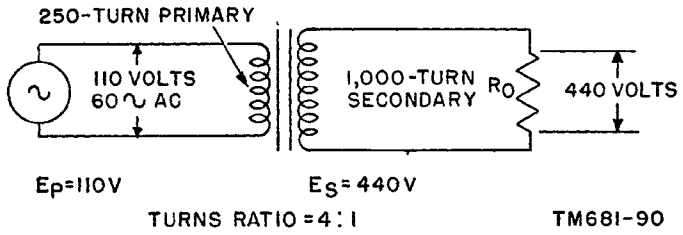


Figure 117. Step-up transformer.

c. Since the voltage induced in the secondary is directly proportional to the number of turns in the secondary compared with the number of turns in the primary, the voltage ratio of secondary to primary may be found by determining the number of volts per turn for a given transformer. Thus, in the example above, 110 volts applied to a primary of 250 turns results in a volts-per-turn ratio of .44, or:

$$\text{Volts/turn} = \frac{110}{250} = .44 .$$

Since the number of volts per turn is a constant for any given transformer, the voltage in the secondary winding may be determined by multiplying this constant by the number of turns in the secondary, or:

$$E_s = 1000 \times .44$$

$$E_s = 440 \text{ volts.}$$

d. The volts-per-turn ratio is also convenient for determining a number of secondary voltages when a transformer carries more than one secondary winding. For instance, figure 118 shows the schematic of a transformer with the primary winding linking a number of secondary windings. The primary consists of 200 turns, secondary winding S_1 is 1,200 turns, S_2 is 850 turns, S_3 is 11 turns, and S_4 is 22 turns. The volts per turn of this transformer is E_p/N_p, or 110/200, and is equal to .55. Winding S_1, then, has an induced voltage of:

$$E_{s1} = 1{,}200 \times .55 = 660 \text{ volts.}$$

And:

$$E_{s2} = 850 \times .55 = 467.5 \text{ volts,}$$

$$E_{s3} = 11 \times .55 = 6.05 \text{ volts,}$$

$$E_{s4} = 22 \times .55 = 12.1 \text{ volts.}$$

80. Turns Ratio: Current

a. In the transfer of electrical power across an ideal transformer, the power absorbed by

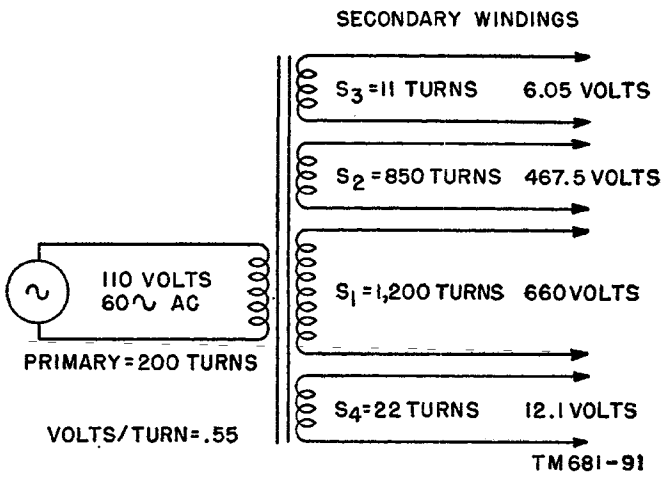


Figure 118. Multi-winding transformer.

the primary winding is equal to the power delivered by the secondary winding. Hence:

$$P_p = P_s.$$

In such an ideal transfer of energy from generator to load, the load appears as a pure resistance to the generator, as noted earlier in this chapter, and the apparent power is equal to the true power. Therefore, the power factor of the transformer is 1 and the phase angle is 0° . Thus, the power in either side of the transformer is equal to the volt-amperes, or:

$$P_p = E_p I_p.$$

And: $P_s = E_s I_s.$

Then: $E_p I_p = E_s I_s.$

Rewritten: $\frac{I_p}{I_s} = \frac{E_s}{E_p}.$

However, the ratio of secondary voltage to primary voltage is equal to the turns ratio:

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}.$$

Therefore, the ratio of current in the primary to current in the secondary is equal to the turns ratio:

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

or: $I_p = I_s \left(\frac{N_s}{N_p}\right)$

b. From another point of view, the relationship of current to the number of turns may be determined from a consideration of the ampere-turns of the transformer considered as an electromagnet. In the detailed study of electromagnetism in TM 11–661, it was shown that the strength of an electromagnet may be expressed in ampere-turns. Therefore, the lines of flux set up in the core of a transformer by a given magnetizing current in the primary are directly proportional to N times I, the ampere-turns. However, by Lenz' Law, the induced voltage in the secondary and its resultant current at any instant set up lines of flux equal and opposite to the ampere-turns of the primary. Since this transformer is conceived as ideal, the flux linkage is complete, and:

$$N_p I_p = N_s I_s.$$

Then: $I_p = N_s$

$I_s = N_p$

or: $I_p = \frac{I_s N_s}{N_p}.$

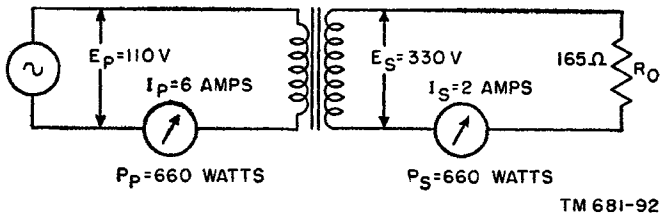


Figure 119. Current ratio in a step-up transformer.

c. Thus, it can be seen that the current across a transformer varies inversely as the number of turns. For example, figure 119 shows a simple power transformer of 300 turns on the primary and 900 turns on the secondary connected to a 110-volt a-c line and a 165-ohm load. As previously, the turns ratio is 3 to 1, and E_s is equal to 330 volts. Then, since the load determines the energy used, the current in the secondary is:

$$I_s = \frac{E_s}{R_o} = \frac{330}{165} = 2 \text{ amperes.}$$

The current in the primary is:

$$I_p = \frac{I_s N_s}{N_p} = 2 \times 3$$

$$I_p = 6 \text{ amperes.}$$

A turns ratio, then, of 3 to 1 increases the applied voltage from 110 to 330 volts, at the same time decreasing current from 6 amperes to 2 amperes. From these observations, it will be noted that the product of current and voltage in one side of a transformer is equal to the product of current and voltage in the other side. Thus, the power in the primary circuit is 110 times 6, or 660 watts; the power consumed by the secondary circuit is 330 times 2, or 660 watts.

81. Transformer Losses: Efficiency

a. EFFICIENCY. Thus far in the discussion of transformers, the ideal transformer has been assumed. Such an ideal transformer would be 100 percent efficient; that is, the ratio of power output compared to power input would be 1. Practical iron-core transformers are not, of course, 100 percent efficient but when carefully designed they show a high efficiency, ranging from 95 to 98 percent. This high efficiency is possible in a transformer because of the careful attention devoted to minimizing the effective losses due to flux leakage, hysteresis, eddy currents, flux saturation of the core, as well as the copper losses of the coil windings and losses from distributed capacitance.

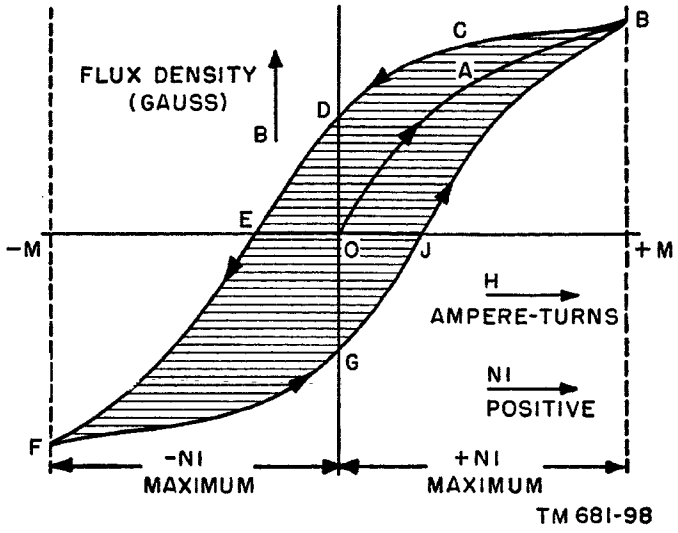


Figure 120. Hysteresis loop.

b. HYSTERESIS. The mutual inductance between windings of a transformer depends directly on the coefficient of coupling between them. Total flux linkage is impossible, but by interleaving windings, that is, by winding first a layer of one coil and then a layer of the other, and by the use of cores of high permeability, flux leakage is held to a minimum. The effort to achieve a high coefficient of coupling at power-line frequencies (60 cycles) requires cores of high permeability but involves in turn the loss from hysteresis, which is the lagging of the magnetization and demagnetization of the soft steel core behind the alternating current in the circuit. The hysteresis loop is a graphical representation of this phenomenon, plotting flux density at any instant against the ampere-turns causing it, as shown in figure 120. Thus, the flux density of a given core material is at point D (well above zero) when H, the force which originally had caused this density, is at zero; the flux density does not reach zero until the point E, when the magnetizing force is in the opposite direction. On the return swing, the flux density is a point G when H is zero, and zero when H is at point J. This persistent lagging of the magnetization behind the current causing it means that if a sine wave of current is applied to an iron-core transformer, the resultant flux-density curve will not be sinusoidal and the maximum effect of the current will not be realized. Figure 121 shows a core specimen, its hysteresis loop, the sine wave of current causing it, and the resultant flux-density curve. Since the width of the hysteresis loop horizontally at the X-axis and vertically at the Y-axis is a sign of the amount of lag of the magnetization behind the current, then the area of the hysteresis loop is proportional to the loss. This loss can be lessened by careful selection of a core material such as soft silicon steel.

c. SATURATION.

(1) In addition to hysteresis losses, soft steel cores show losses due to *saturation*; that is, the number of flux lines in the core reaches a point at which an increase in current causes no additional magnetization or less additional magnetization than the increase in magnetizing force warrants. Figure 122 shows a saturation curve, which is a graph of flux density plotted against a steadily increasing direct current. At the *knee* of the curve, the linear increase in flux density ceases and there is little increase in flux density for a large increase in current. Thus, for a rise in current from O to A, the flux density rises from A to C, but for an equal rise in current from B to D, the flux density rises only from E to F. Hence, increasing current beyond the knee of the saturation curve represents an increasing loss in efficiency, since the same effect of magnetization could be obtained with a much smaller current were the core not saturated. An increase in cross-sectional core area

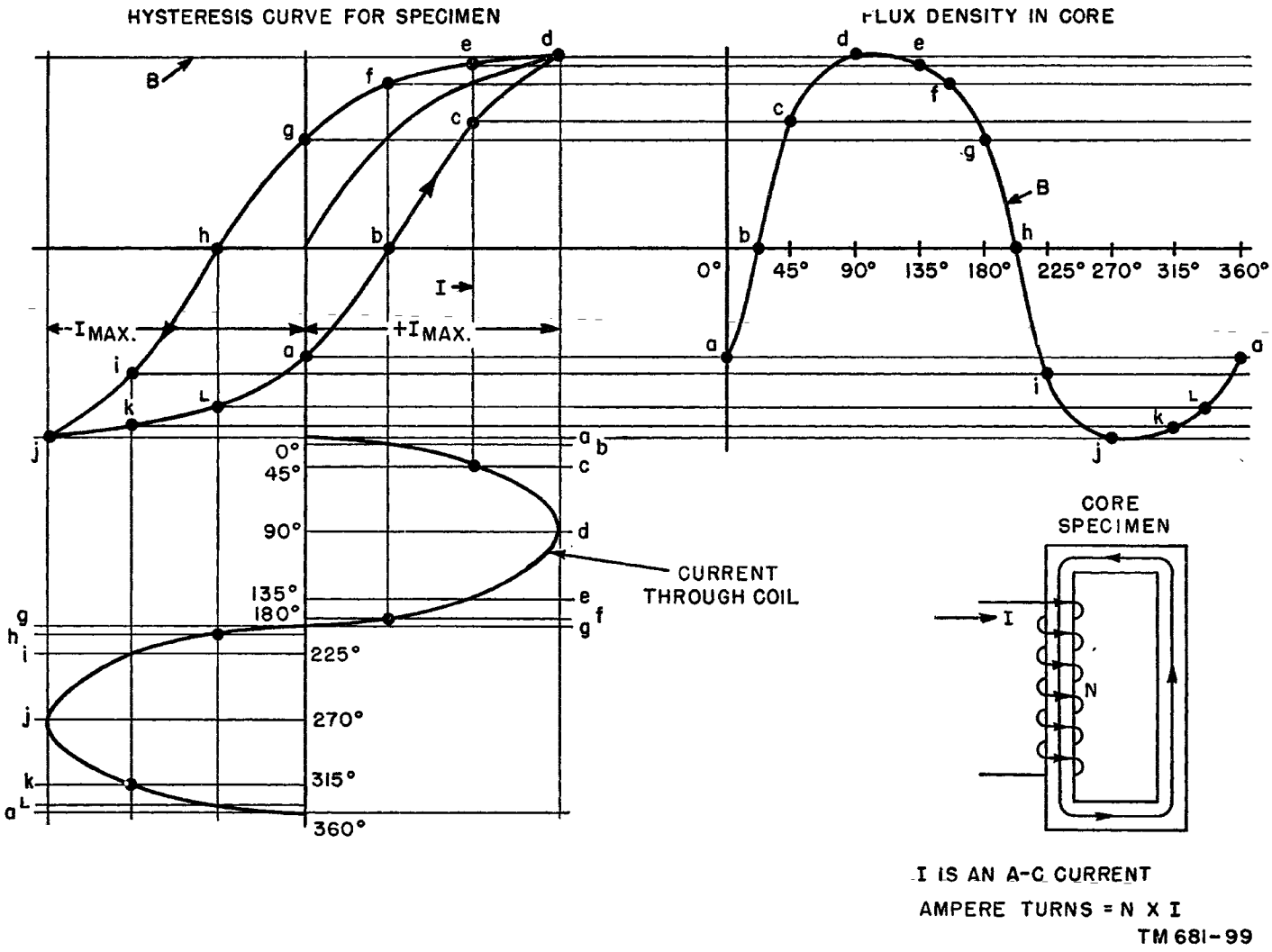


Figure 121. Variation of flux density with sine wave of current.

decreases flux density for a given current but increases weight, cost, and hysteresis loss of the transformer. For this reason, most transformers designed for power transfer at 60 cycles operate at or near the knee of the saturation curve of the core material.

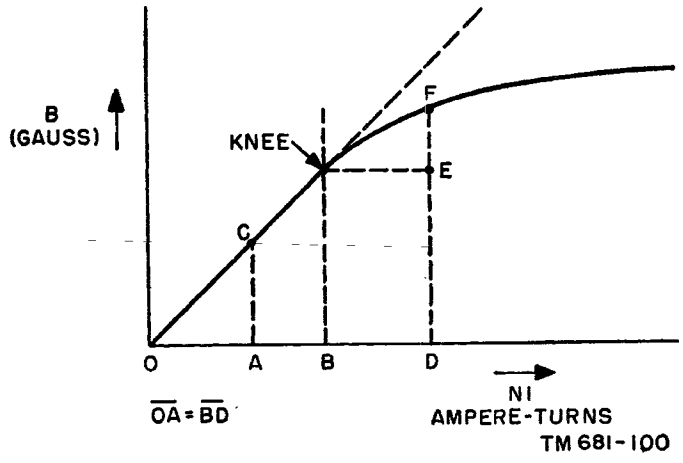


Figure 122. Saturation curve.

(2) In the discussion of iron-core inductances in chapter 2, it was shown that the permeability of a magnetic core falls off when flux density increases beyond an optimum point. This is equivalent to saying that as the core saturates, the ratio of B to H (flux density to magnetizing force), which is the definition of permeability, becomes smaller. In addition, a certain amount of dc flowing in an a-c circuit (a usual condition in power-supply and vacuum-tube circuits) magnetizes the core permanently in one direction

and decreases the magnetizing variation possible for the alternating current present. Because of this fact, iron-core chokes (except swinging chokes or saturable reactors) and iron-core transformers are designed so that a small *air-gap* exists in the magnetic circuit. The reluctance of the air gap is high compared to the reluctance of the soft steel core, and the over-all reluctance of the circuit is increased, reducing inductance but effectively preventing saturation. Physically, the air gap needed is very small and may be achieved by the end-to-end placing of the laminations or by inserting a thin piece of fibre or cardboard through the core at some point. Many additional turns of wire are needed when an air gap is used in order to achieve the same inductance, but more importantly the choke or the transformer action, which depends on flux change in proportion to changing current, is not lost through saturation of the core.

d. EDDY CURRENTS. Magnetic core materials cause additional transformer losses because, as conductors, small short-circuited currents, called eddy currents, are induced in them. To reduce eddy currents to a minimum, transformer cores are laminated—that is, made up of thin strips of steel pressed together. Each strip is sprayed with an insulating coating so that the d-c resistance between strips is very high. Thus, eddy currents find a series of high resistance paths, whereas the magnetic permittivity of the core is not affected. In figure 123, A shows eddy currents in a cross section of unlaminated core and B the same cross section with laminated core.

e. COPPER LOSSES. The current in the primary and secondary windings of a transformer must flow through the d-c resistance of the wire. A certain amount of power is lost through heat, since this is a true power (I^2R) loss. However, transformers carrying considerable amounts of power use wire of large cross-sectional area to cut down this heat loss, which in addition lowers the permeability of the core and raises resistance, thus increasing other losses. However, since a high percentage of

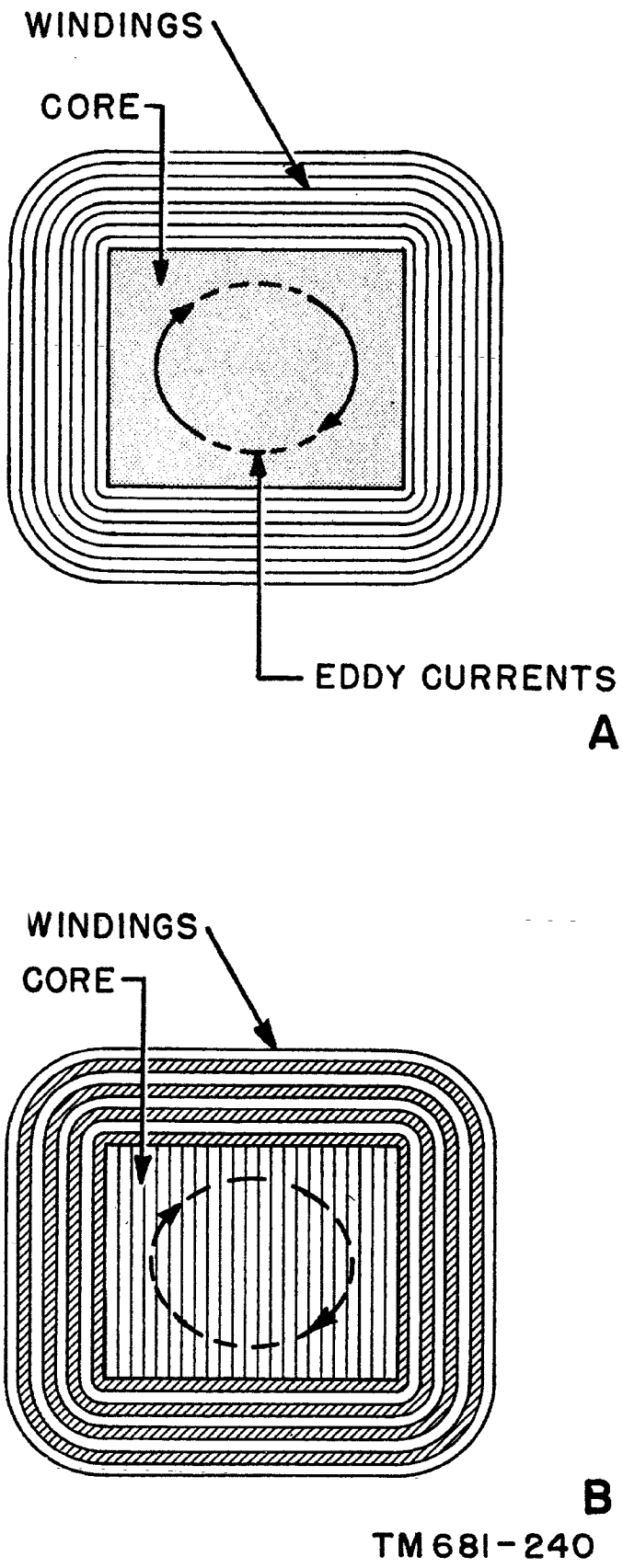


Figure 123. Cross section of core showing eddy currents and laminations.

flux linkage at power-line frequencies requires large inductance, some compromise between the size of the core and the number of windings must be made. Thus, a huge core and small winding for a given inductance would show little copper loss, but the transformer would be heavy and awkward and core losses would be great. On the other hand, a small core reduces core loss and increases copper losses because of the increased number of turns, but, for most applications, the d-c resistance of the wire may be ignored if the ratio of inductive reactance to resistance is 10 to 1, or greater. Skin effect, or the tendency of alternating currents to travel on the surface of a conductor (see chapter 2) also raises the resistance of the wire, but this effect, although present at all frequencies, is more noticeably a loss factor at the higher frequencies. Distributed capacitance, the stray capacitance existing between closely packed windings, is also a loss factor but more noticeable at the audio and radio frequencies than at the power frequencies (pars. 87 and 88).

82. Power Transformers

a. In transmitting large amounts of power over long distances the power consumed by the resistance of the conducting wires is lost as heat. This true power loss varies directly as the resistance and the *square* of the current, since it is equal to I^2R. If current in the line is doubled, the power lost is four times as much, the loss varying as the square of the current. Or, if current in the line is halved, the power lost is only one-quarter as much. Thus, power generated at Niagara Falls or in the Tennessee Valley may be sent across country with relatively small loss if current is kept as low as possible. This is done by means of a power transformer, which converts the power output of the generators to a high-voltage, low-current level for transmission and then reconverts this power to a low-voltage, high-current level for consumption in the home. For instance, if 1,000 watts of power (5 amperes at 200 volts) is sent through 10 ohms of line, the power loss is I^2R (5^2 times 10), or 250 watts. Converting this power to .5 ampere at 2,000 volts, the loss is $.5^2$ times 10, or 2.5 watts—a saving of 247.5 watts.

b. Long-distance power transmission uses voltage levels as high as 300,000 volts with correspondingly small currents. The transformers used to convert power at this level are huge units filled with oil to prevent arcing-over within the windings because of the high voltage. An electric power substation is a series of such transformer units for reconverting power, and the familiar neighborhood pole transformer brings voltage and current to the levels used in the home, 110 to 120 volts at the current demanded (fig. 124).

c. The power transformers used in radio and communications are much smaller units than the power transformers described above, but their function is the same, the efficient transfer of power while changing the voltage-current level. A number of 60-cycle transformers, designed to adapt the 110 to 120 volts at the line to more suitable levels, are shown in figure 125. A transformer such as this should always be operated within 90 percent of its rated load, since, as described earlier, the current in the secondary brings voltage and current in phase, reducing the phase angle and bringing the power factor close to unity. It will be remembered that the power factor of any a-c circuit is the cosine of the phase angle, or the ratio of true power to apparent power, and that it is desirable to keep true power as close as possible to apparent power in order to keep excessive voltage and current out of the circuit. For instance, any power transformer operated at one-fifth its proper load shows considerable primary and secondary inductive reactance, and the induced voltages across it are very high, causing arcing and breakdown of insulation with resultant burning out of the transformer. Under full load, the inductive reactance of the primary circuit is almost completely canceled by the opposing magnetizing force set up by the secondary current, the phase angle is small, and accordingly the power factor is close to unity. Any reactance present is a leakage reactance and hence a function of the efficiency of the transformer. A carefully designed power transformer has very small leakage reactance and small copper and core losses, and therefore shows a high power factor and high efficiency. Commercial power transformers generally show a power factor of .9 or above and an efficiency of approximately 95 percent.

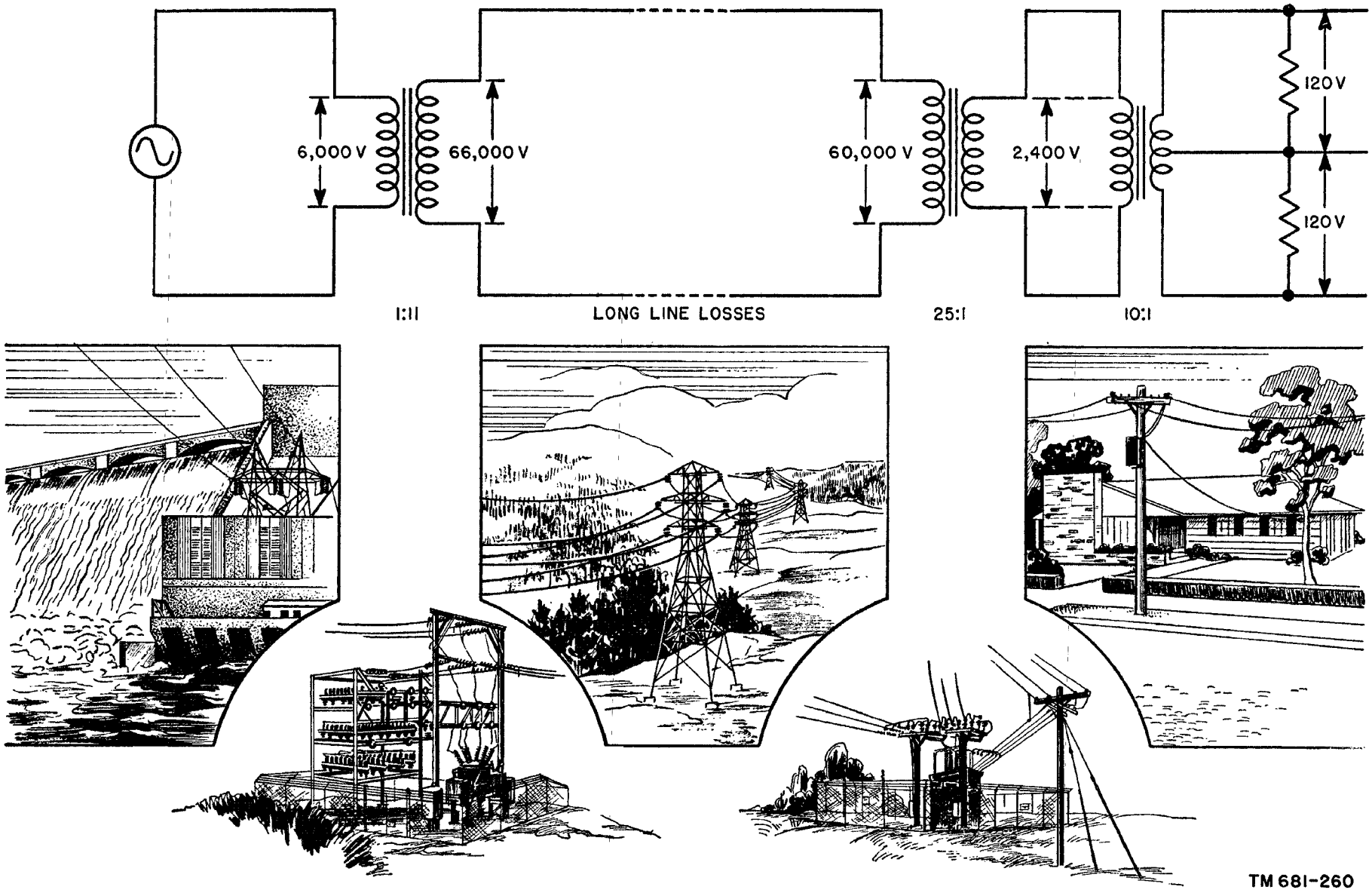


Figure 124. Long-distance power transmission.

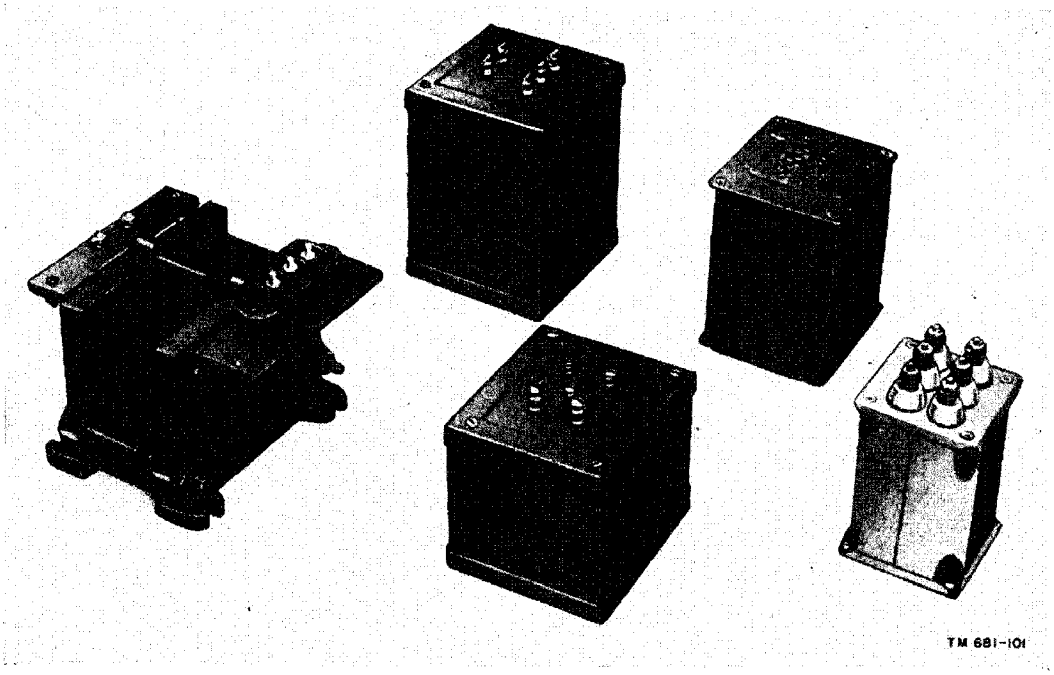


Figure 125. Power transformers.

d. Figure 126 shows the schematic of a power transformer known as a *plate* transformer because it supplies power to the plate circuits of the vacuum tubes of a transmitter. This unit is 95 percent efficient and at its rated load has a power factor of .9. The secondary delivers 250 milliamperes at 1,500 volts either side of the center-tapped secondary. The power output is:

$$P_s = 3000 \times .25 = 750 \text{ watts.}$$

Since efficiency is the ratio of true power out to true power in, then:

$$\text{Eff} = \frac{P_{av} \text{ (secondary)}}{P_{av} \text{ (primary)}}$$

$$P_{av} \text{ (primary)} = \frac{P_{av} \text{ (secondary)}}{Eff} = \frac{750}{.95}$$

$$P_{av} \text{ (primary)} = 790 \text{ watts.}$$

Since the power factor of this transformer is .9, then:

$$\frac{P_{av}}{P_{ap}} = .9$$

$$P_{ap} = \frac{P_{av}}{.9} = \frac{790}{.9}$$

$$P_{ap} = 877 \text{ watts.}$$

Since apparent power is always equal to E_p times I_p, then:

$$I_p = \frac{P_{ap}}{E_p} = \frac{877}{110}$$

$$I_p = 8.0 \text{ amperes.}$$

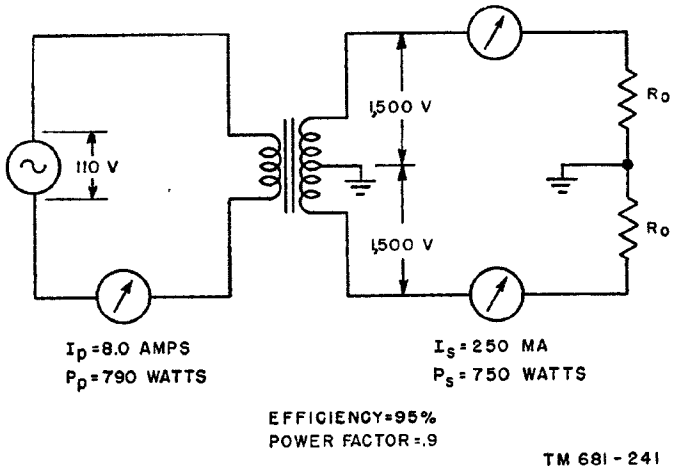


Figure 126. Plate transformer.

e. Where small amounts of power are used, as in special-purpose power transformers, the efficiency and the power factor may be ignored. Figure 127 shows the schematic of a typical filament or step-down transformer used to convert 110 volts at the line to 6.3 volts for heating the cathodes of a number of vacuum tubes. Since the secondary reduces the applied voltage to 6.3 volts, the turns ratio is:

$$\frac{N_s}{N_p} = \frac{6.3}{110} = \frac{1}{17.6} = .057.$$

Then, if 4 amperes flow in the secondary circuit, the current in the primary is:

$$I_p = \frac{I_s N_s}{N_p} = 4 \times .057$$

$$I_p = .228 \text{ ampere.}$$

In this type of transformer, a small current in the primary produces a relatively high current in the secondary at low voltage. The power consumed in the secondary circuit is 6.3 times 4, or 25.2 watts; the power given by the primary is 110 times .228, or 25.2 watts.

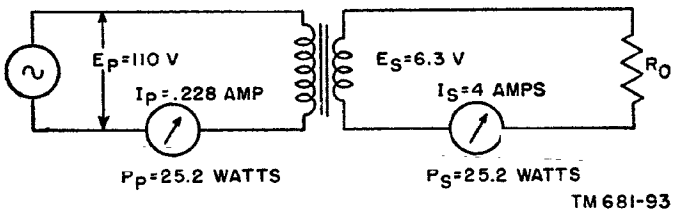


Figure 127. Step-down transformer.

f. In the computation of total power transferred by a transformer with multiple secondary windings, the power in the primary circuit is equal to the arithmetical sum of the amounts of power delivered by the various secondaries. In figure 128 is shown the schematic of a typical power transformer with three secondary windings for use in a conventional radio receiver. As above, the power delivered by S_2 is 25.2 watts; by S_3 is 5 times 2, or 10 watts; by S_1 is 750 times .07, or 52.5 watts. The total power delivered by the transformer is then:

$$P_s = 25.2 + 10 + 52.5 = 87.7 \text{ watts.}$$

Then the power in the primary is approximately the same:

$$P_p = 87.7 \text{ watts.}$$

The current in the primary is:

$$I_p = \frac{P_p}{E_p} = \frac{87.7}{110}$$

$$I_p = .797 \text{ ampere.}$$

The current actually measured in the primary would be approximately 10 percent higher, or .876 ampere, because of the power factor. Then, since true power is equal to $EI \cos \theta$, and E for conventional power transformers is assumed to be 110 volts, a rule of thumb may be used for rapid calculation of true power if primary current is known and a power factor of .9 is assumed:

$$P_{av} = I \times E \times \cos \theta$$
$$= I \times 110 \times .9$$
$$= I \times 100 \text{ (approx).}$$

Thus, multiplying the measured primary current times 100 gives an approximation of the true power as actually delivered:

$$P_{av} = .876 \times 100$$
$$P_{av} = 87.6 \text{ watts.}$$

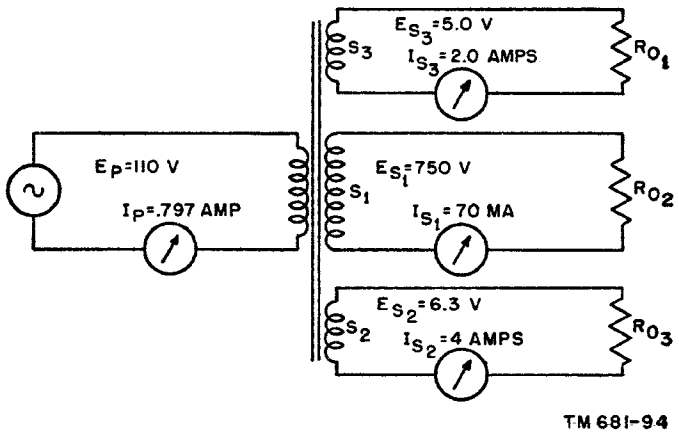


Figure 128. Typical power transformer for radio receivers.

83. Autotransformers

a. The autotransformer, or *self* transformer, is a special type of power transformer which is designed for good voltage regulation under varying loads. A single tapped winding characterizes the autotransformer, as shown in figure 129. When used as a step-up transformer, as in A, all of the primary winding is part of the secondary winding, and when used as a step-down transformer, all of the secondary winding is part of the primary winding, as in B. Voltages across the individual windings follow the turns ratios as previously, but it must be remembered that certain turns are common to both coils, and that generally the coils are tapped for variation of the turns ratio, with resultant change in voltage levels. A disadvantage of this arrangement is that the secondary circuit is not isolated electrically from the primary circuit as it is in a transformer in which the only connection between the source and the load is the magnetic coupling of the coils.

b. It will be remembered that power transformers are designed for a given current load. If the load is lessened, the high voltages induced cause arcing and burning of the transformer. The chief advantage of the autotransformer is that the load may be varied without arcing and with little change in output voltage. This result depends on the fact that primary and secondary currents in any transformer are 180° out of phase and therefore tend to cancel in that part of the transformer which is common to both windings. For example, A of figure 129 shows an autotransformer of 1 to 1½ ratio.

With 100 volts applied to the primary and a 75-ohm load on the secondary, E_s is 150 volts and I_s is 2 amperes. By the turns ratio formula, I_p should be 3 amperes. However, since I_s and I_p are 180° out of phase, the total current in the primary is the algebraic sum of the two, or 1 ampere. If the load on the secondary is increased to 4 amperes, I_p increases to 6 minus 4, or 2 amperes. If the load on the secondary is decreased to 1 ampere, I_p decreases to 1½ minus 1, or ½ ampere. B of figure 129 illustrates this same relationship in terms of a step-down autotransformer with a turns ratio of 1 to ½. With the same voltage applied and a load of 25 ohms, E_s is 50 volts and I_s is 2 amperes. The current in the part of the primary common to both is 1 ampere in the direction of the secondary current. Thus, the primary current of 1 ampere may be conceived as flowing directly to the load and not passing through the primary winding. Any variation in the load causes a smaller change in primary current than is possible with the separate winding power transformer.

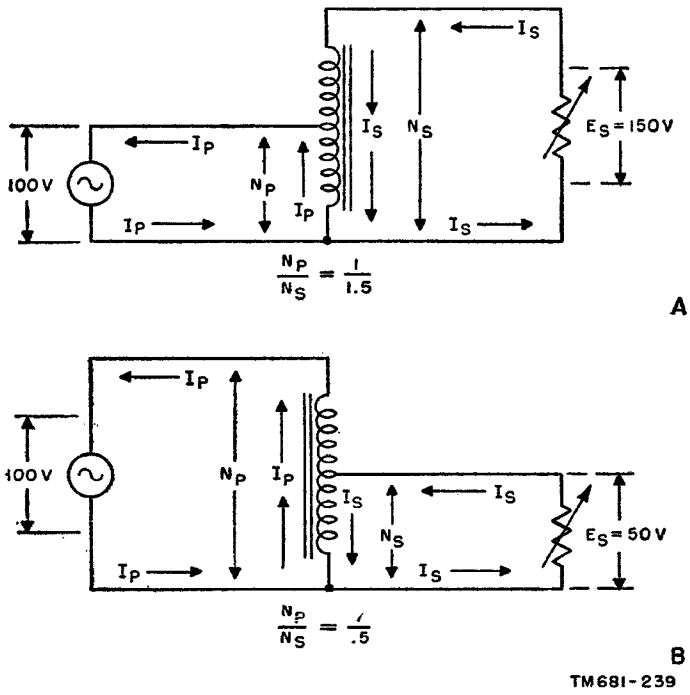


Figure 129. Autotransformer.

c. The further practical effect of these small primary currents for a given load on the secondary is that power loss in the autotransformer is also small. Thus, in the example of A of figure 129, the 1-ohm d-c resistance of the primary winding results in an I^2R loss of 1 watt, 4 watts, and .25 watt for the three loads listed as compared to 9, 36, and 2.5 watts loss in a separate primary. The I^2R loss in the secondary is computed only for that part of the secondary not common to the primary. Hence the fall in voltage, in both primary and secondary, caused by the d-c resistance of the windings under heavy load is much less in an autotransformer. Therefore, these tapped transformers are widely used to vary line voltage for a given load, and to maintain a given output voltage under varying load.

84. Turns Ratio: Impedance

a. In the example of figure 119, a transformer with a primary to secondary turns ratio of 1 to 3 is connected to a 165-ohm load. The applied voltage is 110 volts and the primary current 6 amperes; the secondary supplies 2 amperes at 330 volts. The impedance of the secondary circuit is the impedance of the load, or 165 ohms. The impedance of the primary may be calculated:

$$Z_p = \frac{E_p}{I_p} = \frac{110}{6} = 18.3 \text{ ohms.}$$

The ratio of primary impedance to secondary impedance is:

$$\frac{Z_p}{Z_s} = \frac{18.3}{165} = \frac{1}{9}.$$

Thus, it will be seen that the impedance relationship across this transformer is not equal to the turns ratio of 1 to 3 but to the square of the turns ratio $(\frac{1}{3})^2$ or 1 to 9.

b. The following analysis will show that what is true of this particular example is true generally of all power transformers. Since the voltage ratio is equal to the turns ratio:

$$\frac{E_p}{E_s} = \frac{N_p}{N_s}.$$

And the current ratio is equal to the reciprocal of the turns ratio:

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}.$$

Then by dividing one by the other:

$$\frac{\frac{E_p}{I_p}}{\frac{E_s}{I_s}} = \frac{N_p^2}{N_s^2}.$$

But E_p/I_p is equal to Z_p, the impedance of the primary, and E_s/I_s is equal to Z_s, the impedance of the secondary. Therefore:

$$\frac{Z_p}{Z_s} = \frac{N_p^2}{N_s^2} .$$

Thus, the ratio of impedances across a transformer varies as the *square* of the turns ratio. Then:

$$Z_p = Z_s \times \left(\frac{N_p}{N_s}\right)^2$$

or:

$$Z_s = Z_p \times \left(\frac{N_s}{N_p}\right)^2 .$$

c. For example, a step-down transformer with a turns ratio of 15 to 1 supplies an impedance of 1.5 ohms. The impedance of the primary is:

$$Z_p = 1.5 \times \left(\frac{15}{1}\right)^2$$

$$Z_p = 1.5 \times 225$$

$$Z_p = 337.5 \text{ ohms.}$$

A step-up transformer with a turns ratio of 1 to 50 draws 6 amperes at 6 volts in the primary circuit. The impedance of the primary is:

$$Z_p = \frac{E_p}{I_p} = \frac{6}{6} = 1 \text{ ohm.}$$

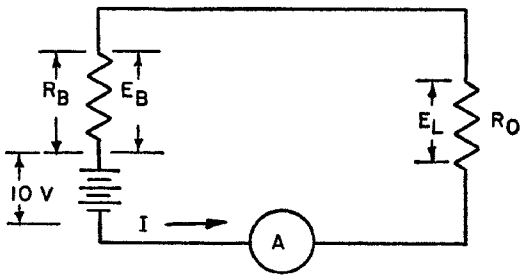
The impedance of the secondary is:

$$Z_s = 1 \times \left(\frac{50}{1}\right)^2$$

$$Z_s = 1 \times 2{,}500 = 2{,}500 \text{ ohms.}$$

85. Impedance Matching

a. In the transfer of power from any electrical source to its load, the impedance of the load must equal, or match, the internal impedance of the source for maximum transfer of power. The simple d-c circuit and table of figure 130 will illustrate this principle. A 10-volt battery with an internal resistance of R_B of 1 ohm feeds an external circuit of variable resistance E_O. When R_O is 4 ohms, the total resistance of the circuit is 5 ohms ($R_B + R_O$), the current in the circuit is 2 amperes, and the voltage drop across the load E_L is 8 volts. Then the power absorbed by the load is E_L times *I*, or 16 watts. The power dissipated in the battery as heat is E_B times *I*, or 4 watts. When R_O is 3 ohms, total resistance is 4 ohms, *I* is 2.5 amperes, and E_L is 7.5 watts. The power absorbed by the load is 18.75 watts, and by the battery is 6.25 watts. When R_O is 2 ohms, total resistance is 3 ohms, I is 3.33 amperes, and E_L is 6.67 volts. The power absorbed by the load is 20 watts, and by the battery is 10 watts. When R_O is 1 ohm, total resistance is 2 ohms, I is 5 amperes, and E_L is 5 volts. The power absorbed by the load is 25 watts, and by the battery 25 watts. When R_O is less than 1 ohm, power falls off in the load as indicated in the table. Thus, *the greatest power is delivered to the load when the impedance of the load is equal to the internal impedance of the source.*



E	R_B	R_O	R_T	I	E_B	E_L	P_L (WATTS)
10	1	4	5	2	2	8	16
10	1	3	4	2.5	2.5	7.5	18.75
10	1	2	3	3.33	3.33	6.67	22.2
10	1	1	2	5	5	5	25
10	1	.5	1.5	6.67	6.67	3.33	2.22
10	1	.33	1.33	7.5	7.5	2.5	18.75
10	1	.25	1.25	8	8	2	16

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Figure 130. Power transfer and impedance.

b. The importance of matching the load to the source for maximum transfer of power makes the transformer useful as an impedance matching device. For example, it often becomes necessary in electronic circuits to match a low impedance load to a high impedance source. A of figure 131 shows such a circuit. The impedance of the source R_G is 10,000 ohms, and of the load 400 ohms. The primary impedance is to match the source, and the secondary impedance is to match the load. The turns ratio of the transformer is:

$$\frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{10{,}000}{400}} = \sqrt{\frac{25}{1}} .$$

Then: $\frac{N_p}{N_s} = \frac{5}{1}$.

If 100 volts are applied to the primary, the secondary voltage is 20 volts. The current in the secondary is 20/400 or .05 ampere; the

current in the primary is 100/10,000 or .01 ampere. B of figure 131 is the equivalent circuit. Since the power in the primary (1 watt) is equal to the power in the secondary, the transformer has matched a 400-ohm load to a 10,000-ohm source with maximum transfer of energy. It may be said then that the source *sees* the primary impedance as a matching impedance, and that the secondary, which by transformer action receives the power of the primary, *sees* the load impedance as a matching impedance.

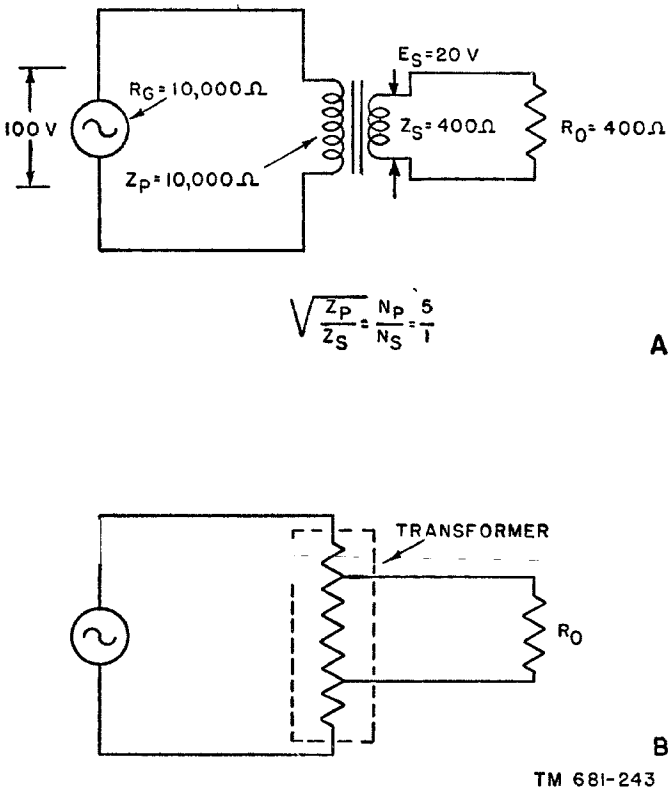


Figure 131. Impedance matching transformer and equivalent circuit.

86. Reflected Impedance

a. In the analysis of transformer circuits thus far in this chapter, the load on the secondary has been assumed to be a pure resistance. Under this condition, the load appears to the generator source as a pure resistance but with the voltage and current of the primary displaced by 180° in the secondary. Because of this 180° phase shift, the load on the secondary is said to be a *reflected* or *image* load in the primary circuit. In terms of a resistance load, this phase shift has no further significance beyond reversed polarity, but if the load is partly inductive or capacitive some interesting possibilities emerge. Thus, if the load on the secondary of a transformer is inductive, the current in the circuit lags the voltage by some phase angle θ_s. This phase angle is reflected back into the primary circuit as the same angle but shifted by 180°, or as an angle of $(180° + \theta_s)$. Because the reactive components of inductors and capacitors act in the opposite manner, always 180° out of phase, it will be noticed that the effect of an inductive reactance in the secondary circuit is to reflect a *capacitive* reactance in *series* with the primary circuit. The effect of a capacitive reactance in the secondary circuit is to reflect an *inductive* reactance in series with the primary circuit.

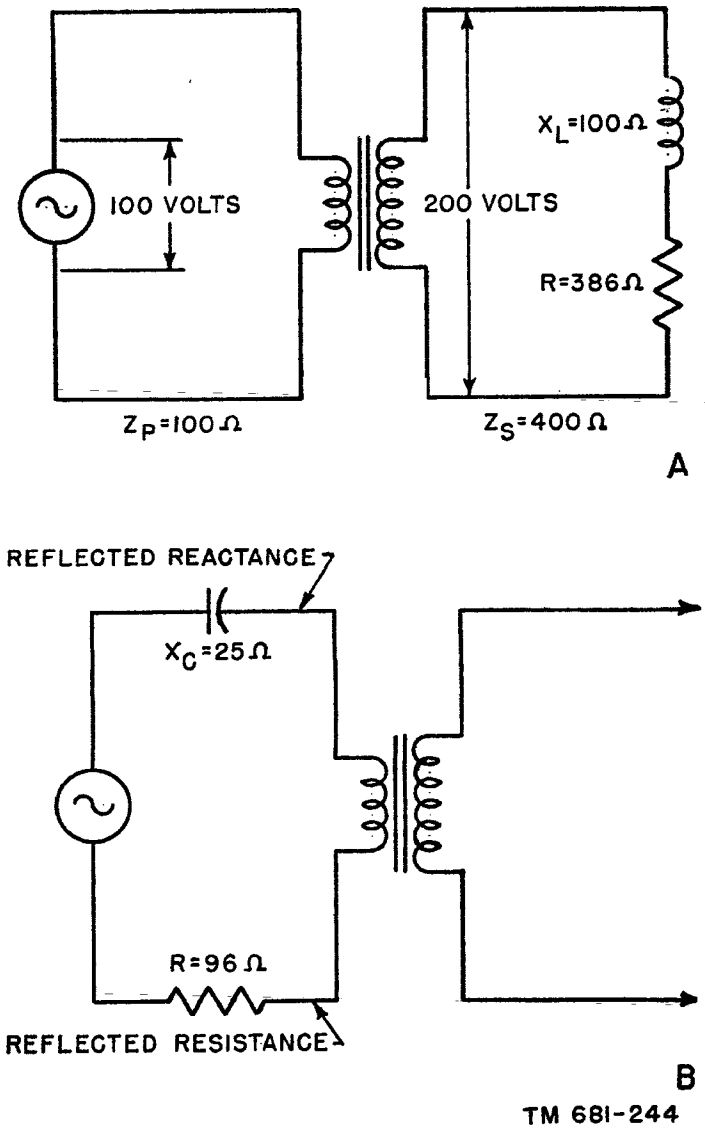


Figure 132. Inductive load and equivalent primary circuit.

b. In A of figure 132 the schematic of a transformer circuit with an L-R load of 400 ohms is shown. The turns ratio is 1 to 2 with 100 volts applied. The inductive reactance is 100 ohms and the resistance 386 ohms. B is the equivalent primary circuit in which the 100 ohms of inductive reactance is shown as a reflected capacitive reactance of 25 ohms, and the resistance is shown as a reflected resistance of 96 ohms, since each of these impedances is reflected back into the primary circuit decreased in magnitude by the square of the turns ratio. Thus, the total impedance of the primary circuit is 100 ohms. It should be noted, however, that although the reflected capacitive reactance cancels part of the primary winding inductive reactance, the lagging current of the secondary does not effect an over-all leading primary current, since in an ideal transformer (perfect coupling: K equal to 1) the X_C reflected can never be greater than an amount that will just cancel the inductive reactance of the primary. A and B of figure 133 show the same circuit in terms of an *R-C* load on the secondary and the effective inductive reactance reflected in series in the primary circuit.

c. This concept of reflected impedance may be explored further. Thus, in previous studies in this manual it was demonstrated that inductances and capacitors always react in opposite ways, and that an inductance *in series* with a circuit is the equivalent of a capacitor *in parallel,* or that a capacitor in series is the equivalent of an inductance in parallel. For example, in B of figure 132, the reflected inductive load was shown as a capacitor in series with the primary circuit. Since the effect of this hypothetical capacitor is to cancel some of the inductance present, it can be conceived of as an inductance *in parallel* with the inductance of the primary (the total inductance of two inductors in parallel is less than the smaller inductance), as shown in A of figure 134. Then, since a capacitive load is reflected as an inductance in series, it may be conceived of as a capacitor in parallel with the primary inductance, since its effect is to add to the total reactance of the circuit (see the analysis of the parallel *L-C* circuit of chapter 5). This circuit is shown in B of figure 134. It should be emphasized, however, that these variations in point of view of the phenomenon of reflected impedance are purely conceptual;

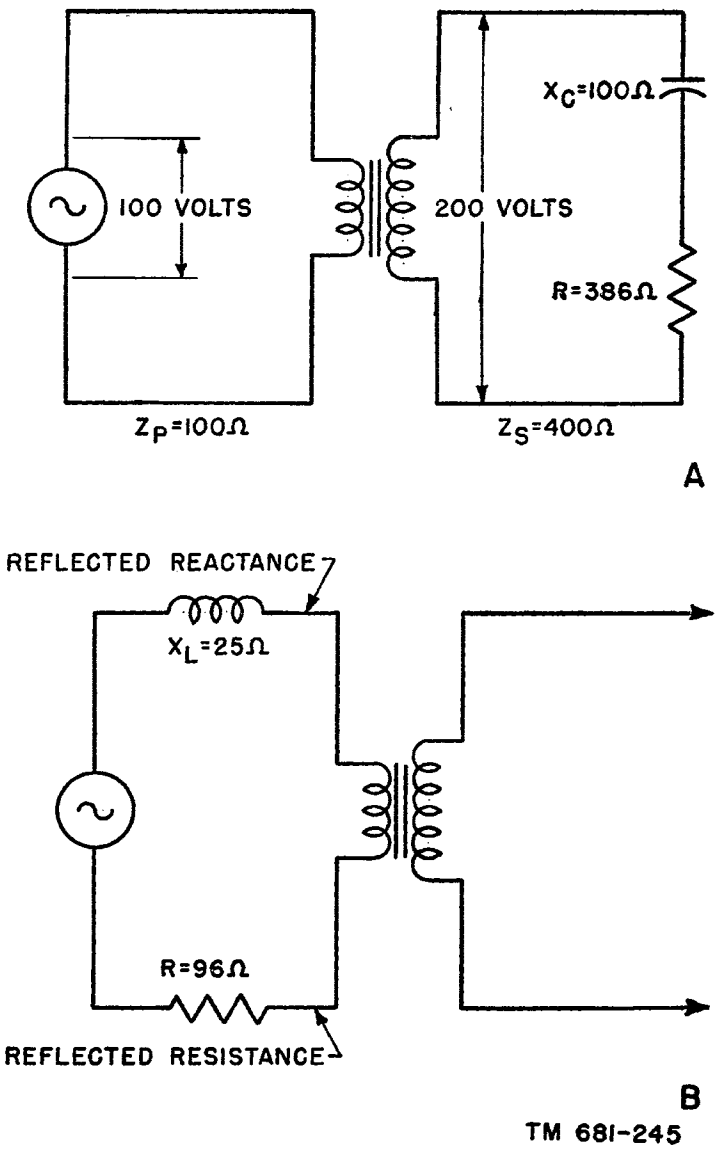


Figure 133. Capacitive load and equivalent primary circuit.

that is, they represent an *as if* situation, not a physical situation, but such concepts are sometimes helpful in the understanding of circuit theory.

d. The power transformer circuit of an automobile receiver illustrates the effect of a reflected impedance and shows the practical importance of the foregoing variations in ways of looking at the transformer. Figure 135 shows this circuit. It will be noticed that the transformer primary is fed pulses of direct current rather than the usual alternating current because the 6-volt car battery is the only source of power. The make-and-break circuit of the vibrator interrupts the direct current to make the pulses, but this mechanical device is

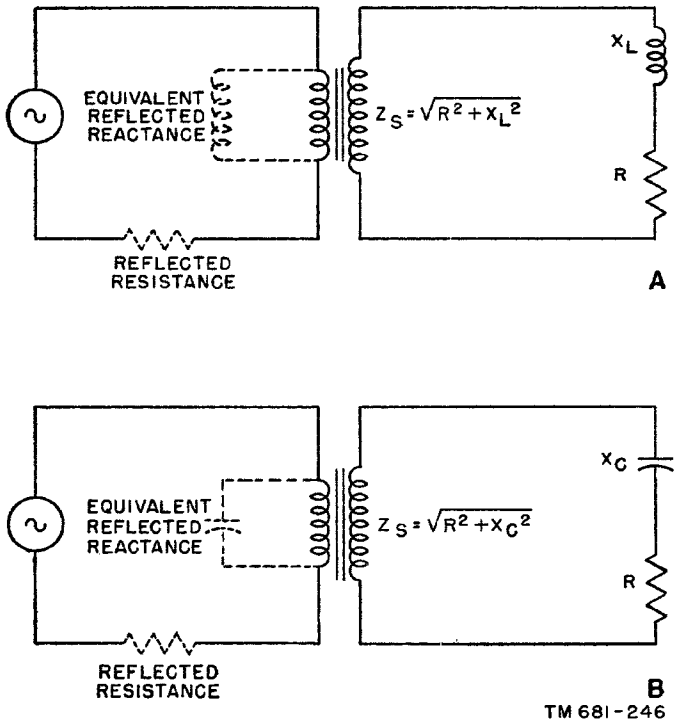


Figure 134. Equivalent reactances for inductive and capacitive loads.

subject to arcing at the points, and the transformer circuit is subject to high instantaneous voltages because of the high rate of change of the current as the circuit is broken. The effect of this sharp peak of voltage may be lessened by the action of a capacitor placed across the primary circuit. This capacitor would be charged while current flows in the primary, and when the circuit is broken would discharge through the primary, thus preventing current in the circuit fom falling instantaneously to zero. However, because of the large amount of current flowing in such a circuit (more than 5 amperes) the capacitor must be very large to provide sufficient current to operate as described. Moreover, two capacitors would be necessary, one for each half of the primary when the vibrator switches from one circuit to the other. This problem has been effectively solved by placing the small capacitor C_1 (called a buffer capacitor) across the secondary of the transformer, resulting in a large reflected capacitance across the primary.

e. For example, the capacitor in the circuit shown is .01 μf. Since the frequency of the usual commercial vibrator is approximately 120 cycles per second, the capacitive reactance developed across this capacitor is:

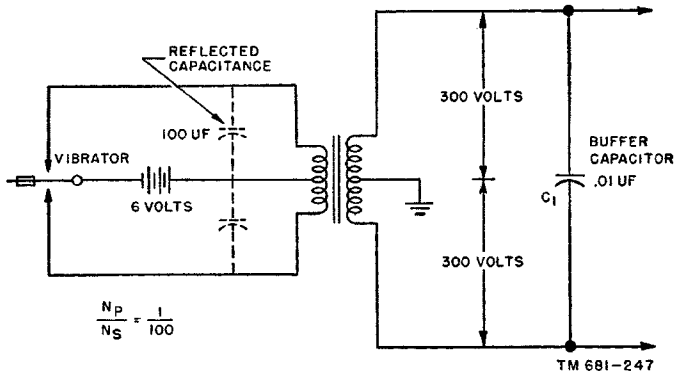


Figure 135. Power transformer of an automobile receiver.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{6.28 \times 120 \times .01 \times 10}$$

$$X_C = \frac{10^6}{7.536} = 133{,}000 \text{ ohms.}$$

The turns ratio of this transformer may be determined by the voltage ratio, since the 6-volt primary develops 600 volts across the whole secondary (300 volts each side of center). Then:

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{6}{600} = \frac{1}{100}.$$

The reflected reactance is:

$$Z_p = Z_s \left(\frac{N_s}{N_p}\right)^2 = 133{,}000 \times \frac{1}{10{,}000}.$$

$$Z_p = 13.3 \text{ ohms.}$$

At this point it must be remembered that although the reactance reflected into the primary is small, the *capacitance* effecting this reactance at the frequency of the vibrator must be very large, since capacitive reactance and capacitance are inversely proportional to each other. From:

$$X_C = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{6.28 \times 120 \times 13.3}$$

$$C = \frac{1}{10{,}011.88} = .0001 \text{ farad}$$

$$C = 100 \ \mu\text{f.}$$

The effect of the buffer capacitor then is to reflect a 100-μf capacitor into the primary circuit. Such a capacitor is 10,000 times the size of the buffer capacitor. This reflected capacitance may be computed directly from the turns ratio by assuming the reactance to be the impedance of the circuit. Then:

$$\frac{X_C \text{ (primary)}}{X_C \text{ (secondary)}} = \left(\frac{N_p}{N_s}\right)^2$$

or:

$$\frac{\frac{1}{2\pi fC_p}}{\frac{1}{2\pi fC_s}} = \left(\frac{N_p}{N_s}\right)^2.$$

Canceling the factor $2\pi f$:

$$\frac{C_s}{C_p} = \left(\frac{N_p}{N_s}\right)^2.$$

And: $$C_p = C_s\left(\frac{N_s}{N_p}\right)^2.$$

It may be concluded then that capacitance across a transformer is *inversely* proportional to the square of the number of turns. In like manner, an inductance in the secondary reflects an inductance across the primary *directly* proportional to the square of the number of turns:

$$\frac{L_p}{L_s} = \left(\frac{N_p}{N_s}\right)^2.$$

And: $$L_p = L_s\left(\frac{N_p}{N_s}\right)^2.$$

Thus, the 100-ohm inductive reactance of A of figure 132 represents a certain inductance at the power line frequency of 60 cycles. Since:

$$X_L = 2\pi fL$$

$$L = \frac{X_L}{2\pi f} = \frac{100}{376.8}$$

$$L = .265 \text{ henry.}$$

The turns ratio of this circuit is 2 to 1, and the reflected reactance is 25 ohms, representing an inductance of .066 henry, or:

$$L_p = .265 \times \left(\frac{1}{2}\right)^2$$

$$L_p = .066 \text{ henry.}$$

87. Audio Frequency Transformers

a. GENERAL.

(1) Thus far in the discussion of transformers and the transfer of power, the 60-cycle frequency of the power

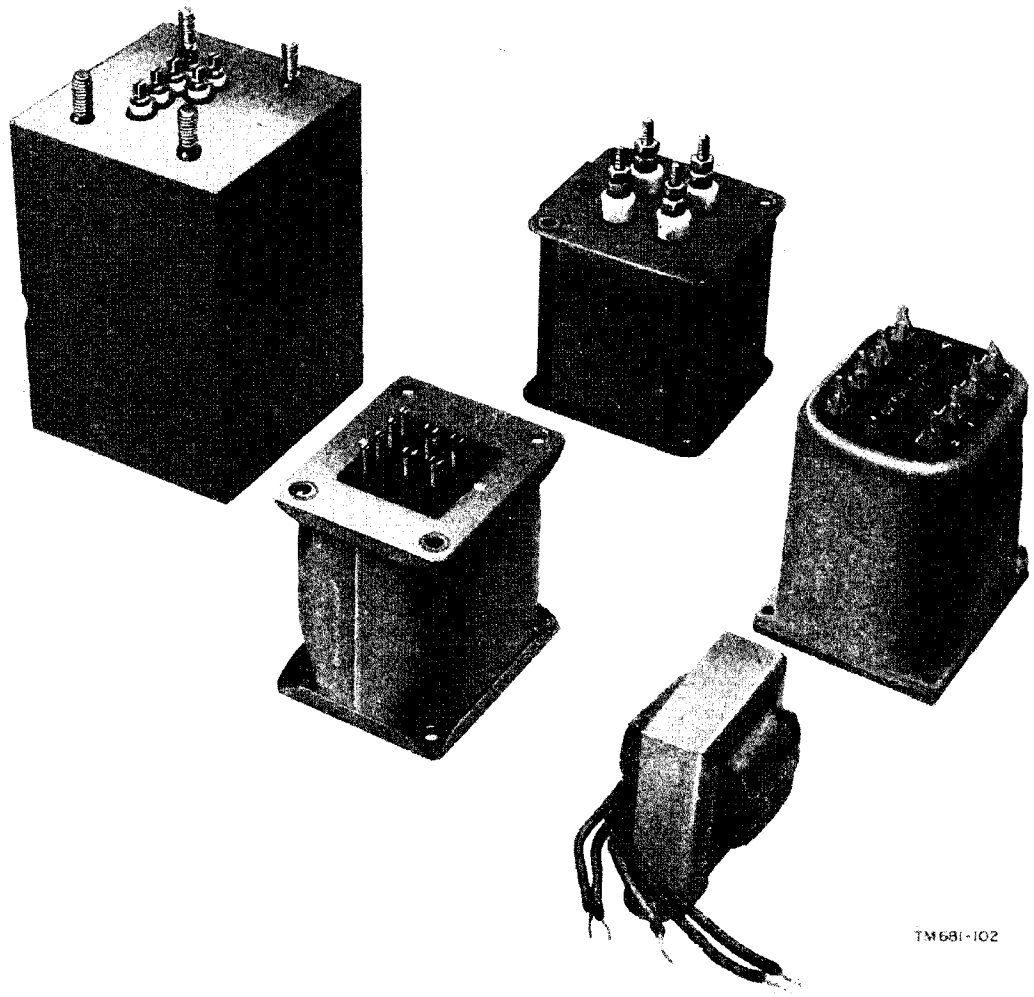


Figure 136. Audio frequency transformers.

line has been assumed. At frequencies between 20 and 20,000 cps (the audio frequencies), iron-core transformers are also used as power transfer and impedance matching devices. A selection of these audio frequency transformers is shown in figure 136. These audio frequency transformers find wide use in vacuum-tube amplifiers as microphone transformers, interstage transformers, output transformers, and as modulation transformers for transmitter units. However, as frequency increases above 60 cycles, the hysteresis and eddy current losses of the core increase, and the disadvantageous effect of distributed capacitance across the windings of the transformer becomes more apparent. The effect of either or both of these losses would be constant for a given audio frequency (for example 1,000 cps), but the frequencies of the human voice and of the musical scale vary constantly within the range of the human ear, and therefore the losses vary as frequency varies, resulting in some frequencies appearing at the output amplified more than others. The transformer, like the amplifier of which it is an essential part, must if possible pass all of these frequencies at the same level of amplification, if the voice or music is to be reproduced in its original form. An ideal audio frequency amplifier would pass all frequencies between 20 and 20,000 cps without differentiation, as shown in A of figure 137, which is a graph of voltage output plotted against frequency. Such an amplifier is said to have a *flat* response between the limits for which it is designed. Since the transformer is an integral part of the amplifier it also must show a flat response. However, such a flat response to all of the audio frequencies is difficult to achieve, and transformers which pass the principal part of the band (30 to 15,000 cps) are considered excellent. Good but less expensive transformers pass 60 to 10,000 cps, and those used in the ordinary well made radio receiver pass 100 to 7,000 cps. Audio transformers designed for voice transmission only may pass a band of frequencies from 150 to 3,000 cps, which is sufficient range for human speech although an unfair treatment of even the worst orchestra. Both the bass notes and the upper treble notes are attenuated and rendered inaudible, as shown in the graph of B of figure 137 (the dotted line ideal response curve is included for comparison). On such transformers, 60-cycle and 120-cycle hum from the power supply is inaudible.

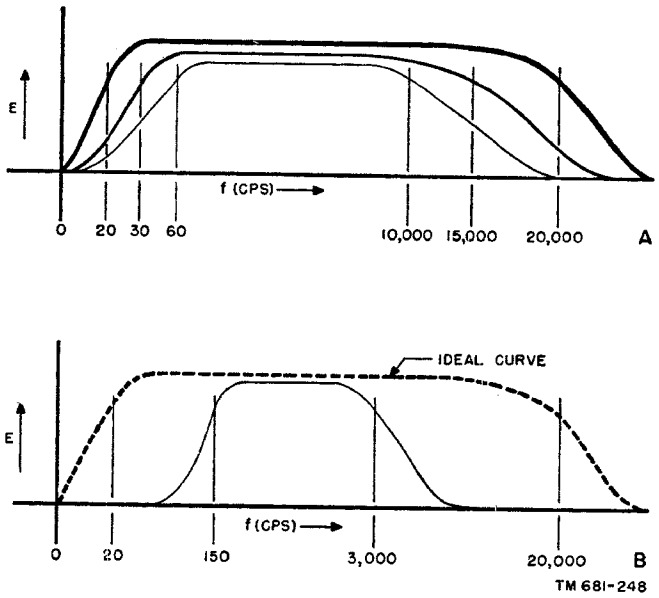


Figure 137. Frequency response curves.

(2) In the general analysis of power transformers, it will be remembered that ideally a transformer should contain as much inductance as possible to insure the greatest possible amount of inductive coupling (flux linkage) between the coils. Since inductance depends directly on the permeability and size of the core and on the number of turns, every iron-core transformer, as was noted earlier, must be a compromise between the size of the core and the number of turns. Too large a core increases core losses and too many turns increase copper losses. As fre-

quency increases, hysteresis and eddy current losses in the core increase appreciably, much more than the increase in skin effect in the windings, and therefore audio transformers in general contain small iron cores and thousands of turns of wire, achieving inductances from 50 to 100 henrys for a given winding. If the transformer carries a small amount of power, this wire may be very fine (#36 gage). At the low audio frequencies, however, the primary shows less reactance, not less inductance, than at the high audio frequencies, and more current must flow into the primary to maintain flux linkage. This increase in magnetizing current represents a loss in voltage or power, and the lower frequencies are accordingly passed through the transformer at less voltage than the higher frequencies. At the higher audio frequencies, the distributed capacitance accumulated in the great number of turns of wire in both primary and secondary windings attenuates the higher audio frequencies. In particular, the distributed capacitance of the secondary, approximately 100 $\mu\mu$f, is reflected back into the primary as a shunt capacitance multiplied by the square of the turns ratio. Thus, as was shown in the discussion of reflected impedance, a step-up transformer with a turns ratio of 1 to 50 reflects 100 $\mu\mu$f as a capacitance of 250,000 $\mu\mu$f (.25 μf) into the primary circuit. Such a capacitance directly across the primary effectively short-circuits the higher audio frequencies, having a reactance less than 50 ohms at 10,000 cps.

b. MICROPHONE TRANSFORMER. The carbon button microphone, which is widely used in communications equipment, has an internal impedance of 100 to 200 ohms. The small amount of energy developed by this microphone must be amplified in order to be heard, but the low impedance microphone cannot be connected directly to the input circuit of a vacuum-tube amplifier, which may have an impedance of approximately 200,000 ohms. An impedance matching transformer, called a microphone transformer, is placed between the microphone and the first vacuum tube, as shown in figure 138. The distributed capacitance of the secondary winding and the capacitance reflected into the primary are shown as dotted-line capacitors. The battery is necessary as a source of power for the microphone. Since the impedances to be matched are as shown, the necessary turns ratio of the transformer may be determined:

$$\frac{N_p}{N_s} = \frac{Z_p}{Z_s} = \frac{100}{200{,}000}$$

$$\frac{N_p}{N_s} = \frac{1}{2000} = \frac{1}{44.6}.$$

The 100-$\mu\mu$f distributed capacitance of the secondary is reflected into the primary as 100 times the square of the inverse of the turns ratio, or:

$$C_p = 100 \times 2{,}000 = 200{,}000\ \mu\mu\text{f}$$

$$C_p = .2\ \mu\text{f}.$$

Thus, the low impedance of the microphone has been effectively raised to the high impedance of the vacuum tube for maximum transfer of the small energy available.

c. INTERSTAGE TRANSFORMER. The amplification of the microphone signal as shown in the schematic diagram is usually insufficient for use and the output signal of the vacuum tube is coupled into a second stage of amplification by means of an interstage transformer (fig. 139). In this case, the problem of energy transfer is somewhat different. Since the input circuit of the second vacuum tubes draws little or no power, the principal object is not power transfer but high voltage transfer. High voltage may be obtained by increasing the load impedance to a value greater than the internal impedance of the source, as shown previously in the table of figure 130. For instance, when R_L is greater than R_B, then E_L, the voltage drop across R_L, is greater than at any other value, although power transfer is less. Hence, if the impedance of the primary circuit of the interstage transformer of figure 139 is higher than the internal output impedance of the vacuum tube, a greater voltage will appear across the primary and, by induction, across the secondary. Thus, the usual 1-to-3 turns ratio transformer with 500,000 ohms impedance across the secondary (the input impedance of the second tube) shows a high primary impedance:

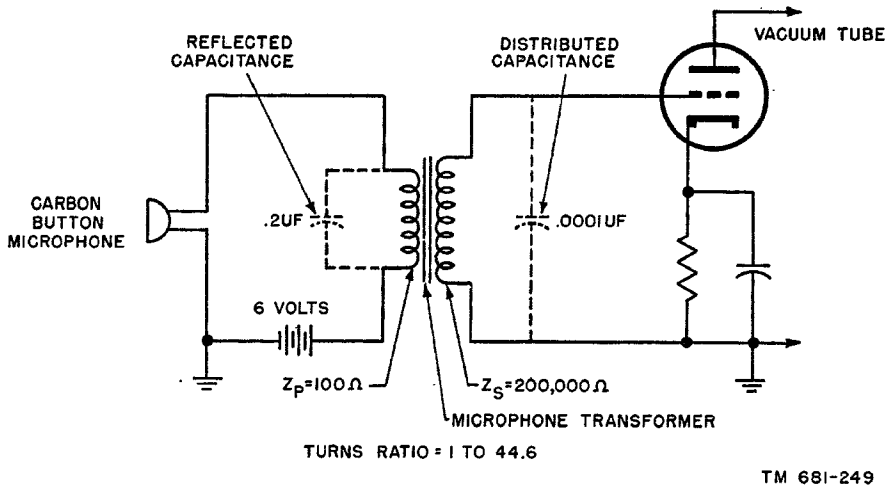


Figure 138. Microphone transformer.

$$Z_p = Z_s \left(\frac{N_p}{N_s}\right)^2 = 500{,}000 \times \left(\frac{1}{3}\right)^2$$

$$Z_p = \frac{500{,}000}{9} = 55{,}555 \text{ ohms.}$$

Assume that the output impedance of the first vacuum tube is approximately 10,000 ohms. Thus, the deliberate mismatch of this 10,000 ohms to the 55,555 ohms of the primary gives a large voltage across the primary, and this voltage is tripled in the secondary, achieving a high voltage input to the second vacuum tube. The effect of the distributed capacitance reflected into the primary is small because the turns ratio is small, being 900 $\mu\mu$f or .0009 μf.

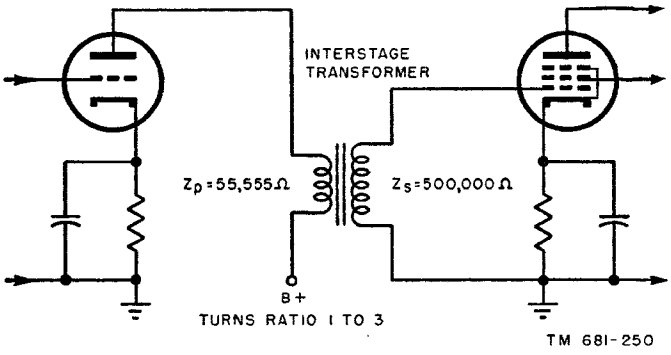


Figure 139. Interstage transformer.

d. OUTPUT TRANSFORMER. The problem of matching the output of the second tube to the loudspeaker is once again a problem in the transfer of power, not of voltage alone. Since the cone of the speaker has physical mass and moves against air, power is required to move it. Therefore, the power output of the tube must be delivered to the cone of the speaker with as little loss as possible. Figure 140 shows the schematic of this circuit. The output impedance of the tube is 7,000 ohms, and the impedance of the coil attached to the cone is 8 ohms. The turns ratio of the transformer should be:

$$\frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}} = \sqrt{\frac{7{,}000}{8}} = \sqrt{\frac{875}{1}}$$

$$\frac{N_p}{N_s} = \sqrt{\frac{875}{1}} = \frac{29.4}{1}.$$

At this turns ratio, the 8 ohms impedance of the speaker appears as a 7,000 ohms impedance to the output of the vacuum tube. Figure 141 is the complete schematic of the amplifier, showing microphone, interstage, and output transformers in their proper sequence.

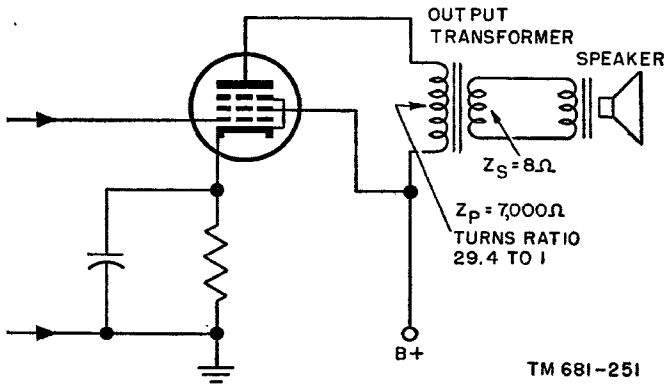


Figure 140. Output transformer.

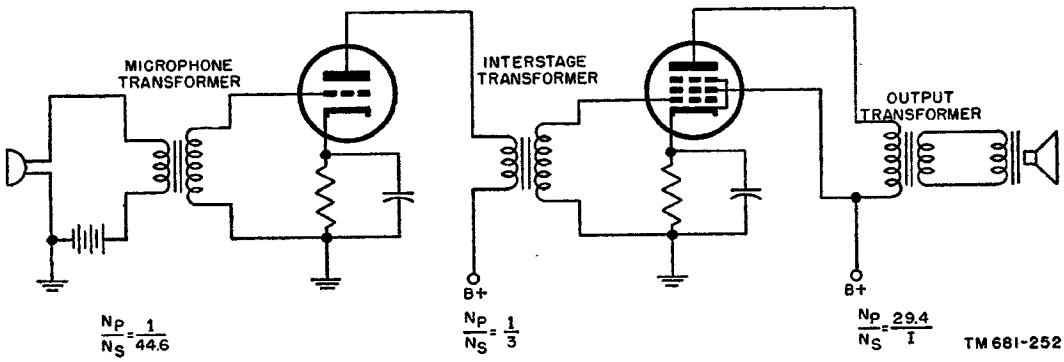


Figure 141. Transformer-coupled audio frequency amplifier.

88. Radio Frequency Transformers

a. In the previous section of this chapter, the power and audio frequency transformers described are primarily power transfer or impedance matching devices in which very high mutual coupling is obtained by means of the large number of turns and the soft steel core. At radio frequencies, eddy current and hysteresis losses in a magnetic core become prohibitive, and therefore r-f transformers are generally of air-core construction. Eddy current loss is the more important factor since it increases as the square of the frequency; hysteresis loss, being constant for each cycle, increases directly with frequency. With air-core construction, the tightest possible coupling results in a coefficient of coupling of about .65 compared to the .98 of well designed iron-core transformers. A large part of the primary and secondary windings of these partially coupled transformers constitutes a leakage reactance and therefore voltage, current, and impedance ratios cannot be calculated by the turns ratio equations, which are based on an assumption of almost perfect coupling. Hence, the turns ratios of radio-frequency transformers have no exact significance. In fact, the windings of these transformers are generally very loosely coupled (K may be as small as .005), and they function primarily as devices coupling two circuits rather than as voltage or current level transformers. At radio frequencies, the reactance of even a small inductance is very high because the rate of change of the current is high, and therefore the high mutual reactance between primary and secondary makes these transformers effective for impedance matching, for power transfer at low loss, and for high voltage gain through resonance, as will be noted in the following discussion.

b. The typical radio frequency transformer consists of an untuned primary, which is connected to the output circuit of a vacuum tube, and a tuned secondary connected to the input circuit of the following vacuum tube (A of fig. 142). The two coils are wound next to each other, or one over the other, for close coupling, and with the number of turns in each dependent on the function of its particular circuit. Thus, the primary winding has many turns because its principal function is to offer high impedance to the first vacuum tube; the secondary winding is determined by the amount of inductance needed for the band of frequencies to be tuned. When the resistance of the primary circuit is much greater than the inductive reactance of the primary, the resonance curve of current in the secondary circuit is as shown by the solid line curve of B of figure 142. When the reactance of the primary is greater than the resistance of the primary circuit, the Q of the secondary circuit falls and its resonant frequency is shifted to a higher frequency, as shown by the dotted line curve in B. This effect is caused by the capacitance of the secondary circuit—the load on the secondary winding of the transformer—being reflected into the primary circuit as an inductance in series with the primary winding. Such an inductance reduces the voltage in the primary winding and hence in the secondary winding, lowering the Q of the circuit. This reduction in effective primary inductance lowers the effective secondary inductance, causing the circuit to resonate at a higher frequency than that indicated by its values of L and C.

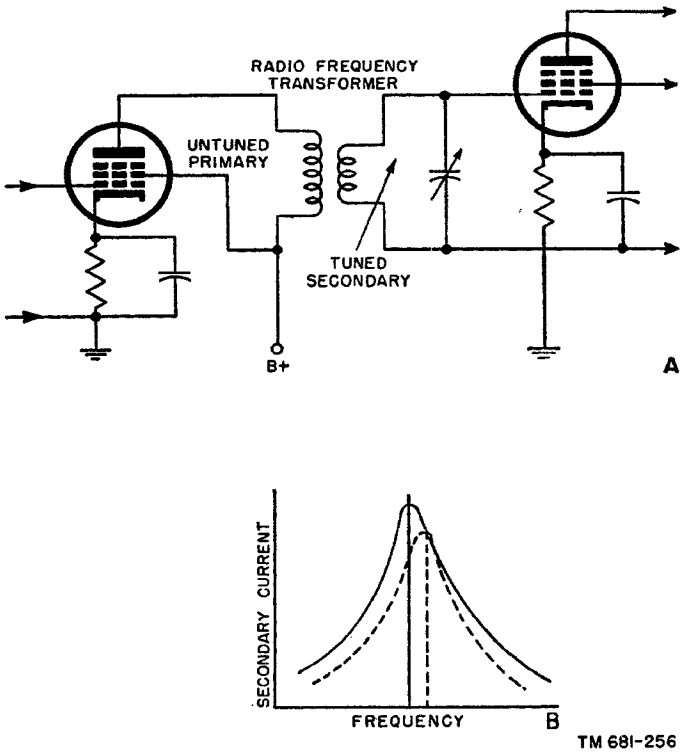


Figure 142. Radio frequency transformer with untuned primary.

c. The use of loose coupling at radio frequencies is illustrated by the i-f (intermediate frequency) transformer of a typical radio receiver. Such a transformer, shown in figure 143 is designed to pass 455 kilocycles with a 10-kc bandwidth. It will be noted that the primary winding is an inductance that is part of a parallel resonant circuit, and that the secondary is part of a series resonant circuit, as explained in the discussion of resonance in chapter 5. If these windings were tightly coupled, the energy transfer would be maximum, but the leakage reactance would be small—the inductances of the windings effectively cancelled by the transformer action; when loosely coupled, each winding is largely a leakage inductance and operates effectively as the inductance of a tuned circuit. Since each winding is part of a resonant circuit, the voltage gain of the circuit is determined by the Q of the circuit and not by the turns ratio of the windings. Hence, the turns ratio is usually 1 to 1, and the inductances and capacitances for resonance are equal.

d. The extent of the coupling between the windings of an i-f transformer is determined by the band pass of the circuit. Thus, A of figure 144 shows the usual resonance curves

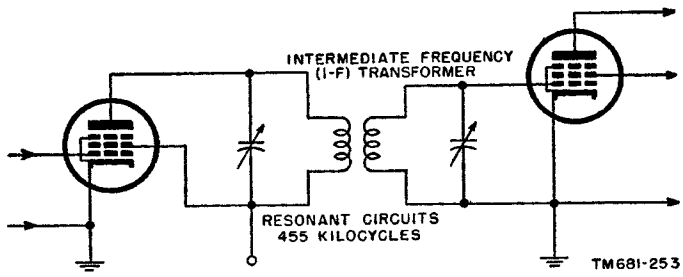


Figure 143. Intermediate frequency transformer.

of primary and secondary currents superimposed and with little coupling between them (K equal to .005). Since coupling is small, the amount of energy induced in the secondary circuit is low, and the band pass is narrow. As coupling is increased (K equal to .01), the Q of the primary circuit is lowered and the band pass is broadened, with a resultant dip in current at resonances because of the resistance reflected by the secondary into the primary circuit. Above and below resonance, the secondary reflects a reactance into the primary circuit which cancels some of the reactance of the primary, and hence larger currents flow at these points, as revealed by the peaks in the primary resonance curve shown in B. The secondary current curve is now taller and broader because of a greater induced energy, and the band pass of 10 kc is achieved between the .707 points on the curve. This coupling is known as the critical, or optimum, coupling. C and D show the effects of overcoupling in that attenuation of the center frequencies is revealed by the dips in the current curves. Such a transformer is a means of connecting together two resonant circuits by mutual reactance and in actuality is not so much a transformer as a band-pass filter, as described in chapter 5.

e. The construction of i-f transformers is shown in figure 145. The distance between the two coils is an indication of the coupling; overcoupled coils are close together, and loosely coupled coils are widely spaced on the shaft of insulating material. The variable capacitors across the transformer in the schematic of figure 143 are built into the assembly so that the whole unit may be shielded from other circuits by the can cover shown in figure 146. Some i-f transformers use fixed capacitors and vary the inductance of the coil by so-called *slug* tuning. In this type of transformer, a magnetic core

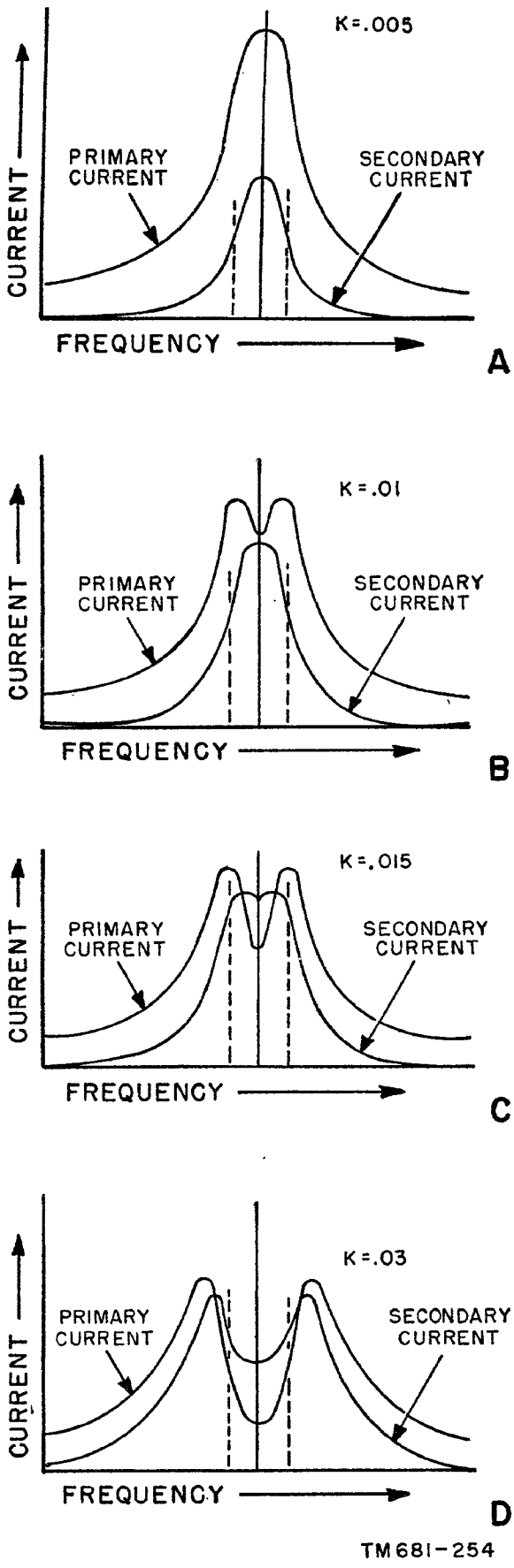


Figure 144. Coupling curves of an i-f transformer.

of powdered iron particles, mixed with a nonmagnetic binder and cast into a slug, may be moved in and out of the inductance, thus changing inductance by varying the geometry of the core. The particles of iron in this core are separated by the binder, and accordingly eddy current and hysteresis losses are held to a minimum. In addition, a given inductance requires fewer turns of wire when a magnetic core is used, thus reducing a-c resistance from the resistance of the wire itself and from skin effect, and increasing the Q of the circuit. Both the capacitor-tuned i-f transformer and the slug-tuned i-f transformer are built into metal cans for shielding, and provision is made for varying capacitance or inductance through the top of the can, as shown in the pictorial of figure 146. The function of the shield is to prevent the electromagnetic fields of the coils from inducing voltages in circuits physically near the i-f transformer. The metal can, being itself a conductor, represents a single-turn secondary into which is induced a voltage that sets up a current counter to the original force, thus preventing the flux lines from passing beyond the can. From another point of view, the metal can short-circuits the electric field which always accompanies a magnetic field, thus effectively preventing both from interacting with other circuits. It should be noted that a well made, well grounded shield has negligible resistance, and the reactance reflected is only the amount due to a single turn.

f. Radio frequency transformers which handle large amounts of power may be illustrated by the antenna-coupling circuit of a transmitter, shown by A of figure 147. The primary resonant tank in the plate circuit of the last power amplifier of the transmitter is inductively coupled through the transformer to the series resonant circuit of the antenna. The factors of critical and optimum coupling, noted in the discussion of i-f transformers, are present in this circuit since both primary and secondary circuits are resonant; that is, the coupling is adjusted for proper bandwidth, and the power delivered to the antenna is a function of the Q of the secondary circuit and of the coupling possible at a given bandwidth. In tuning such a coupling circuit, the primary tank is brought to resonance at the desired frequency (line current falls to a minimum); then the

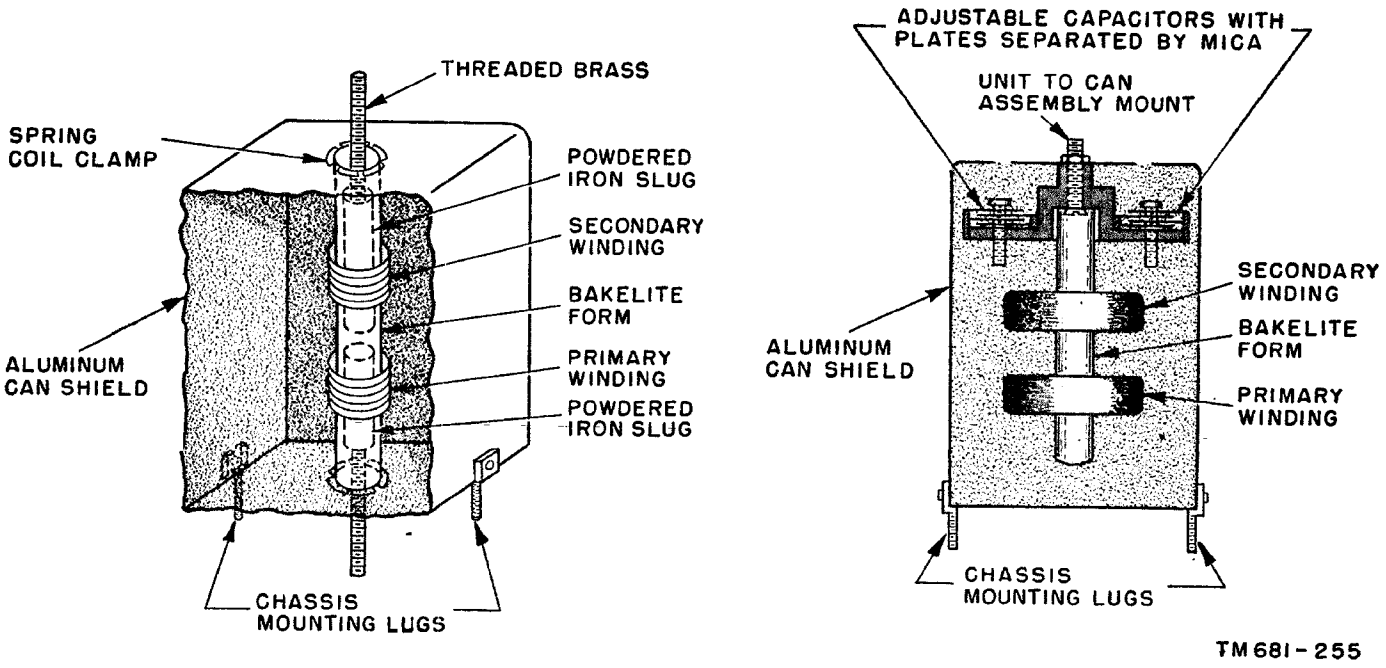


Figure 145. Details of i-f transformer construction.

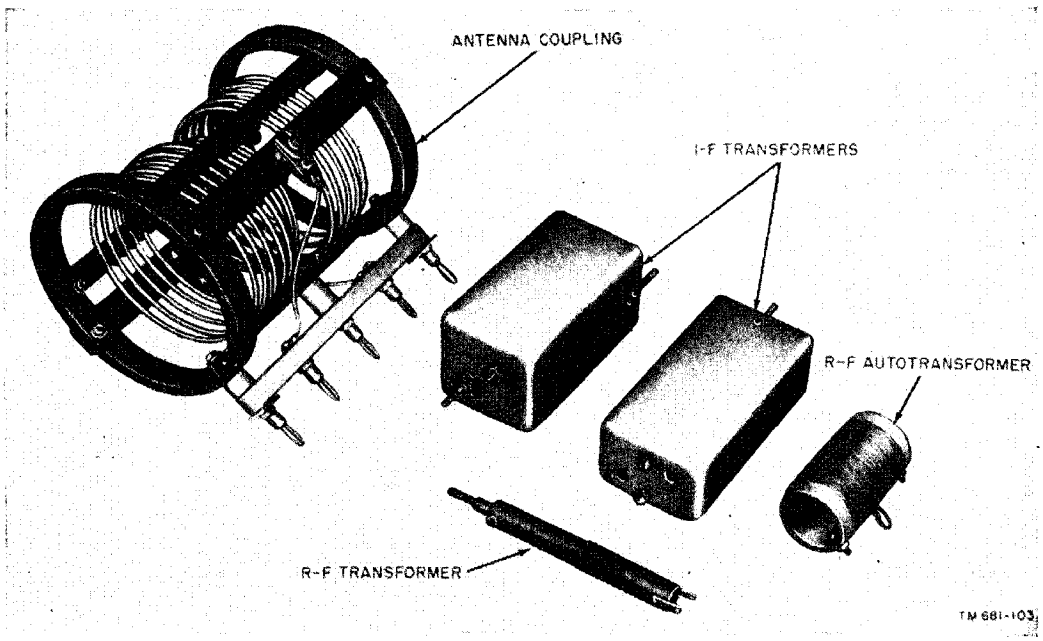


Figure 146. Radio frequency transformers.

antenna circuit is brought to resonance as indicated by a maximum rise in current in the antenna circuit. This action reflects both the resistance and the reactance of the antenna circuit into the primary circuit, thus loading and, at the same time, detuning the primary. The reactance reflected into the primary by the secondary must be compensated for by retuning the primary to resonance, or until the reactance of the secondary is just canceled by the reactance of the primary and a resonant condition set up across the transformer. B of figure 147 shows the equivalent circuit in which the inductive reactance (points *T–E*) of the antenna circuit is just balanced by the capacitive reactance (points *I–T*) of the primary. Then, the pure resistance of the generator (points *H–I*) sees the pure resistance of the antenna circuit R_L, and maximum power is transferred across the transformer.

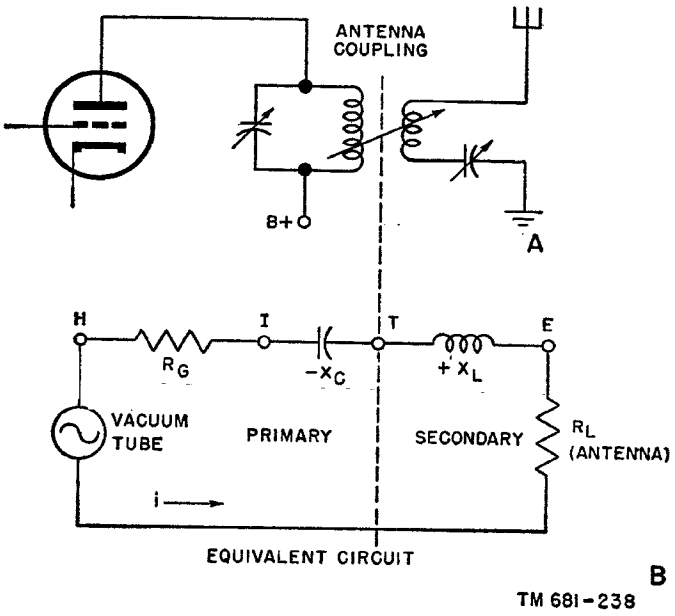


Figure 147. Antenna-coupling transformer.

89. Impedance Matching at Radio Frequencies

a. In the discussion of power transfer and impedance matching earlier in this chapter, the effects noted were dependent on almost complete mutual coupling as it exists in power transformers and audio frequency transformers. At radio frequencies, the coefficient of coupling is smaller and power transfer is accordingly less. In the previous section, however, it was noted that resonance offset the effect of loose coupling and considerable power transfer takes place. Impedance considerations are also important in these double-tuned circuits. For instance, the primary tuned circuit of the i-f transformer of figure 143 is a parallel resonant circuit offering extremely high impedance to the preceding vacuum tube—a desirable condition, since the internal impedance of the tube is high and a large percentage of the signal voltage available is developed across the primary. The tuned secondary circuit of the same schematic, although a series resonant circuit, shows also a very high impedance across the capacitor alone, and therefore a high impedance to match the input of the second vacuum tube.

b. The maximum transfer of power from one circuit to another at radio frequencies is, however, often necessary. In such cases, a radio-frequency impedance matching transformer is used, but as has been noted, the turns ratio of a radio-frequency transformer has no particular significance, and so the impedance match cannot be determined by means of the familiar formula. In general, however, the smaller number of turns is the low-impedance side, and the greater number of turns is the high-impedance side of the transformer. Figure 148 shows a 600-ohm impedance transmission line from an antenna system matched to the output circuit of a transmitter. Under ideal conditions, the characteristic impedance of the line is 600 ohms of pure resistance, which is reflected into the primary circuit as a higher impedance matching the output of the vacuum tube. Since the impedance reflected is dependent on the coupling, the distance between the coils of the transformer is adjusted until the resistance reflected into the primary loads the vacuum-tube source for maximum power output. If the transmission line shows a reactive component, this component is reflected into the primary as its image reactance and may be compensated for by either increasing or decreasing the main tuning capacitance, or by placing a capacitor across the secondary, as shown by the dotted line of figure 148. It should be noted, however, that a capacitor across the secondary does not tune the secondary to resonance but merely cancels the reactive component present in the transmission line.

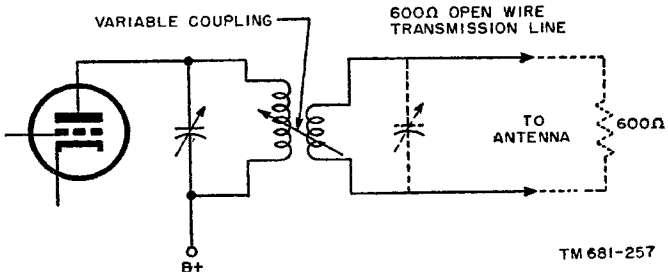


Figure 148. Antenna coupling: impedance match.

c. Two nonresonant impedances may be matched without appreciable loss by means of a matching transformer, or a transmission line considered as a matching transformer, or by means of a resistor network. Figure 149 shows the schematic of a 300-ohm transmission line carrying a signal from a television antenna to the input circuit of a receiver. A small air-core transformer is used to match the 300 ohms impedance of the line to the 75 ohms input impedance at the terminals of the receiver. The

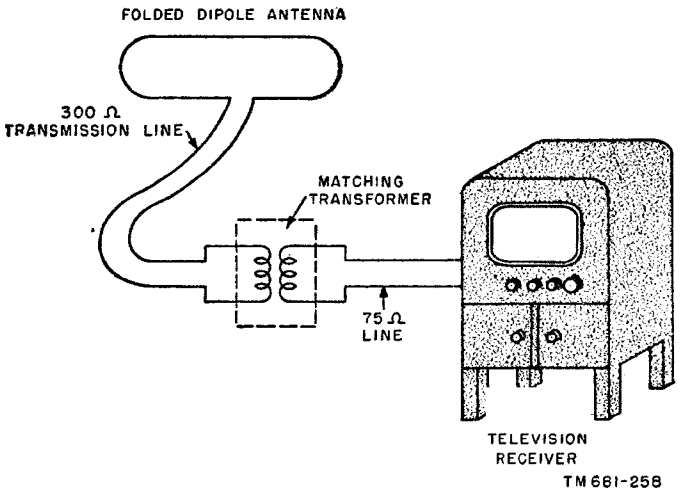


Figure 149. Impedance matching r-f transformer.

function of this transformer is like that described above except that the coupling is close because neither side is resonant. If a transmission line is used as a matching transformer, as shown in A of figure 150, the impedance of the matching section, which is cut to a quarter-wavelength of the frequency on the line, is determined by the geometric mean between the extremes to be matched. This matching impedance is given by the formula:

$$Z_o = \sqrt{Z_1 Z_2}$$

where: Z_o = the matching impedance

Z_1, Z_2 = the impedances to be matched.

Thus, the quarter-wavelength matching section must have an impedance of 150 ohms when matching a 300-ohm line to the 75-ohm input of the receiver:

$$Z_o = \sqrt{300 \times 75} = \sqrt{22{,}500}$$

$$Z_o = 150 \text{ ohms.}$$

B of figure 150 shows a resistor network for matching these same impedances. Note that the 300-ohm transmission line is terminated in a 300-ohm resistor R_1, the 75-ohm receiver is matched by the 75-ohm resistor R_2, and the 150-ohm resistor is common to both.

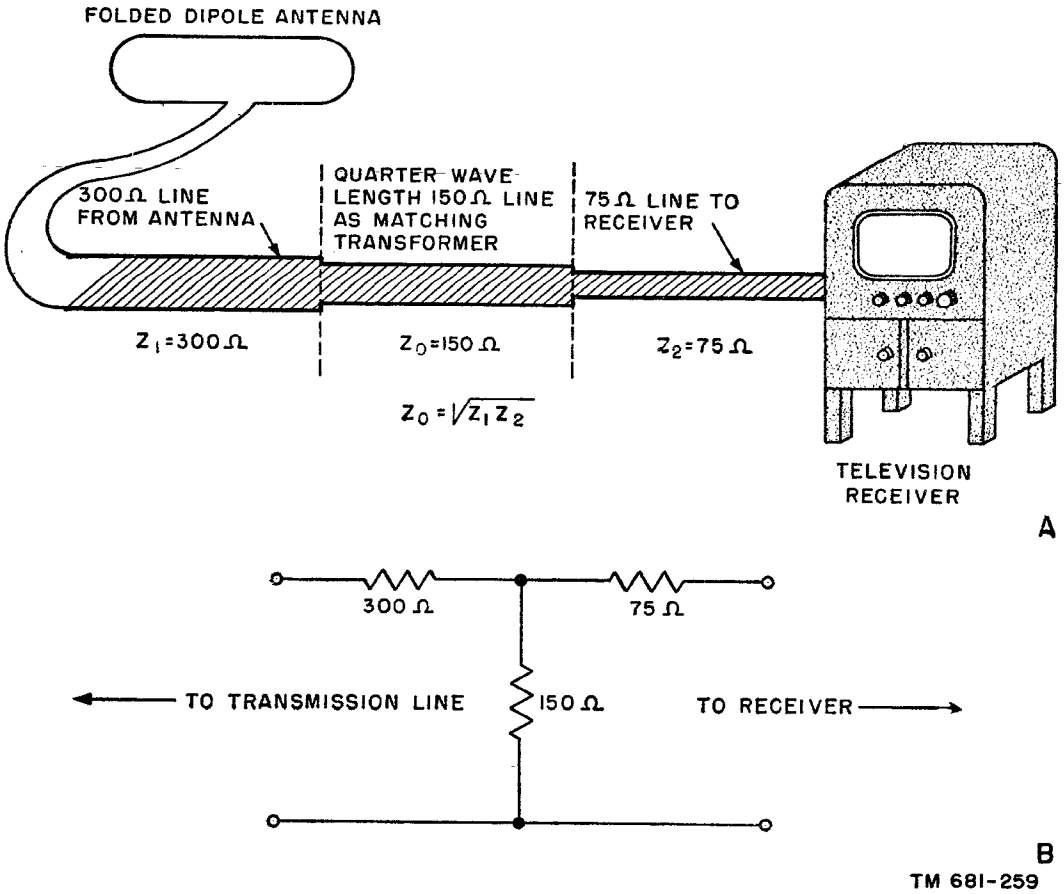


Figure 150. Impedance matching networks.

90. Summary

a. A transformer is a device which converts electrical power from one voltage-current level to another voltage-current level. It consists of any two inductors so placed physically that the changing magnetic field set up by an alternating current in one induces an alternating emf in the other.

b. The coil connected to the source of power is the primary winding; the coil connected to the load is the secondary winding.

c. The connection between the two coils is the flux linkage of the magnetic circuit, which produces high mutual inductance. At power-line frequencies (60 cycles), soft iron cores of high permeability reduce flux leakage, increase inductance and inductive reactance, and accordingly reduce magnetizing current.

d. Current in the secondary circuit sets up flux lines opposed to the primary flux lines, thereby increasing primary current. Under full load, voltage and current in the primary circuit tend to be in phase with each other and 180° out of phase with voltage and current in the secondary circuit.

e. The voltage induced in the secondary of a transformer is proportional to the number of turns in the secondary compared to the number of turns in the primary:

$$\frac{E_p}{N_p} = \frac{E_s}{N_s} .$$

f. A *step-up* transformer is one in which the *voltage* induced in the secondary is greater than the voltage applied to the primary; a *step-down* transformer is one in which the *voltage* induced in the secondary is less than the voltage applied to the primary.

g. The volts per turn for any given transformer is a constant and is determined by dividing the applied voltage by the number of turns in the primary.

h. The power in the primary circuit of a transformer is always equal to the power in the secondary circuit (regardless of the ratio of voltages).

i. The current in either winding of a transformer is *inversely* proportional to the number of turns:

$$\frac{I_p}{I_s} = \frac{N_s}{N_p} .$$

j. Practical transformers show efficiencies above 95 percent. Losses occur because of flux leakage, hysteresis, eddy currents, flux saturation of the core, copper losses, and distributed capacitance.

k. Power transformers convert electrical power to very high voltage with correspondingly low currents for long-distance transmission, and re-convert this power to low voltage at high current for use by the consumer.

l. Power transformers used in communications equipment generally have a power factor of .9 and efficiencies above 95 percent.

m. The total power delivered by a transformer with multiple secondary windings is the arithmetical sum of the amounts of power delivered by the various secondaries.

n. The autotransformer is a special type of low-loss power transformer which shows good voltage regulation under varying loads.

o. The impedance in the primary circuit of a transformer is directly proportional to the *square* of the ratio of the turns in the primary to the turns in the secondary:

$$\frac{Z_p}{Z_s} = \frac{N_p^2}{N_s^2} .$$

p. For maximum transfer of power in any electrical circuit, the impedance of the load must equal, or match, the internal impedance of the source.

q. A transformer of the proper turns ratio may be used as an impedance matching device by means of which a load of a certain impedance is matched to a source of different impedance.

r. An impedance in the secondary of a transformer is said to be reflected into the primary circuit. If the load is a pure resistance, it is reflected as a pure resistance but displaced by 180°; if the load is reactive it is reflected as its opposite reactance—that is, as an *image* reactance.

s. The effect of an inductive reactance in the secondary circuit is to reflect a capacitive reactance in series with the primary circuit; the effect of a capacitive reactance in the secondary circuit is to reflect an inductive reactance in series with the primary circuit.

t. An inductive reactance in series may be conceived of as a capacitive reactance in parallel; and a capacitive reactance in series may be

conceived of as an inductive reactance in parallel.

u. An inductance in the secondary reflects an inductance across the primary *directly* proportional to the square of the number of turns; a capacitance in the secondary reflects a capacitance across the primary *inversely* proportional to the square of the number of turns.

v. Audio frequency transformers are high-inductance iron-core transformers used as power-transfer and impedance matching devices at frequencies between 20 and 20,000 cps.

w. A microphone transformer matches the impedance of a low-impedance microphone to the high-impedance input circuit of a vacuum tube. An interstage transformer matches the output circuit of one vacuum tube to the input circuit of another tube. An output transformer matches the output circuit of a vacuum tube to the low-impedance circuit of a loudspeaker.

x. Radio frequency transformers are low-inductance air-core transformers—iron-core losses are prohibitive at radio frequencies—for use at frequencies between 50 kcs and 30 mcs.

y. Radio frequency transformers employ loose coupling and inductances determined by the function of the circuits. The untuned primary, tuned secondary transformer shows high impedance to the output circuit of one vacuum tube and a secondary resonant at the frequencies to be passed. Intermediate frequency transformers are loosely coupled; both primary and secondary windings are tuned and become, in effect, a band-pass filter. Antenna-coupling transformers are power transformers at radio frequencies; by balancing of the reactances present, a resonant condition across the transformer allows maximum transfer of power.

z. The turns ratio of a radio frequency transformer has no particular significance because of the loose coupling, but impedances are matched by means of a suitable ratio matching transformer, a transmission line considered as a matching transformer, or by a resistor network.

91. Review Questions

a. What is a transformer? How is the primary winding distinguished from the secondary winding?

b. What is the connection between the windings of a transformer?

c. Why are soft iron cores necessary at power line frequencies?

d. What determines the number of turns on the primary winding of a transformer?

e. What is magnetizing current? Why must it be kept to a minimum?

f. What is flux linkage? Flux leakage?

g. Describe the effect of primary current on flux lines set up in the core; of secondary current.

h. What is the phase relationship of voltage and current across a given winding at no load? At full load?

i. What is the phase relationship between voltage and current on one side of a transformer and voltage and current on the other side of the transformer?

j. Give the formula for determining voltages across a transformer in terms of the turns ratio.

k. What is a step-up transformer? A step-down transformer?

l. What does the expression *volts per turn* mean? How may it be used for calculation of voltages in a multiple winding transformer?

m. What is the relationship between power in one side of a transformer and power in the other side?

n. Give the formula for determining the current in any winding of a transformer in terms of the number of turns.

o. How is the efficiency of a transformer determined? What is the usual efficiency of a practical transformer?

p. Describe the sources of loss in an iron-core transformer. Why are laminated cores used?

q. Describe the use of power transformers in long-distance power transmission. What is the special function of the pole transformer?

r. What is usually taken as the power factor of transformers used in communications equipment? As the efficiency?

s. How is the total power delivered by a multiple winding transformer determined?

t. What is an autotransformer? When is it used?

u. What is the prime disadvantage of an autotransformer?

v. Give the formula for determining impedances across a transformer in terms of the turns ratio.

w. What is the prime condition for maximum transfer of power in any electrical circuit?

x. How is a transformer used as an impedance matching device? Give an example.

y. Why is the load on the secondary said to be a *reflected* or *image* load in the primary?

z. What effect does an inductive load have on the primary circuit? A capacitive load?

aa. Give the formulas for determining the amount of inductance or capacitance reflected across the primary of a transformer in terms of the turns ratio.

ab. What are audio frequency transformers?

ac. Describe separately the function of microphone transformers, interstage transformers, and output transformers.

ad. Why are soft iron cores not used in radio frequency transformers?

ae. Explain the importance of coupling in i-f transformers. What is optimum coupling?

af. How is power transferred at radio frequencies?

ag. Describe how a quarter-wave transmission line is used as an impedance matching transformer at radio frequencies.

CHAPTER 7

D-C MACHINES

Section I. D-C GENERATORS

92. Motors and Generators

In the previous chapters of this manual, the fundamental generation of an emf was described, and then the a-c generator as a source of power was assumed in analyzing the characteristic of the components common to a-c circuits. The purpose of this chapter and the following chapter is the description, both mechanical and electrical, of the generation of both ac and dc from the point of view of electrical power-producing machines and of these same machines considered as mechanical power producers, or motors. Thus, a generator is defined as a machine which converts mechanical energy into electrical energy; and a motor is defined as a machine which converts electrical energy into mechanical energy.

a. The mechanical energy required for the operation of a generator can be supplied by a gasoline engine, a Diesel engine, an electric motor, a steam turbine, or some other kind of prime mover. The electrical energy developed by the generator can be either ac or dc depending on the constructional features of the machine. Under certain circumstances, a generator can be used as a motor, in which case electrical energy is supplied to the terminals of the machine and mechanical energy is delivered to some kind of load.

b. Motors and generators are used extensively in the military services and are integral parts of many communications equipments. For example, hand generators and dynamotors are used in many field radio and telephone equipments. Motors ranging in size from less than 1 horsepower to over 25 horsepower are used to furnish power for a variety of equipments.

c. The fundamental principle for the generation of an emf is explained in chapter 1 of this manual; that is, the *generation* of emf depends on the relative motion between a conductor and a magnetic field. The magnetic field, or flux, may be stationary and the conductor moved or the conductor may be stationary and the flux moved; in either case, the result is the same. The *amount* of emf induced in a conductor moving in a magnetic field depends on three factors:

(1) The strength of the field.
(2) The length of the conductor, or the size and number of turns in a coil when the conductor is wound in the form of a coil.
(3) The speed at which the conductor sweeps through the magnetic field, or the speed of rotation of a coil in a magnetic field.

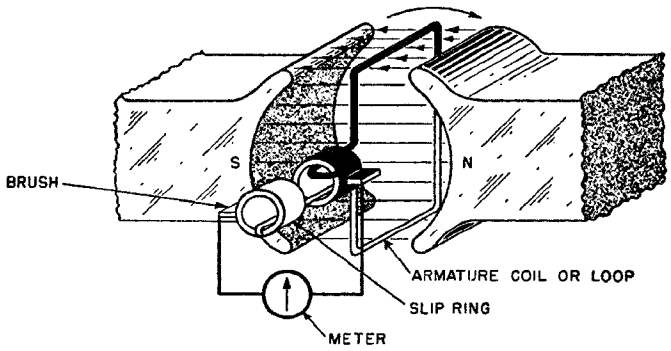


Figure 151. Basic a-c generator.

d. All constructional features in the design of generators provide for a balanced relationship among these three factors.

e. The basic a-c generator (fig. 151) is made up of a permanent magnet to produce a magnetic field, a single-turn loop which is rotated in the field and in which is induced an emf, and a slip-ring and brush assembly which makes possible the connection of generated emf to an external load. This simple generator, although helpful in the understanding of the basic theory, has little practical application because the amount of electrical energy generated is very small. Practical genera-

tors, on the other hand, are capable of supplying electrical energy in almost any desired amount. The practical generator may use one or more electromagnets instead of the permanent magnet, an iron-core coil composed of many turns of wire instead of the single-turn loop, and a superior though similar slip-ring and brush assembly. For d-c motors, the slip-ring and brush assembly is replaced by a commutator and brush assembly, as will be explained in the following paragraphs. The housing and parts of practical generators are designed for maximum speed consistent with use and mechanical limits of safety.

zero volts). Note that the coil ends are connected to the slip rings, which rotate simultaneously with the loop. The stationary carbon brushes make contact with the slip rings and are used to conduct the generated voltage to the external load—in this case, a meter.

b. After the loop has rotated 90° from its initial position and is passing through the position shown in B, the black coil side is moving downward and the white side upward. Both sides are cutting a maximum number of flux lines and the induced emf, as indicated by the meter, is at a positive maximum.

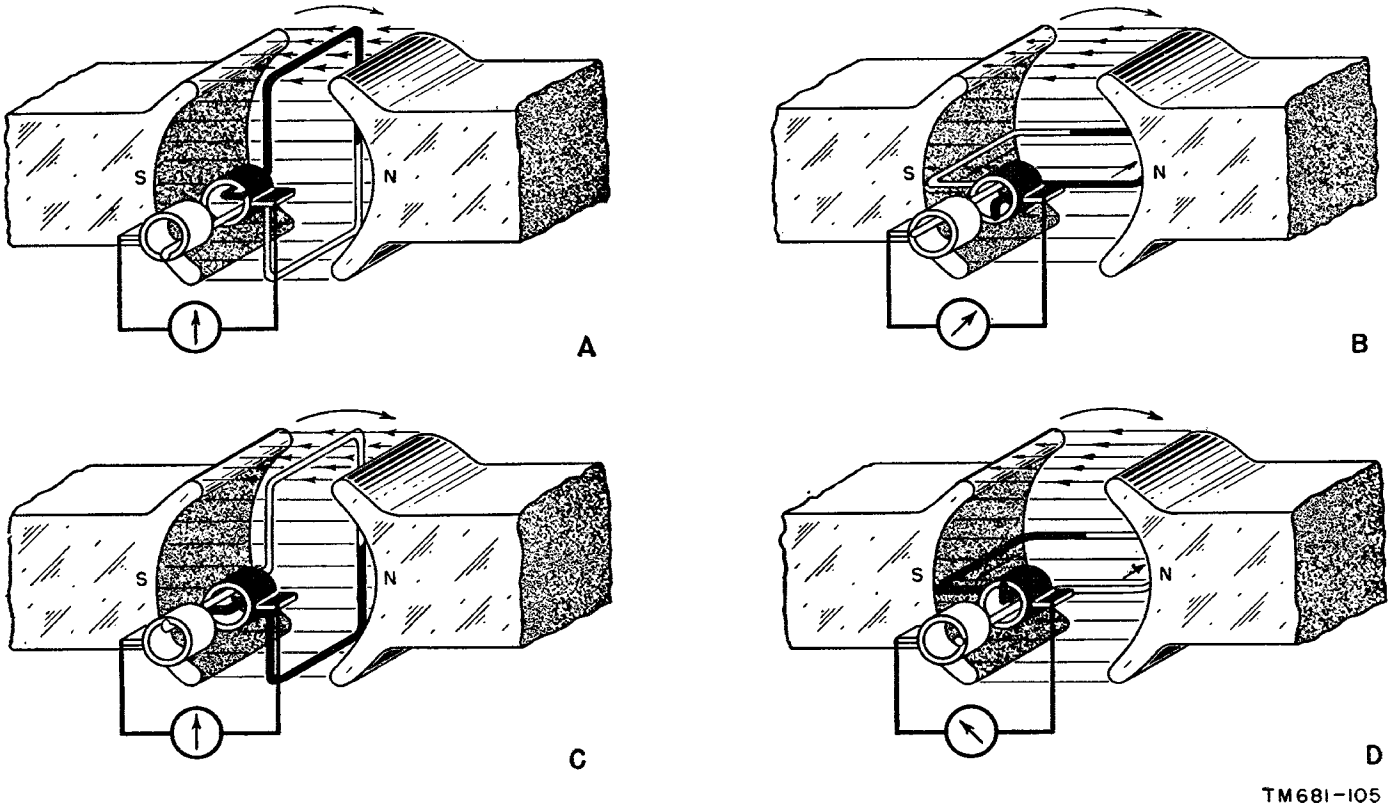


Figure 152. Generation of an emf.

93. Principle of Operation

Consider the four positions of the single-turn loop and slip rings as they rotate clockwise in a uniform magnetic field produced by the poles of a magnet (fig. 152).

a. When the loop rotates through the position shown in A, the black coil side is moving toward the north pole and the white side is moving toward the south pole. Because the coil sides are moving parallel to the direction of the field, no flux lines are cut and the emf induced in the loop is zero. This condition can be proved by connecting a suitable meter across the output terminals as shown (center-scale position of needle indicates

c. As the loop passes through an angle of 180° in C, the coil sides are again cutting no flux lines and the generated emf is again zero.

d. As the loop passes through an angle of 270°, as in D, the coil sides are cutting a maximum number of flux lines and the generated emf is at a negative maximum.

e. The next 90° revolution of the loop completes the 360° revolution and the generated emf falls to zero.

f. The preceding facts may be summarized as follows: As the loop makes one complete revolution of 360°, the generated emf passes from zero to a positive maximum, to zero, to a negative

maximum, and back to zero. If a constant speed of rotation is assumed, the emf generated is a sine wave (fig. 153).

94. Direct-Current Generator

By replacing the slip rings on the basic a-c generator with two semicylindrical segments, called a commutator, a basic d-c generator is obtained (fig. 154). In this figure, note that the black coil side is connected to the black semicylindrical segment and the white coil side is connected to the white segments. These segments are insulated from each other. The two stationary brushes are placed on opposite sides of the commutator and are so

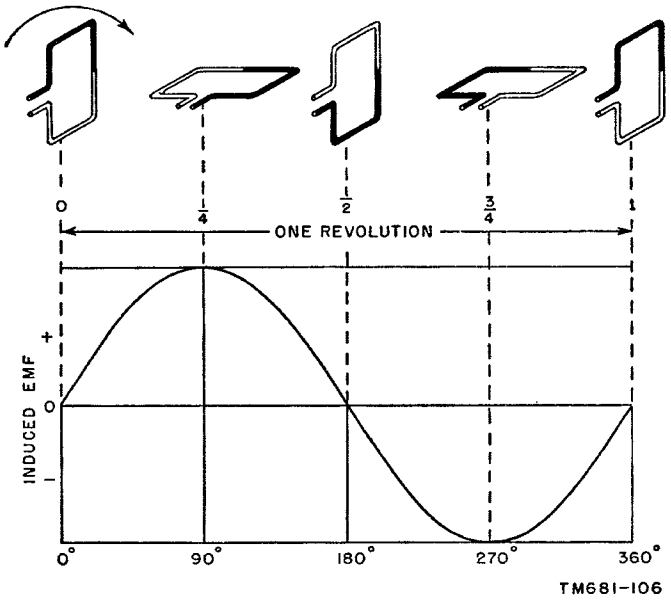


Figure 153. Generation of a sine wave.

mounted that each brush contacts each segment of the commutator as the latter revolves simultaneously with the loop. The rotating parts of a d-c generator (coil and commutator) are called an *armature.*

a. The generation of an emf by the loop rotating in the magnetic field is the same for both a-c and d-c generators, but *the action of the commutator produces a d-c voltage output.*

(1) The loop in position A of figure 155 is rotating clockwise, no flux lines are cut by the coil sides, and no emf is generated. Note that the black brush is just coming into contact with the black segment of the commutator and the white brush is just coming into contact with the white segment.

(2) In position B, the flux is being cut at a maximum rate and the induced emf is maximum. At this time, the black brush is contacting the black segment, and the white brush is contacting the white segment. The deflection of the meter is to the right, indicating the polarity of the output voltage.

(3) At position C, the loop has completed 180° of rotation. Again, no flux lines are being cut and the output voltage is zero.

(4) The important condition to observe at position C is the action of the segments and brushes. The black brush at the 180° angle is contacting both black and white segments on one side of the commutator, and the white brush is contacting both segments on the other side of

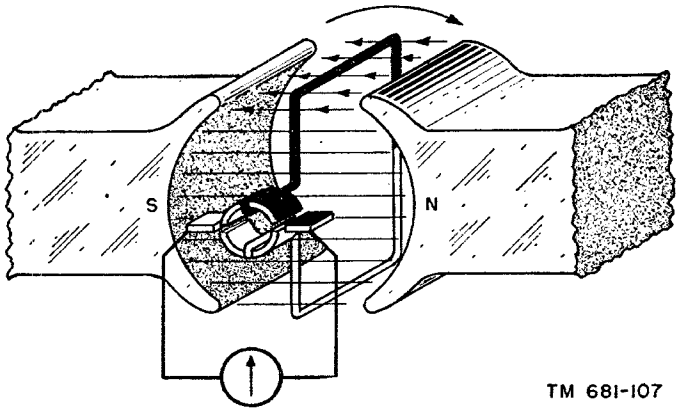


Figure 154. Basic d-c generator.

the commutator. After the loop rotates slightly past the 180° point, the black brush is contacting only the white segment and the white brush is contacting only the black segment.

(5) Because of this switching of commutator elements, the black brush is always in contact with the coil side moving downward, and the white brush is always in contact with the coil side moving upward. Though the current actually reverses its direction in the loop in exactly the same way as in the a-c generator, commutator action causes the current to flow always in the same direction through the external circuit or meter.

b. A graph of 1 cycle of operation is shown in figure 155. The generation of the emf for positions A, B, and C is the same as for the basic a-c generator, but, at position C, commutator action

reverses the current in the external circuit, and the second half-cycle has the same waveshape as the first half-cycle. For this reason, the process of commutation is sometimes called rectification. To summarize, the current within the loop continues to reverse its direction every 180° but the combined action of the commutator and brushes causes the current in the external circuit to flow in one direction only.

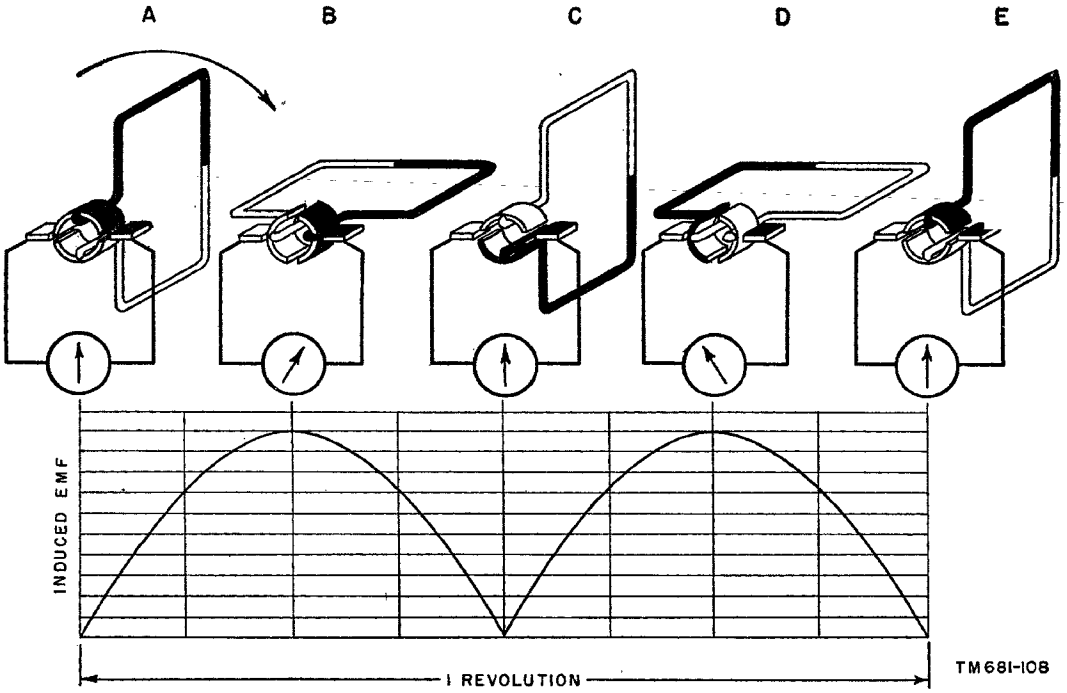


Figure 155. Operation of a basic d-c generator.

c. At the instant that each brush is contacting two segments on the commutator (positions A, C, and E in fig. 155), a direct short circuit is produced. If an emf is generated at this time, a high current will flow in the short circuit, causing an arc and thus damaging the commutator. For this reason, the brushes must be placed in the exact position where the short will occur when the generated emf is zero. This position is called the *neutral plane.*

95. Ripple

The voltage generated by the basic d-c generator varies from zero to its maximum value twice for each revolution of the loop (fig. 155). This variation of d-c voltage is called *ripple,* and may be reduced by using more loops, or coils, as shown in figures 156 and 157. As the number of loops is increased, the variation between maximum and minimum values is decreased, and the output voltage of the generator approaches a steady d-c value. In figures 156 and 157, note that the number of commutator segments is increased in direct proportion to the number of loops; that is, there are two segments for one loop, four segments for two loops, and eight segments for four loops.

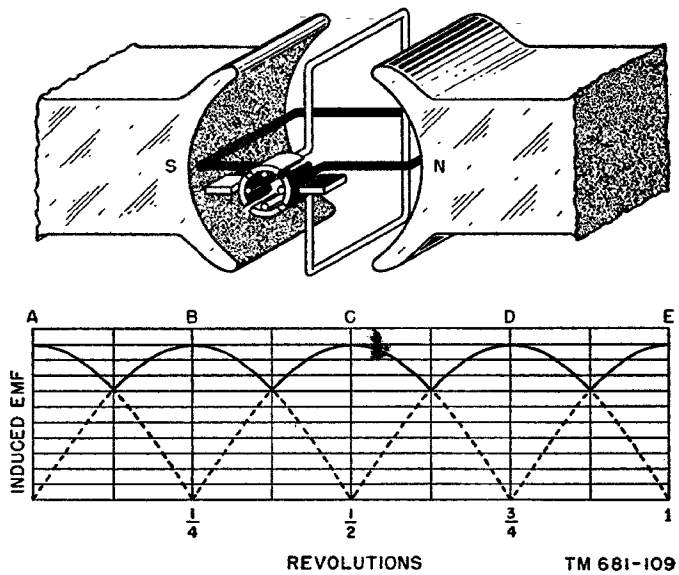


Figure 156. Output voltage from a two-coil armature.

96. Output Voltage

The voltage induced in a single-turn loop is small. Increasing the number of loops does not

increase the maximum value of generated voltage, but increasing the number of turns in each loop will do so. Within narrow limits, the output voltage of a d-c generator is determined by the product of the number of turns per loop, the total flux per pair of poles in the machine, and the speed of rotation of the armature.

Note. Up to this point, only the basic d-c generator, using a single pair of poles (one north and one south pole), has been considered. In following paragraphs, the functioning of other generators, using more than one pair of poles, will be described.

97. Practical D-C Generators

As explained in preceding paragraphs, the major parts of a d-c generator are stationary magnetic

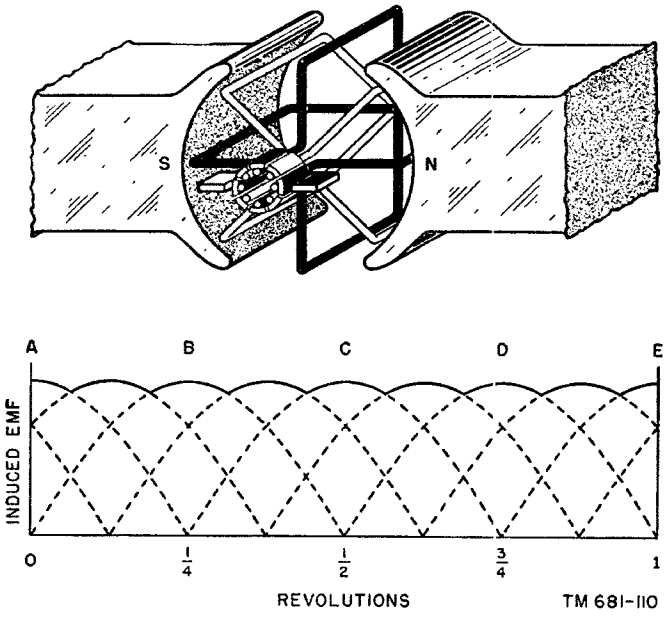


Figure 157. Output voltage from a four-coil armature.

poles, a rotating armature, and a stationary brush assembly. The major parts of practical generators are the same as for the basic generators thus far described and illustrated, but the practical generator differs in appearance and design details.

a. MAGNETIC POLES. The practical generator uses electromagnets instead of permanent magnets in order to produce a much stronger magnetic field without increasing the physical size of the magnets.

(1) Figure 158 shows assemblies used for two-pole and four-pole d-c generators. The *yoke,* or frame, has two functions: It completes the magnetic circuit between poles and acts as a mechanical support for the rest of the machine. In small generators, the yoke is made of one piece of iron, but in larger generators the yoke is usually made up of two parts bolted together. The yoke has high magnetic properties, and together with the pole pieces forms the major part of the magnetic circuit.

(2) The field poles are bolted to the inside of the yoke and form a core on which to mount the field coil windings. The poles usually are laminated to reduce eddy current losses, and serve the same purpose as the iron core of an electromagnet-concentration of the lines of force produced by the field coils. The entire yoke, both frame and field poles, is made from high-quality magnetic iron or sheet steel.

(3) The field coils are made up of many turns of insulated wire and are usually wound on a form which fits over the iron core of the pole and is securely fastened to it. The exciting current which flows through the field coils is obtained from an external source or from the generated dc of the machine. No electrical connection exists between the windings of the field coils and the pole pieces.

(4) Most field coils are connected in such a manner that the poles show alternate polarity. There is always one north pole for each south pole in the machine, and therefore there must always be an even number of poles in any generator.

(5) Note that the pole pieces in figure 158 project from the yoke. This design reduces the length of the air gap between the poles and the rotating armature and increases the efficiency of the machine, as air offers much reluctance to the magnetic field. When the pole pieces are made to project as shown, they are called *salient poles.*

b. ARMATURES. As stated previously, the rotating part of a d-c generator is called an armature. Thus, the armature consists of armature coils wound on an iron core, a commutator, and associated mechanical parts.

(1) The iron core acts as an iron conductor in the magnetic field and, for this reason, is laminated to prevent the circulation of eddy currents (see ch. 6).

(2) There are two kinds of armatures, the ring and the drum. Figure 159 shows a ring-type armature made up of an iron core, an eight-section winding, and an eight-segment commutator. This kind of armature is rarely used; most generators have the drum-type armature.

(3) On the drum-type armature, the coils are placed in slots in the core, but there is no electrical connection between the coils and core (fig. 159). The use of slots increases the mechanical safety of the armature. Usually, the coils are held in place in the slots by means of wooden or fiber wedges. The connections of the individual coils, called coil ends, are brought out to individual segments on the commutator.

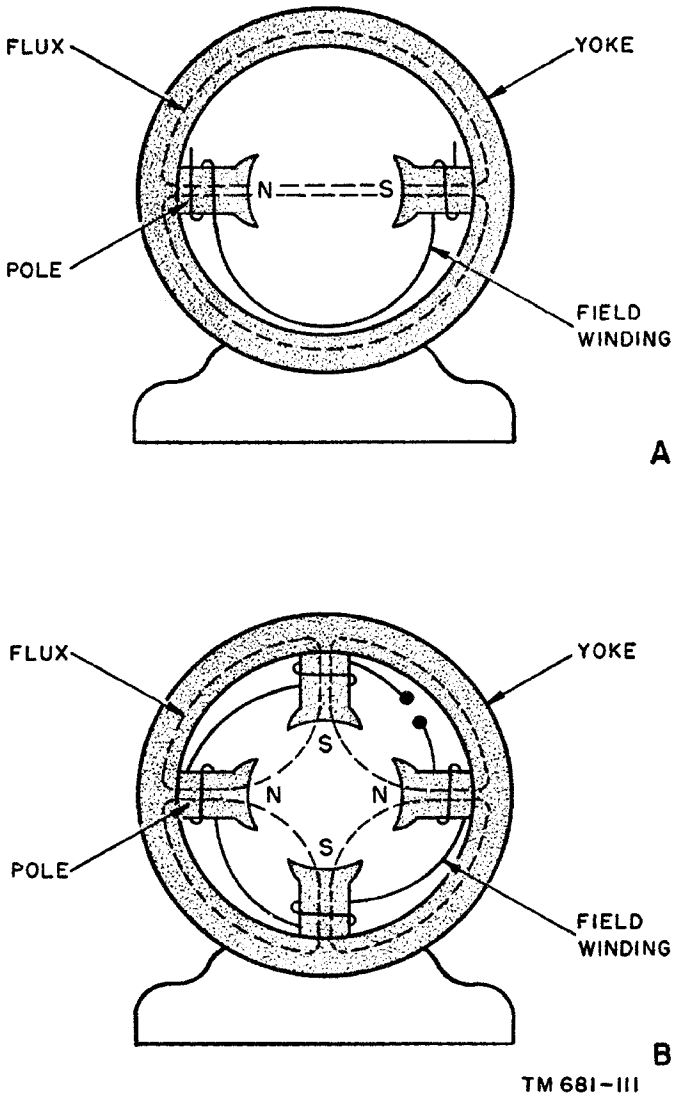


Figure 158. A, two-pole assembly. B, four-pole assembly.

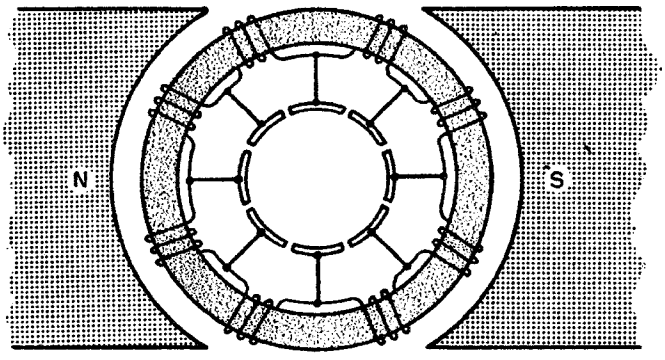


Figure 159. An eight-section, ring-type armature.

c. COMMUTATORS. Figure 161 is a cross-sectional view of a typical commutator. The commutator is located at the end of the armature (fig. 160) and consists of wedge-shaped segments of hard-drawn copper, insulated from each other by thin sheets of mica. The segments are held in placed by steel V-rings or clamping flanges fitted with bolts. Rings of mica insulate the segments from the flanges. The raised portion of each segment is called a riser and the leads from the armature coils are soldered to the risers. When the

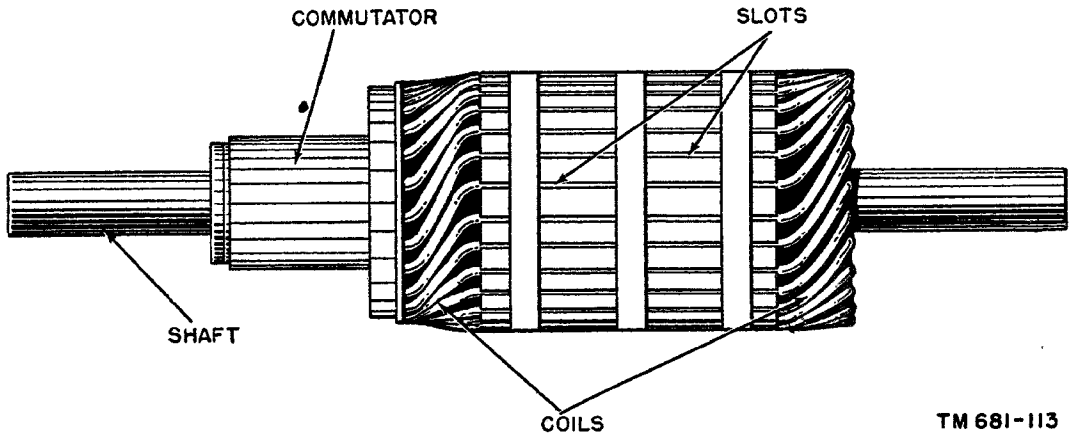


Figure 160. Drum-type armature (partly assembled).

segments have no risers, the leads are soldered to short slits in the ends of the segments.

(1) The brushes ride on the surface of the commutator and form the electrical contact between the armature coils and the external circuit. The brushes are made of high-grade carbon, and are held in place by brush holders, insulated from the frame. The brushes are free to slide up and down in their holders so that they can follow any irregularities in the surface of the commutator.

(2) A flexible, braided copper conductor, commonly called a pigtail, connects each brush to the external circuit.

(3) The brushes are usually adjustable so that the pressure of the brushes on the commutator can be varied and the position of the brushes with respect to the segments can be adjusted.

98. Commutation

As an armature revolves in a d-c generator, the armature coils, passing under the pole pieces, have a voltage induced in them which appears at the brushes of the machine. As the commutator segments to which the coils are connected pass under the brushes, two actions occur: First, current is drawn from the segments as a result of the voltages induced when the coils are passing under the poles; second, those coils which are in the interpolar spaces are short-circuited momentarily and the connections to the coils are reversed.

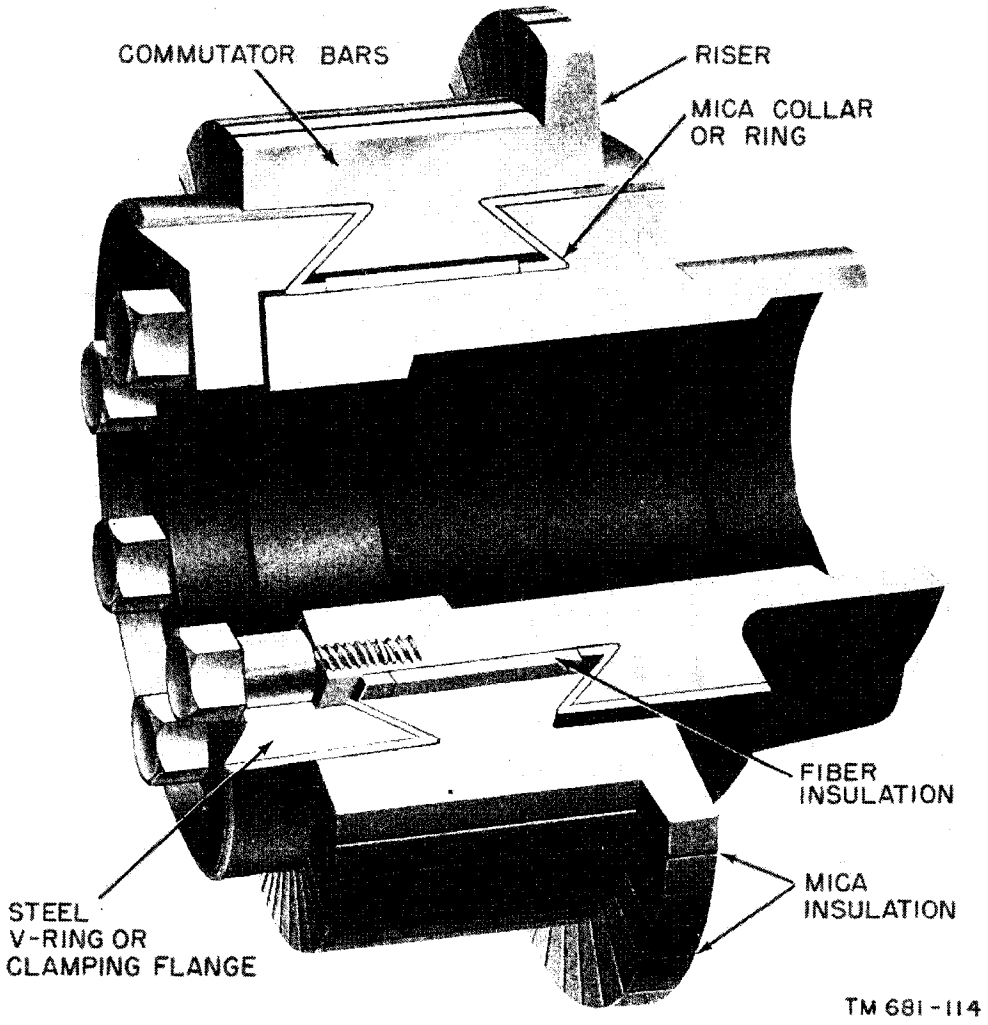


Figure 161. Cross-sectional view of a typical commutator.

a. The constant making and breaking of connections to the coils in which a voltage is being induced necessitates the use of a brush material having a definite contact resistance. Furthermore, it must be of such a nature that the friction between the commutator and the brush is low; otherwise, the commutator would wear excessively.

For these reasons, the material commonly used for brushes is high-grade carbon. This carbon must be soft enough to prevent undue wear of the commutator and yet be hard enough to provide reasonable brush life. Since the contact resistance of carbon is fairly high, the brush must be made quite large to provide a large area of contact. The commutator surface is highly polished to reduce friction as much as possible. Oil and grease must never be used on a commutator, and extreme care must be used in cleaning it to avoid marring or scratching the surface.

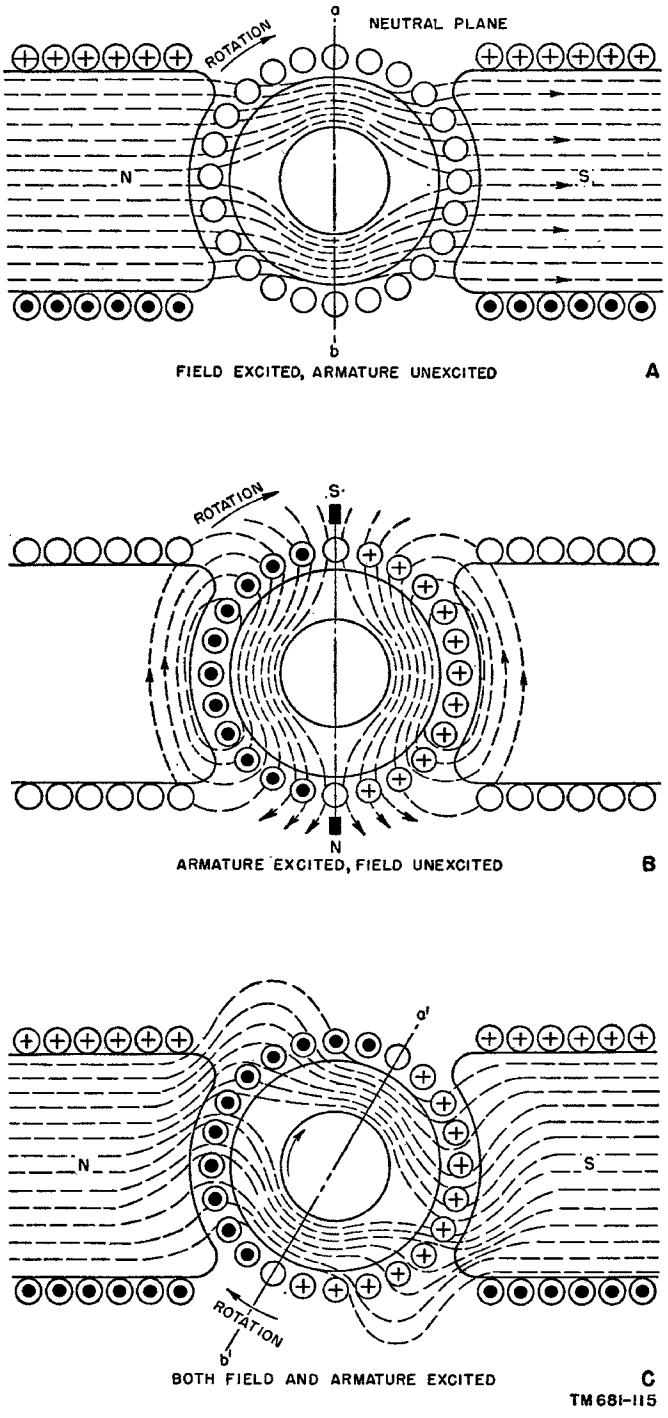


Figure 162. Armature reaction.

b. Theoretically, no sparking should occur at the commutator brushes. However, the current flowing in the armature coils sets up a magnetic field of its own which opposes the magnetic field set up by the field structure. The presence of this opposing field causes a shift in the neutral plane (par. 94*c*). Figure 162 illustrates how this effect, called armature reaction, is produced. A shows the flux lines produced by the pair of poles when no current is flowing through the armature coils. The curvature in the flux lines is produced by the presence of the iron-core armature between the poles and is a normal distortion of the magnetic field in a generator. The line *ab* indicates the zero axis of the field. B shows the flux lines produced by current flowing through the armature coils alone; that is, it is assumed that the field coils are unexcited and no field is produced by the poles. C shows the resultant field of two fluxes: the flux of A produced by the poles and the flux of B produced by the current-carrying armature. Note that the zero axis of the resultant field is displaced as indicated by the line *a'b'*. This displacement results in a shift in the position of the neutral plane. The shift of the main field brings the armature coils which are being short-circuited by the brushes under the influence of an additional slight field, which induces a low voltage in them. This low voltage is short-circuited by the brushes, causing sparking, which results in the burning and pitting of the commutator. As the load current and resulting armature reaction increase, this effect becomes more pronounced.

c. Various expedients are in use to nullify this shift of the neutral plane:

(1) The entire brush assembly can be adjusted to bring the brushes in line with the shifted neutral plane. Since the neutral plane shifts with the load, it follows that the brush assembly must be rotated or shifted each time the load on the machine is changed. Such a procedure is impractical in most cases, and other means must be used for reducing the effects of armature reaction.

(2) The poles can be chamfered slightly; that is, the radial distance between the pole face and the armature can be increased slightly at the edges of the poles (fig. 163). The increased air gap at the edges of the poles offsets to some extent the

tendency of the field to shift because of armature reaction.

(3) Commutating poles can be placed in the interpolar spaces. These poles are smaller and narrower than the main field poles and the winding on them is in series with the armature and so connected that the field set up by them opposes the field caused by armature reaction.

(4) The faces of the main field poles may be slotted longitudinally and windings placed in the slots. These windings are then connected in such a manner that the field produced by them opposes the field set up by the armature.

d. To summarize, the effect of armature reaction can be minimized or overcome by shifting the brush assembly, by using chamfered poles, by using commutating poles, by the use of pole face windings, or by combinations of two or more of these corrective methods.

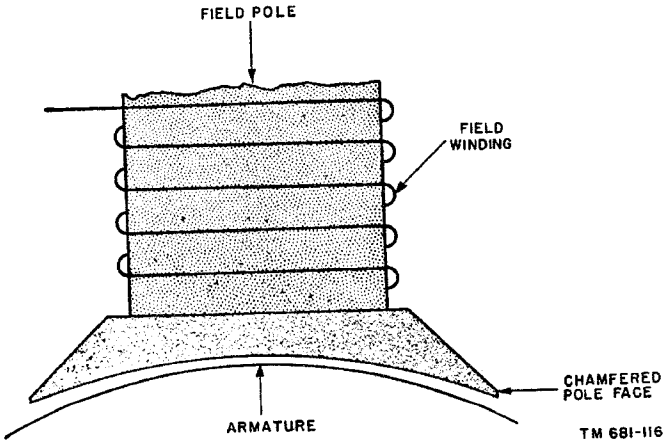


Figure 163. Chamfered pole.

99. Types of D-C Generators

D-c generators are classified according to the method used to supply the exciting current to the field windings. When the field current is obtained from a separate source, the generator is said to be *separately excited.* Usually this separate source is a small auxiliary generator called an exciter. If the field current is supplied by the generator itself, the generator is said to be *self-excited.* The self-excited classification is further divided into shunt, series, and compound types, all of which are explained in the following paragraphs.

a. SHUNT GENERATOR. The poles of a generator retain some magnetism, called residual magnetism, when the machine is not in operation. Advantage is taken of this fact in the self-excited shunt generator.

(1) This residual magnetism produces a weak magnetic field and, when the generator is started, a small voltage is induced in the armature and appears at the output terminals. Then, because the armature output voltage is connected across the field windings (B of fig. 164), a small current flows in the windings. This field current, in turn, strengthens the magnetic field, the output voltage increases accordingly, and a larger current flows in the windings. This action is cumulative, and the output voltage continues to rise to a point

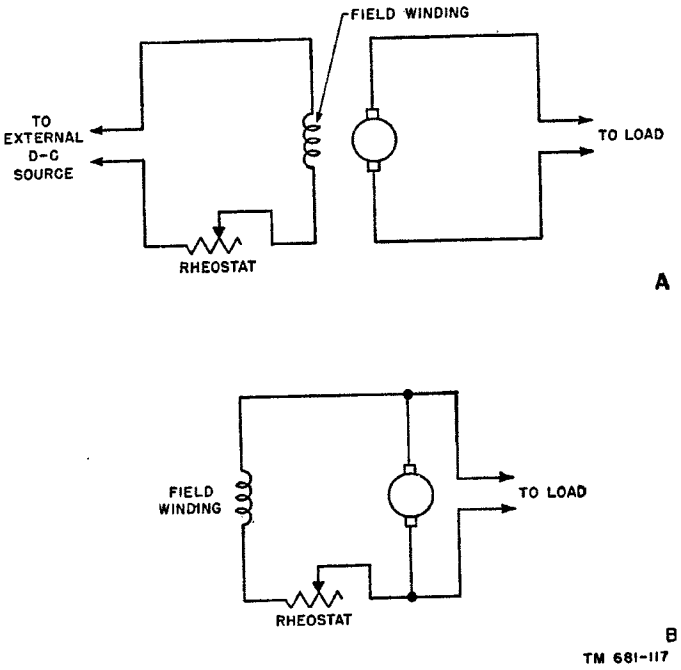


Figure 164. Shunt generators, simplified diagrams.

called field saturation, where no further increase in output voltage occurs.

(2) If the initial direction of armature rotation is wrong, the cumulative action does not occur, since the small induced voltage opposes the residual field and there is no build-up of output voltage.

(3) When the field windings of a shunt generator are excited by a separate source (A of fig. 164), the machine is said to be separately excited.

(4) The terminal voltage of a shunt generator can be controlled by means of a rheostat inserted in series with the field windings. As the resistance is increased, the

field current is reduced and, consequently, the generated voltage is reduced also. For a given setting of the field rheostat, either for a separately excited or self-excited machine, the terminal voltage at the armature brushes will be approximately equal to the generated voltage minus the IR drop produced by the load current in the armature. Consequently, the voltage available at the terminals of the machine will drop as the load is applied. This effect is greater in a self-excited generator than in a separately excited generator. Certain voltage-sensitive devices are available which automatically adjust the field rheostat to compensate for variations in load and, when these devices are used, the terminal voltage remains essentially constant.

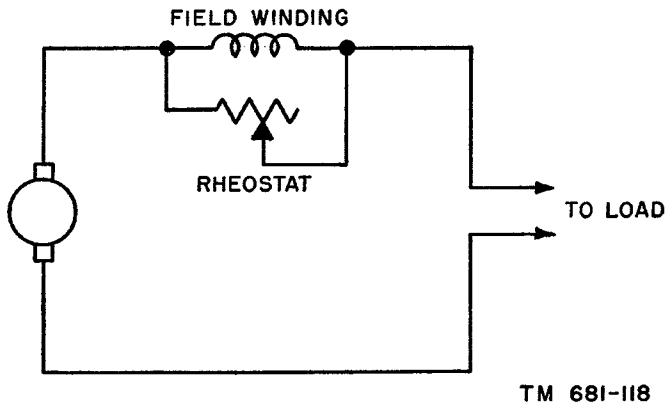


Figure 165. Series generator, simplified diagram.

(5) Shunt generators are used primarily in applications, such as battery charging, when a constant voltage under varying current conditions is required. Separately excited shunt generators are often used in certain speed-control systems.

b. SERIES GENERATOR. The field winding of a series generator is connected in series with the armature output voltage (fig. 165).

(1) The field coils of the series generator are made up of a few turns of heavy wire, and the magnetic field is more dependent on the amount of current flowing through the coils than on the number of turns in the coils. The wire must be large because all of the current in the external circuit passes through the coils.

(2) Series generators have very poor voltage regulation under changing load, as the greater the current through the field coils to the load, the greater will be the induced emf and the greater will be the output voltage of the generator. Therefore, when the load is increased, the voltage will increase, and when the load is decreased, the voltage will also decrease. Because the series generator has such poor regulation, only a few are in actual use.

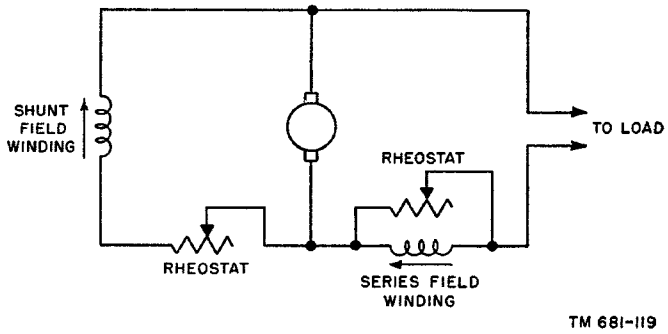


Figure 166. Cumulative-compounded connections.

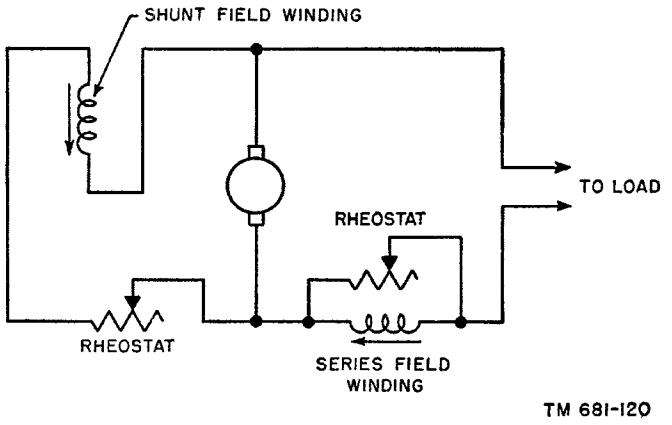


Figure 167. Differentially compounded connections.

c. COMPOUND GENERATORS. A compound generator has both a series and a shunt field, both windings being on the same pole structure. The series field may be connected either to aid or oppose the shunt field.

(1) If the series field aids the shunt field, the generator is said to be cumulative-compounded (fig. 166); if the magnetizing turns in the series field are such that the voltage rises with increasing load, the machine is over-compounded; if the voltage remains essentially constant for all values of load, it is flat-compounded;

and if the voltage drops with increasing load, the machine is under-compounded.

(2) If the series field opposes the shunt field, the machine is said to be differentially compounded, or is called simply a differential generator (fig. 167).

(3) Compound generators are usually designed to be over-compounded. This feature permits the degree of compounding to be varied by connecting a variable shunt across the series field. Such a shunt is sometimes called a diverter. Compound generators are used where voltage regulation is of prime importance.

(4) Differential generators have somewhat the same characteristics as series generators in that they are essentially constant-current generators. However, they generate rated voltage at no load, the voltage dropping materially as the load current increases. This constant-current characteristic makes them ideally suited as power sources for electric arc welders and find almost universal application in this work.

(5) If the shunt field of a compound generator is connected across both the armature and the series field, it is known as a long-shunt connection. If the shunt field is connected across the armature alone, it is called a short-shunt connection. These connections produce essentially the same generator characteristics.

d. THREE-WIRE GENERATORS. Some d-c machines, called three-wire generators, are designed to deliver 240 volts, with a neutral connection which provides 120 volts on either side of neutral. This is accomplished by connecting a reactance coil to opposite sides of the commutator (fig. 168), with the neutral connected to the midpoint of the reactance coil. Such a reactance coil acts as a low-loss voltage divider; if resistors were used, the I^2R loss would be prohibitive unless the two loads were perfectly matched. The coil may be built into the machine as part of the armature and the midpoint connected to a single slip ring with which the neutral makes contact by means of a brush, or the two connections to the commutator may be in turn connected to two slip rings, in which case the reactor is located external to the machine. In either case, the load unbalance either side of the neutral must not be more than 25 percent of the rated current output of the generator. The three-wire generator permits simultaneous operation of 120-volt lighting circuits and 240-volt motors from the same generator.

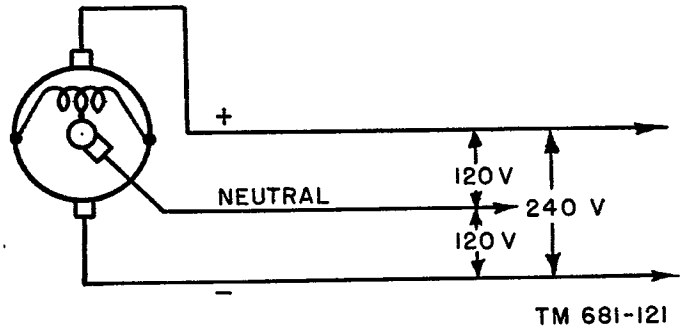


Figure 168. Three-wire generator.

Section II. D-C MOTORS

100. D-C Motors

Motors, like generators, are simply a means of transforming energy or power. Motors convert electrical power into mechanical power. The essential features and parts of the d-c generator and d-c motor are the same, both machines having field coils, armature coils, a commutator, and a brush assembly. As stated in paragraph 92, a d-c generator can be used as a d-c motor. When a voltage is applied to the terminals of a generator, the currents in the field coils and armature coils, respectively, set up magnetic fields which react on each other and cause the armature to revolve. When this happens, a d-c motor is in operation.

101. Principle of Operation

An understanding of basic motor action can be obtained by considering the following simple facts concerning magnetic fields.

a. A of figure 169 shows the uniform magnetic field which exists between the poles of a magnet when its field coils are connected to a d-c source. The lines of flux are directed from the north pole to the south pole.

b. B represents a cross section of a current-carrying conductor when the direction of current flow is into the paper (away from the observer). By applying the left-hand rule for a current-carrying conductor, it can be found that the direc-

tion of the field is counterclockwise, as indicated by the arrows on the lines of flux in B.

c. Now suppose the conductor in B is placed in the magnetic field of A. When this is done, a resultant field is produced as shown in C. Above the conductor, the field is weakened because the field produced by the poles and the field produced by the conductor are opposite in direction and tend to cancel each other. Below the conductor, the

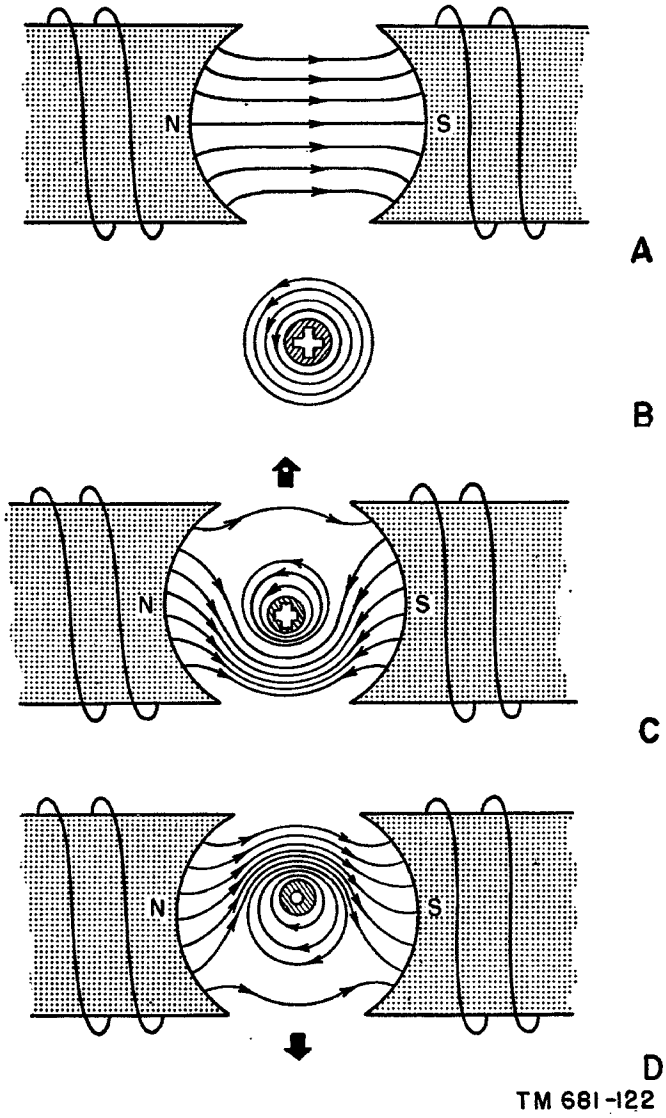


Figure 169. Principle of motor action.

two fields are in the same direction and the resultant field is strengthened. Because magnetic lines of force act to push each other apart, those lines below the conductor tend to push the conductor up and those above the conductor tend to push the conductor down. However, the field below is much stronger, having many more lines; so the push upward is greater, with the result that the conductor in C is moved upward.

d. If the current in the conductor is now reversed, the direction of motion of the conductor will also be reversed because, in this case, the field above the conductor is strengthened and the field below is weakened (D of fig. 169).

e. The principle on which the preceding experiment is based, and on which the operation of a motor depends, can be stated as follows: *A conductor carrying current in a magnetic field tends to move at right angles to (across) the field.*

102. Basic D-C Motor

The basic d-c motor, like the d-c generator, consists of a pair of poles which set up a magnetic field, an armature made up of a single-turn loop and commutator, and a brush assembly. Each coil side

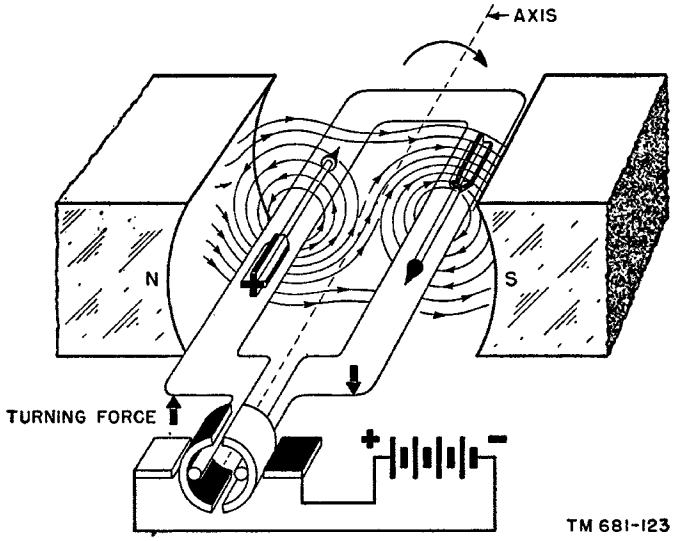


Figure 170. Basic d-c motor action.

of the loop lies in the magnetic field, as shown in figure 170.

a. When a d-c voltage is applied to the brushes, a current flows around the loop, and the magnetic field produced by the flow of current interacts with the magnetic field produced by the poles of the magnet. The resultant magnetic field is represented by flux lines in figure 170.

b. The loop rotates clockwise under the influence of the resultant magnetic field. It can be noted that the force acting in one direction (on one side of the loop) and the force acting in the other direction (on the other side of the loop) combine to cause the coil to turn on its axis. The loop thus acts as if it were a lever with a turning force, or *torque*, at each end. Because of this lever arrangement, the force at each end is magnified

by the distance from the center. Thus, the torque is equal to the combined force on the two sides of the loop multiplied by the distance of the conductors from the axis about which the loop rotates. The greater the torque, the more pull the motor has to drive a mechanical load.

103. Right-Hand Rule

There is a definite relationship between the direction of the magnetic field produced by the poles, the direction of current flow in the conductor, and the direction in which the conductor tends to move. This relation is expressed in the Right-hand Motor Rule which states: Place your right hand in a position so that the lines of force from the north pole of the magnet enter the palm of

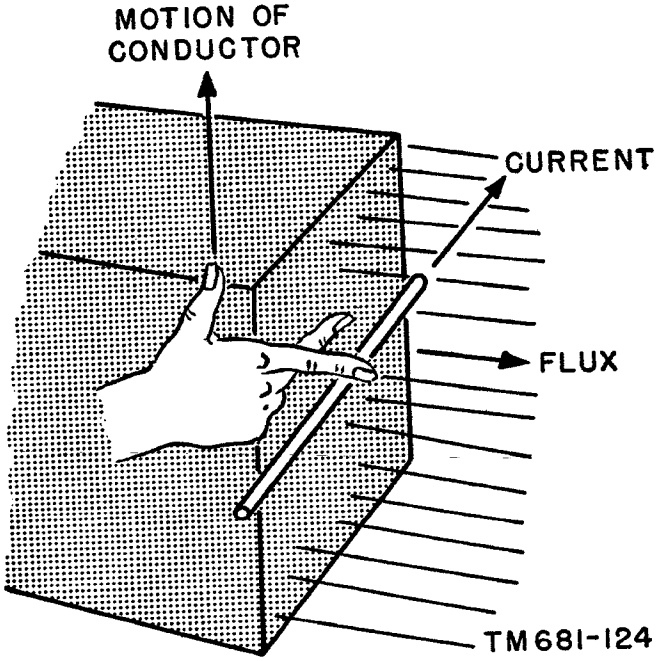


Figure 171. Right-hand rule for motors.

the hand. Let the extended fingers point in the direction of the current in the conductor; then the thumb, placed at right angles to the fingers, points in the direction of motion of the conductor (fig. 171). From this rule, the direction of rotation of a d-c motor can be determined if the direction of the field flux and direction of current flow are known.

104. Basic D-C Motor Theory

In order to understand how the armature of a d-c motor rotates continually, consider the basic d-c generator in figure 172.

a. In A, note that the direction of current is indicated by the arrows placed alongside the loop conductors. The black coil side is carrying current away from the observer and, by applying the motor rule, it is found that this coil side tends to

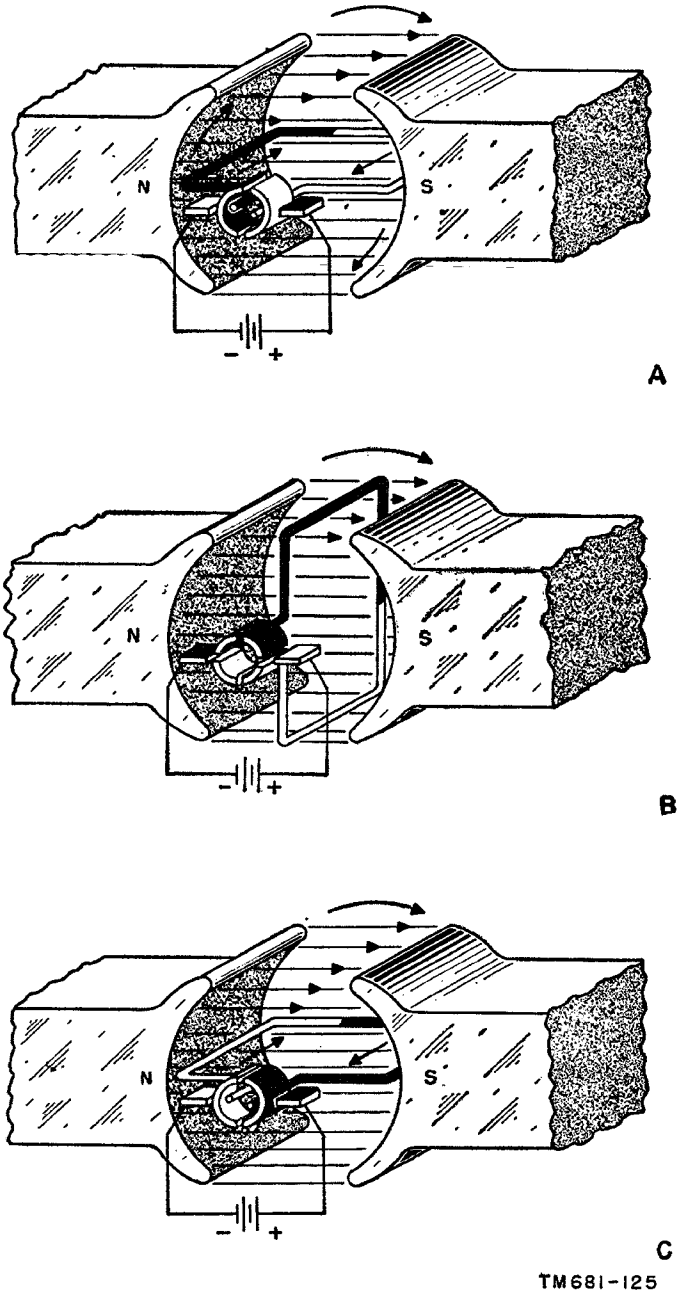


Figure 172. Continual rotation of an armature in a d-c motor is made possible by commutator action.

move upward. Conversely, the white coil side is carrying current toward the observer, and the motor rule shows the motion of this side to be downward. Thus, if the white coil side moves down and the black coil side up, the entire coil is

caused to rotate about its axis in a clockwise direction.

b. The forces acting against the conductors tend to move the coil sides in straight lines—that is, downward and upward—but, because the coil is assumed to be pivoted, they cannot move in a straight line; they must move in an arc or part of a circle. When any force produces a circular or twisting motion, a torque is produced. The torque on the loop of figure A continues until the coil is in the position shown in B. In this position, the coil has reached the neutral plane and no torque is produced. With zero torque, it might be thought that armature rotation would stop. However, this will not happen unless the motion of the coil is very slow. Normally, the momentum of the armature causes the coil to swing past the neutral plane and, simultaneously, commutator action reverses the direction of current flow through the conductors. When this happens, the forces in the field produce further clockwise rotation of the coil, as shown in C. Application of the right-hand rule to the conductors of C shows that the white side of the coil is now forced upward and the black coil side downward.

c. This same sequence of events occurs each time the coil passes a neutral plane; that is, the current in the coil reverses because of commutator action and a continuing rotation of the armature in a clockwise direction results.

105. Multi-Loop Motors

The basic d-c motor having a single-turn loop is often used to explain the theory of motor operation. However, the torque developed by such a simple motor is too small for practical purposes. Also, each time the loop passes through a neutral plane, the torque is zero and a jerk is introduced in the rotation of the armature. To develop more torque and to eliminate the effect of the zero torque points on armature rotation, additional loops are added to the armature. For example, figure 173 shows an armature with two loops. Note that only the loop in the best position to produce torque is connected to the brushes. Thus, the loop passing through a neutral plane is never conducting current. By adding more loops to the armature, additional torque is developed and still smoother operation of the armature results. For this reason, practical motors, like d-c generators, use a large number of loops on the armature and a corresponding large number of segments in the commutator. Both ring- and drum-type armatures are used on practical d-c motors.

106. Shunt Motor

The shunt motor is structurally the same as the shunt generator, and its schematic diagram is drawn in the same manner (fig. 164). In the shunt motor, the field coils are connected directly across the d-c input terminals and, when the supply voltage remains constant, the current through the field coils remains constant. As a result, the magnetic field is also constant and the torque of the shunt motor varies only with the current through the armature windings.

a. GENERATOR ACTION IN A MOTOR. Whenever an armature rotates in a magnetic field, an emf is induced in the armature coils. During rotation, the armature windings cut lines of force and an emf is induced in the windings. Thus, there is motor and generator action at the same time.

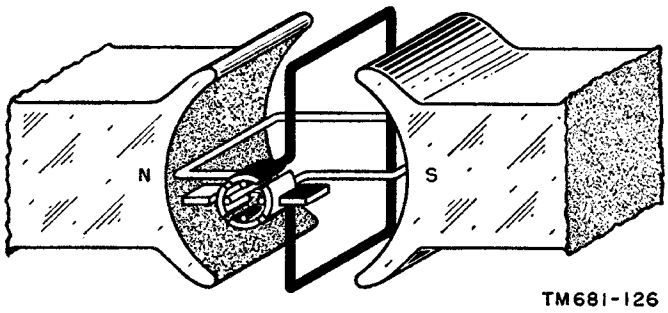


Figure 173. Two-loop motor.

The induced emf is in such a direction as to oppose the applied emf and, for this reason, is a counter emf.

b. ARMATURE CURRENT. In a shunt motor, the amount of current which flows through the armature depends upon the resistance of the armature windings and the difference between the applied voltage and the induced voltage (cemf). The cemf, in turn, depends solely upon the armature speed because the armature coils are fixed to the armature core and the field does not vary. Therefore, the following formula is used for motor calculations:

$$I_A = \frac{E\text{applied}-\text{cemf}}{R_A}$$

where: R_A is the armature resistance, and I_A is the armature current.

(1) For example, assume that a motor develops a cemf of 100 volts at 1,000 rpm, the armature resistance is 1 ohm, and the

applied voltage is 105 volts. Find the armature current at 1,000 rpm.

$$I_A = \frac{105 - 100}{1} = 5 \text{ amperes at 1,000 rpm.}$$

(2) Find the armature current if the speed is reduced to 900 rpm. In this case, the cemf is reduced by a factor of 900/1,000 of its value at 1,000, or 90 volts. Therefore, the current becomes:

$$I_A = \frac{105 - 90}{1} = 15 \text{ amperes at 900 rpm.}$$

c. STARTING RESISTANCE. The cemf developed is proportional to the speed of the armature. Consequently, no cemf is developed when the armature is not turning, even though a voltage is applied to the input terminals. Under such a condition, the armature current is equal to the applied voltage divided by the ohmic resistance. For example, if the applied voltage were 100 volts and the ohmic resistance were 1 ohm, the armature current would be 100 divided by 1, or 100 amperes. Such a high current would burn out the armature windings. Also, if the field windings were not excited, no magnetic field would be produced by the poles, no flux lines would be cut when the armature was rotating, and the current again would be limited only by the value of applied voltage and the ohmic resistance of the armature windings. The current again would be excessive since no cemf would be produced. To prevent such excessive current from flowing in the armature windings, a resistance is placed in series with the windings while the armature speed is building up to its proper value. This resistance is usually inserted by means of a starting box. In addition to protecting the motor during starting, the starting box also has provisions for breaking the circuit if the field circuit should open or if the power supply should fail.

d. STARTING BOX. The starting box shown in figure 174 has the starting lever in the off position. When starting the motor, the operator first moves the lever onto the first contact of coil C. This action closes the circuit to the field coil through electromagnet H, and also closes the circuit to the armature windings through the high-resistance coils in the starting box. As a result, maximum current flows through the field coil, and the magnetic field produced by the poles has maximum strength, but the current through the armature is limited by the series resistance. As the armature gradually builds up speed, the operator moves the lever successively to other contacts in order to reduce the value of series resistance. Finally, the operator moves the lever to the extreme right position and the electromagnet holds it in this position. At the extreme right position, the series resistance is removed from the circuit. If the power should fail, or if the field coils should open for any reason, the electromagnet becomes deenergized and the lever is returned automatically to the off position by spring P.

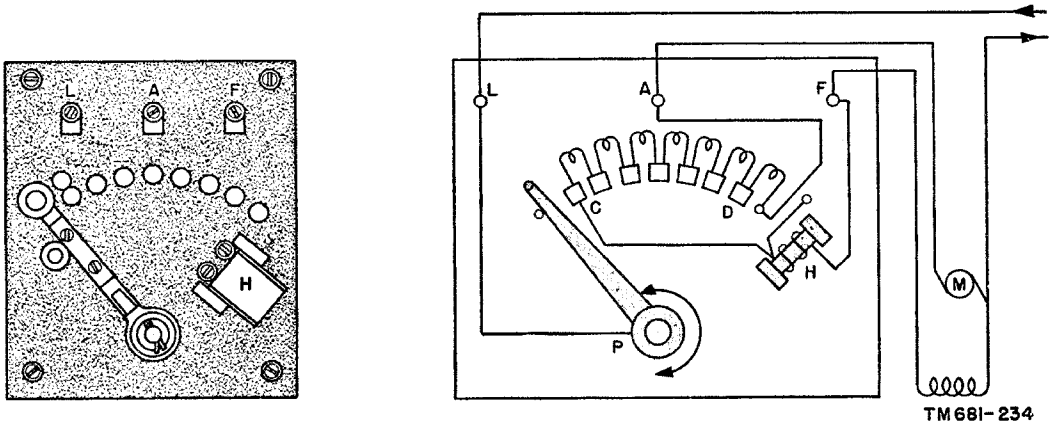


Figure 174. Motor starting box.

e. ARMATURE SPEED. The speed of a shunt motor is fairly constant under conditions of changing load. As a load is applied, the speed of the armature decreases, thus decreasing the cemf and increasing the current. The increase in current boosts the coupling between field and armature and increases the torque, causing the motor to resume approximate running speed.

f. SERIES RHEOSTAT. The speed of a shunt motor may be controlled by means of a rheostat in series with the armature windings, or a rheostat in series with the field winding, or both. The rheostat in series with the field winding is the most commonly used method.

(1) Adding resistance in series with the arma-

ture reduces the armature current, consequently decreasing the torque and causing the motor to slow down until the current increases sufficiently to give the desired torque. Adding resistance in series with the field winding reduces the field strength, decreasing the cemf, thereby permitting more current to flow in the armature. The increased current speeds the motor because of the increased torque until the increasing cemf reduces the current to an amount sufficient to produce the torque required.

(2) A rheostat in series with the field winding is preferable to a rheostat in series with the armature because the field current is much smaller than the armature current, and the power loss in the field rheostat is much less than the power loss in an armature rheostat.

g. ARMATURE REACTION. Because the current flow through a motor armature is opposite to the current through a generator armature, the armature reaction tends to shift the neutral plane back to the vertical plane rather than forward as in the case of generators (C of fig. 162). Therefore, the brushes of a shunt motor are set back of the neutral plane instead of forward.

h. A–C OPERATION. The shunt motor will operate only on direct current. If an a–c voltage is applied accidentally to a d–c shunt motor, the operation of the motor becomes unpredictable.

107. Series Motor

The series motor is structurally the same as the series generator (par. 99*b* and fig. 165).

a. The series motor is adapted for giving a very high starting torque. Actually, the torque of this motor varies approximately as the square of the current which flows through its series-connected armature and field coils. If the armature current is doubled, the flux is also doubled. Hence the torque, which is proportional to the current times the flux, is increased many times.

b. When the armature is at rest, the armature current (and therefore the torque) is at a maximum, because no cemf is generated in the coils and the current is limited only by the applied voltage and the ohmic resistance of the armature and field coils. As the armature gains speed, the cemf increases, decreasing the armature current and torque. If additional load is applied to the motor, the armature slows down, the generated cemf is decreased, a greater current flows, and a greater torque is produced. The speed of the motor is controlled by the load and, if the load is removed, the motor will race dangerously.

c. Series motors are generally used only where the load is constantly applied, and a good starting torque is required. Series motors are used in such applications as hoists, street cars, etc.

d. The series motor will operate on either dc or ac. When it is operated on dc, the brushes are set back of the neutral plane to compensate for armature reaction. If the motor is operated on ac, both the field and the armature change polarity at the same time. The brushes are set in the vertical or neutral plane; and the field core must be laminated to prevent eddy current losses. The theory of this a–c operation is explained in chapter 8.

e. A starting box is generally used with large series motors to limit current flow through the armature and field coils when starting the motor. This starting box has a rheostat which can be connected in series with the motor windings. All the resistance is inserted in the circuit when the motor is being started, and the value of resistance is reduced gradually as the speed of the motor increases.

108. Compound Motors

The compound motor, like the compound generator, has a combination of a series winding and a shunt winding, and combines the separate characteristics of series and shunt motors. By making the field strength of one winding more powerful than that of the other winding, a compound-wound motor may be made to have the approximate characteristics of either a shunt or a series motor. There are two types of compound windings: the *cumulative-compound* and the *differential-compound.*

a. In the cumulative-compound motor, the coils are wound upon the core in the same direction so that the magnetic fields set up by the series and shunt windings tend to aid each other; from this fact comes the name, cumulative-compound motor. A motor thus connected will have a very good starting torque, but poor speed regulation. Motors of this type are used for machinery where speed regulation is not necessary but where great torque is desired to overcome sudden applications of heavy loads.

b. The differential-compound motor has its

windings wound in opposite directions so that the series-field winding opposes the shunt-field winding. A motor of this type may be made to operate at almost constant speed under varying load. As the load increases and the motor tends to slow down, the current in the series-field winding increases because of the decrease in cemf. The shunt-field winding draws a constant current as it is connected directly across the line. Since the series winding opposes the shunt winding, this increase in current through the series field reduces the strength of the combined field. A decrease in the combined field causes the motor to speed. If the number of turns on the series winding is properly chosen, the combined effects of the increased load and weakened resultant field neutralize each other and the speed remains constant.

109. Summary

a. A generator is a machine which converts mechanical energy into electrical energy. Conversely, a motor is a machine which converts electrical energy into mechanical energy.

b. The generation of emf depends only on the relative motion between a conductor and a magnetic field.

c. The basic a-c generator is made up of a pair of poles, a single-turn loop, slip rings, and a brush assembly.

d. As the loop of the basic a–c generator completes one revolution (360°), the generated emf passes from zero to a positive maximum, to zero, to a negative maximum, and back to zero. If the loop rotates at a constant speed, the emf is a sine wave of voltage.

e. The basic d-c generator consists of a pair of poles, a single-turn loop, a commutator, and a brush assembly.

f. The action of the commutator produces a d-c output voltage.

g. Ripple can be reduced by using more coils on the armature of a d-c generator.

h. Output voltage of a d-c generator can be increased by using more turns in each coil of the armature, by increasing the total flux per pair of poles in the machine, or by increasing the speed of rotation of the armature.

i. The yoke of a generator has two functions: It completes the magnetic circuit between poles and acts as a mechanical support for the rest of the machine.

j. When pole pieces project from a yoke, they are called salient poles.

k. There are two kinds of armatures: the ring and the drum.

l. Brushes are usually adjustable on d-c generators.

m. The effects of armature reaction in a d-c generator are minimized by shifting the brush assembly, by using chamfered poles, by using commutating poles, by the use of pole face windings, or by combinations of two or more of these corrective methods.

n. D-c generators can be separately excited or self-excited. The self-excited type can be either shunt-, series-, or compound-wound.

o. Shunt generators are used in applications, such as battery charging, when a constant voltage under varying current conditions is required.

p. Series generators are seldom used because of their inherently poor regulations.

q. When a generator uses both a series winding and a shunt winding, it is called a compound generator.

r. The basic d-c motor consists of a pair of poles, an armature made up of a single-turn loop and commutator, and a brush assembly.

s. Turning force is called torque.

t. Adding coils to the armature of a d-c motor produces additional torque and smoother operation of the armature.

u. Generator action occurs in a motor because the armature windings cut flux lines and, consequently, an emf is induced. In a motor, the direction of this induced emf is opposite to that of the applied emf. For this reason, the induced emf is called a cemf.

v. Shunt motors have good starting torques and run at fairly constant speed under varying load conditions.

w. Series motors have excellent starting torque under load but will speed dangerously if the load is removed.

x. Starting boxes are used to limit the starting current in a shunt motor and to break the armature circuit if the field circuit should open or if the power source should fail.

y. Properly designed series d-c motors will also operate on ac. The operation of a shunt d-c motor is unpredictable if ac is applied.

z. Compound motors have a combination of a series winding and a shunt winding. The cumulative-compound motor has a good starting torque but poor speed regulation. A differential-com-

pound motor, on the other hand, has less starting torque but can be made to operate at almost constant speed under varying load conditions.

110. Review Questions

a. Upon what factors do the amount of emf induced in a rotating conductor depend?

b. What kind of emf is induced in the coils of the armature of a generator?

c. What are slip rings and where are they located?

d. What are the principal parts of the armature of a generator?

e. Would a generator which has four poles have to turn at the same rate of speed as a two-pole generator to produce current of the same frequency? Why?

f. What is done to the armature core of a generator to reduce the effects of eddy currents?

g. Mechanically, what is the weakest part of a d-c generator?

h. Why are the brushes of a d-c generator set ahead of the normal neutral plane?

i. Why are carbon brushes used for a d-c generator?

j. What is meant by the term *ripple*?

k. Does the armature reaction of a d-c generator cause the neutral plane to shift forward or backward, with respect to the direction of rotation of the armature?

l. The ability of a self-excited generator depends upon what property to build up voltage?

m. Make a schematic diagram of a shunt generator and explain why it is so named.

n. How may commutator ripple be reduced?

o. What is the disadvantage of a shunt generator?

p. What is a compound generator?

q. What are the three principal parts of a generator?

r. What is the disadvantage of a series generator?

s. If a generator is connected to storage batteries, how is its charging rate varied?

t. What is meant by armature reaction?

u. If a d-c generator that has been reconnected fails to build up its voltage, what can be tried? Why?

v. What is meant by the counter or back emf of a motor?

w. What effect has armature reaction upon the neutral plane of a motor?

x. Will a shunt motor operate satisfactorily on alternating current?

y. What equipment is used in starting a shunt motor?

z. Will decreasing the speed of a shunt motor increase the torque of the motor?

aa. How is the speed of a shunt motor controlled?

ab. Is a series motor able to start under a load?

ac. What is the most important characteristic of the series motor?

ad. Will the differential- or cumulative-compound motor give the more constant speed?

ae. In general, where are differential-compound motors used?

af. Do the two fields oppose or aid each other when a motor is differentially compounded?

ag. Draw the schematic diagrams for shunt, series, and compound motors.

ah. State the rule for determining the direction of rotation of motors.

ai. Determine the current through the armature when the applied voltage is 125 volts; the armature resistance is 10 ohms; and the counter emf generated at 1,000 rpm is 100 volts. Assume that the armature speed is 800 rpm.

aj. Under what conditions will a shunt motor run away?

ak. Under what conditions will a series motor run away?

CHAPTER 8

A-C MACHINES

Section I. A-C GENERATORS

111. Single-phase Generators

a. The elementary single-phase *alternator* (a-c generator) was described in chapters 1 and 7 of this manual. This single-loop machine has little practical value outside of the laboratory or classroom because the amount of energy generated is quite small. The armature of the practical alternator is wound with many turns of wire and its magnetic field is produced by electromagnetism. The emf generated by this machine is then limited only by the number of coils in the armature, the strength of the magnetic field, and the speed of rotation of the armature. In physical construction the alternator is similar to the d-c generator previously described, except that the energy delivered is taken from relatively simple slip rings, and therefore the physical and electrical difficulties encountered with commutators (wear of brushes and segments, sparking, shifting of the neutral plane) are absent. The coils on the armature are connected to each other in series so that the induced voltages are additive, and slip rings are connected to the first and last coil.

b. Since the output of an alternator is ac, and since dc is required for the field windings, the generated voltage cannot be used for field excitation as with d-c machines. Therefore, all a-c generators are separately excited. However, a small d-c generator is usually mounted on the a-c armature and its commutator and brushes are placed alongside the a-c slip rings. Both units are generally included under a single housing, such as one type of single-phase alternator (110 volts at 60 cycles) powered by a gasoline engine and widely used by the armed forces in portable installations.

c. Slip rings operate satisfactorily at low power but are a source of loss when large current is drawn from the armature. For this reason, an alternator may use a stationary armature and a rotating field. In this case, the coils of the armature are set about the circumference of the yoke, and the poles of the field (with their coils) rotate within the armature. Since the field current is small compared to the current delivered by the alternator, it is supplied through the slip rings, and the load current is taken from stationary conductors. A stationary armature makes possible the generation of voltages unattainable with generators using rotating armatures, because stationary conductors are not subject to the centrifugal force of rotation and may be more effectively insulated.

112. Polyphase Generators

a. GENERATION OF TWO-PHASE EMF.

(1) The armature coils of a single-phase alternator are connected in series, as explained above, so that at any given instant of time, the voltage induced is the sum of the voltages induced separately in each coil. If the coils were not connected in series, and if their connections were brought out separately, the voltages induced would vary in phase with relation to each other since each coil is in a different position in respect to the flux lines of the field. A single-coil basic generator will illustrate this principle. When windings identical in size, and of the proper phase relationships, are added to the single-phase alternator a voltage output will be obtained from each coil added. These voltages will differ in phase but not in amplitude, and the output from such a generator is known as a polyphase output. Systems of this type are generally two- or three-phase, with the three-phase being the more widely used.

(2) In the two-phase system an additional

coil identical in size and placed at right angles to the original coil (fig. 175) is added to the armature of the single-phase alternator. When the armature is rotated in a counterclockwise direction, winding a_2b_2 attains a maximum induced voltage 90° before winding a_1b_1 and, as the armature continues to rotate, the output of a_1b_1 will always lag the output a_2b_2 by 90°. This action results in two

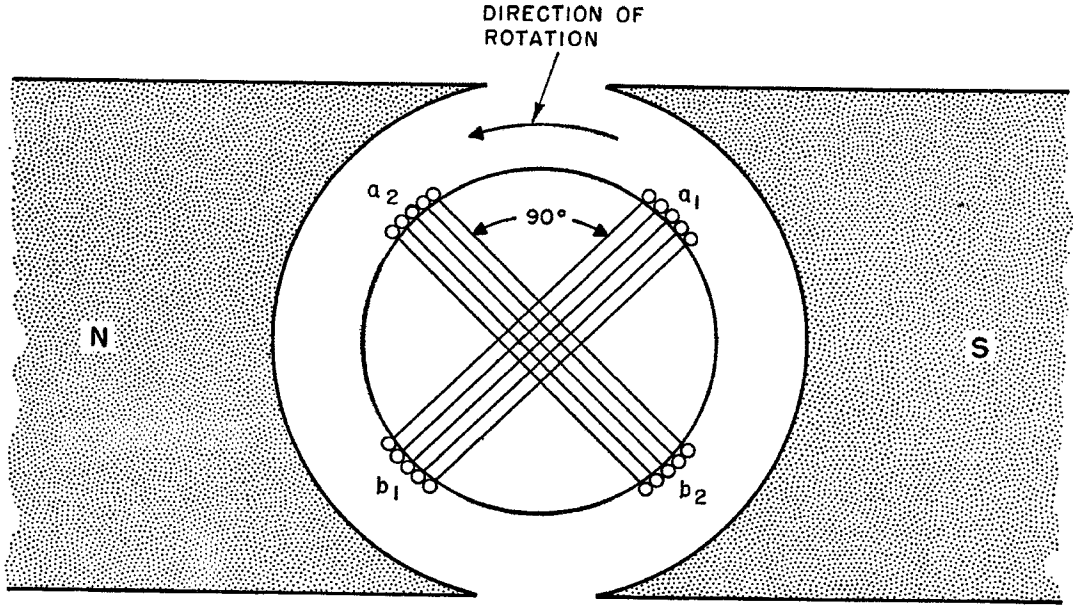


Figure 175. Elementary two-phase alternator.

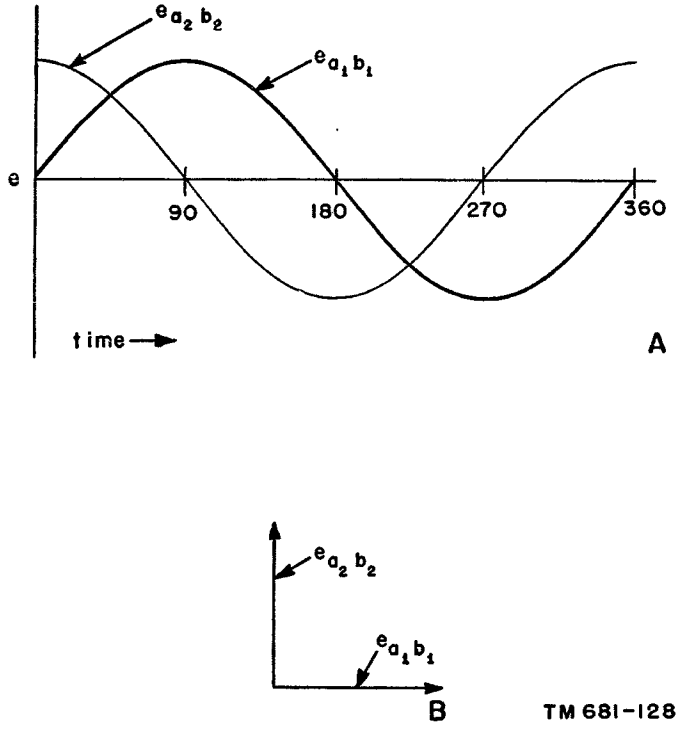


Figure 176. Voltage graph of two-phase alternator.

voltage outputs of equal amplitude, differing in phase by 90 electrical degrees. The phase relationships of these voltages are shown in A, the graph of figure 176, and in the vector diagram in B. The two-phase system illustrates two important facts:

(*a*) The induced voltages have equal magnitude when the coils are identical.

(*b*) The two voltages have the same frequency and amplitude but are 90° out of phase.

b. GENERATION OF THREE-PHASE EMF.

(1) A three-phase system is one in which the voltages have equal magnitudes and are displaced 120 electrical degrees from each other. But merely stating that three voltages e_1, e_2, and e_3 are 120° out of phase is not sufficient. The phase sequence of the voltages must be given. The vector diagrams in figure 177 illustrate the different possible phase sequences. In a two-pole machine, when

three windings are placed on the armature (fig. 178) 120 mechanical degrees apart, three separate output voltages 120° out of phase may be obtained. The winding a_1b_1 represents the first phase, winding a_2b_2 the second phase, and winding a_3b_3 the third phase. Conductors a_1, a_2, and a_3 are the beginnings of the three

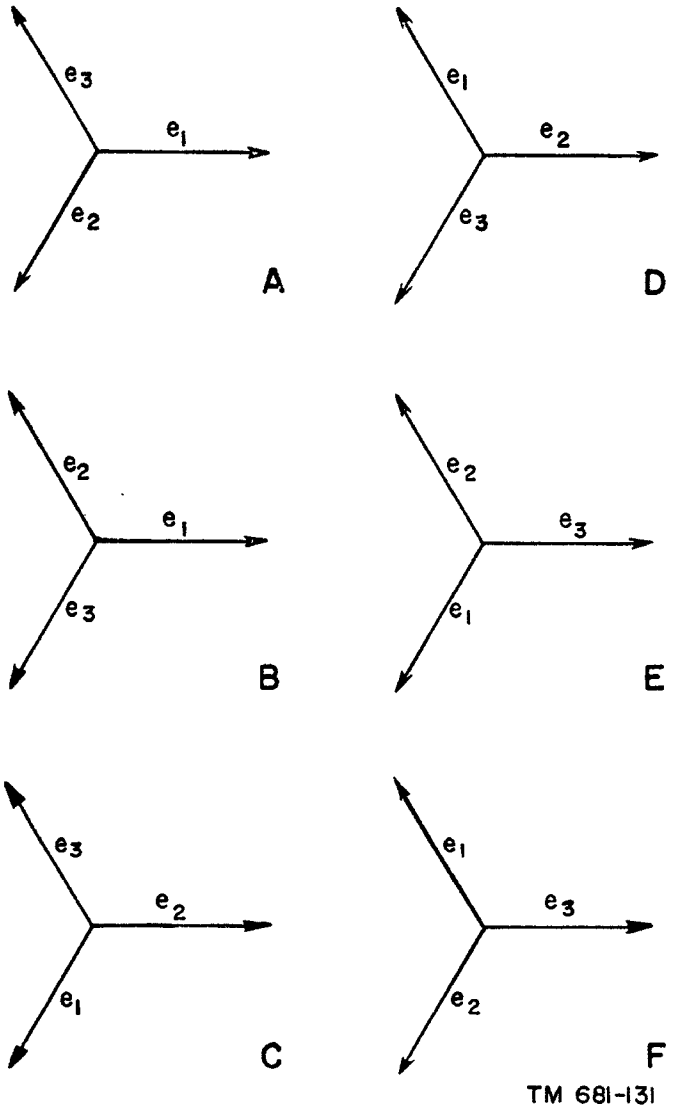


Figure 177. Phase sequence in three-phase system.

phases. As the armature is rotated in a counterclockwise direction the phase sequence of the induced voltages is $e_{a_3b_3}$, $e_{a_2b_2}$, and $e_{a_1b_1}$. Electrically, the voltage $e_{a_3b_3}$ leads the voltage $e_{a_2b_2}$ by 120 electrical degrees, and the voltage $e_{a_2b_2}$ leads the voltage $e_{a_1b_1}$ by the same amount. The output voltages are equal in amplitude but 120° out of phase. These relationships are shown in figure 179.

(2) Three single-phase generators with equal voltage magnitudes are shown in figure 180–B. To obtain a three-phase system of voltage from these generators, they must be connected in the proper phase sequence.

(*a*) If the voltage rise, e_1, from o_1 to a_1 leads the voltage rise, e_2, from o_2 to a_2 by 60°, and the voltage rise, e_3, from o_3 to a_3 lags e_2 by the same amount, the generators are not of the proper sequence to produce a three-phase output since the phase displacement is 60° instead of the required 120° (A of fig. 180).

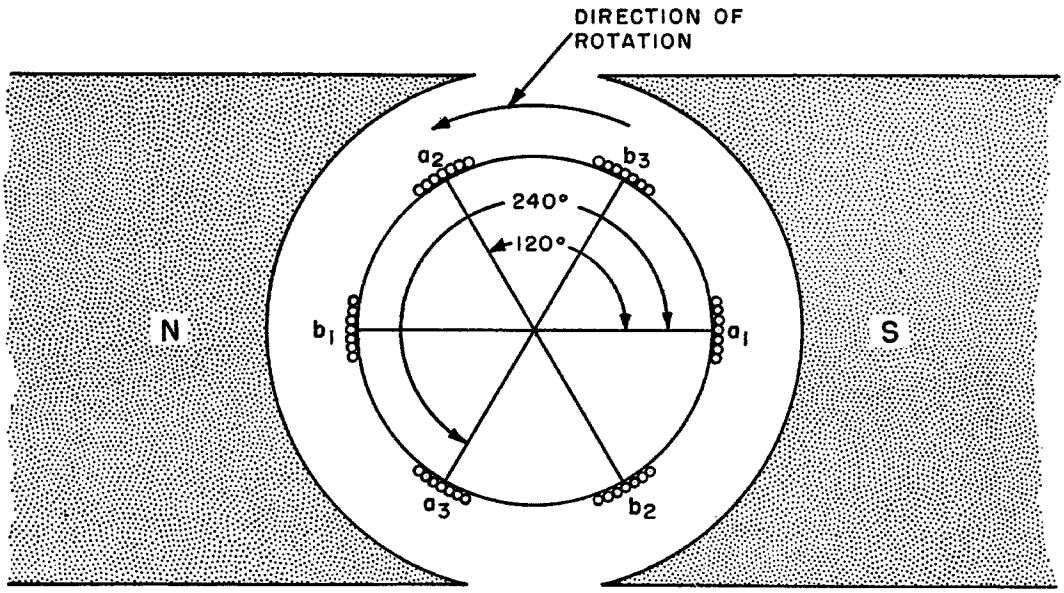
(*b*) If the phase of voltage e_2 is changed 180°, the proper relationships will exist and the three-phase output can be obtained. This change is shown in figure 181 where the polarity of generator e_2 has been reversed in order to obtain phase relationships shown in the vector diagram. Note that terminals o_1, a_2, and o_3 are tied together. With this arrangement, the voltages between this neutral point and lines 1, 2, and 3 from a three-phase system.

(3) Figure 182 shows an elementary four-pole generator that is used as a three-phase system. When the winding a_1b_1 has rotated 180° it has passed through a north pole and a south pole and 1 complete electrical cycle has been generated. Although the coil has passed through only 180 mechanical degrees, the result is 1 complete cycle or 360 electrical degrees. *For every mechanical degree, 2 electrical degrees result.* It is evident that a four-pole generator will rotate at one-half the speed of a two-pole generator since the winding moves an angular distance equal to the angle formed by two consecutive poles. An eight-pole generator will rotate at one-half the speed of the four-pole generator. The frequency of the generator in cycles per second is then:

$$f = P \times \frac{N}{60} \times \frac{1}{2} = \frac{PN}{120}$$

where P is the number of poles and N is the revolutions *per minute* of the armature.

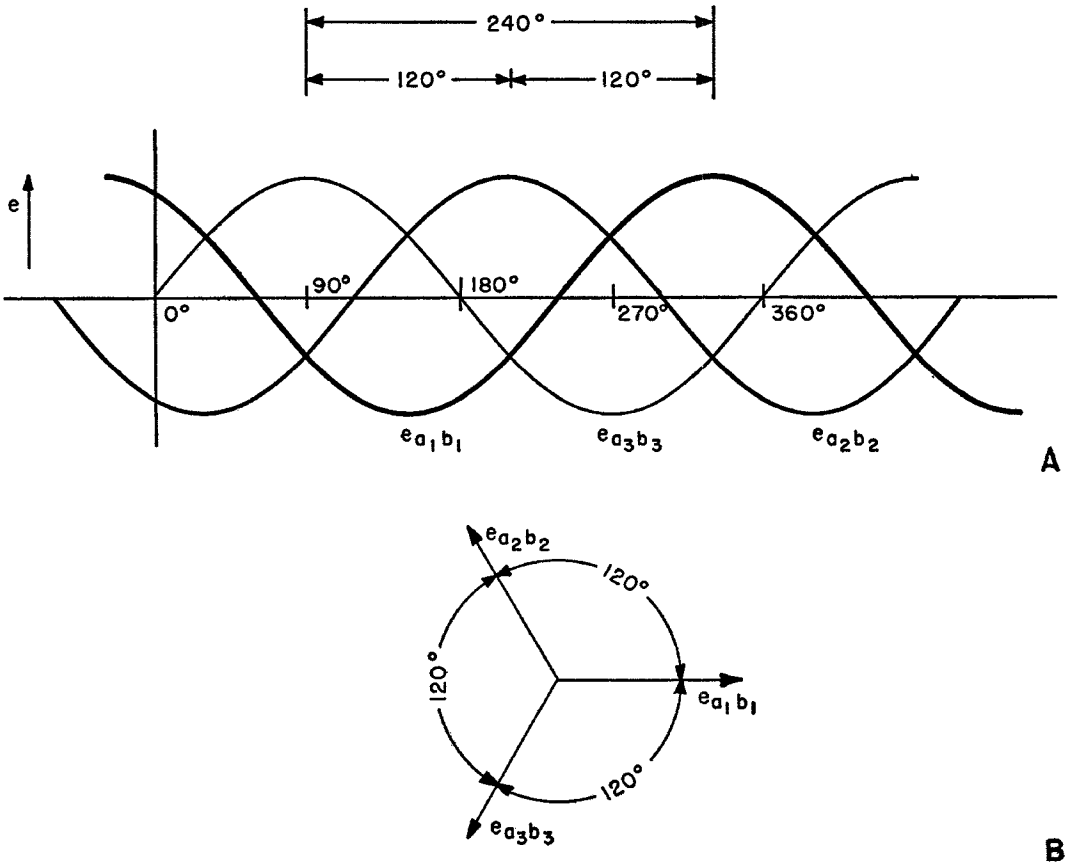
(4) In a three-phase generator, 1 mechanical



NOTE:
WINDINGS, $a_1 b_1$, $a_2 b_2$, AND
$a_3 b_3$ ARE INDIVIDUAL WINDINGS.

TM 681-129

Figure 178. Elementary three-phase alternator.



NOTE:
PHASE SEQUENCE = $e_{a_3 b_3}, e_{a_2 b_2}, e_{a_1 b_1}$.

TM 681-130

Figure 179. Three-phase alternator, voltage graphs.

degree is equal to the number of poles in the generator divided by 2, or $P/2$ electrical degrees. Since each phase is to be 120° apart electrically, and since 1 mechanical degree is equivalent to $P/2$ electrical degrees, then the mechanical separation of the windings is equal to $\frac{120}{P/2}$ or $240/P$. P is the number of poles in the generator, and the figure 240 is a constant.

For example, an 8-pole machine is to be used to generate a 60-cycle, three-phase system of voltages. Find the following: (*a*) the mechanical separation between windings; (*b*) the speed of rotation.

The mechanical separation $= 240/P$ mechanical degrees
$= 240/8$ or 30 mechanical degrees.

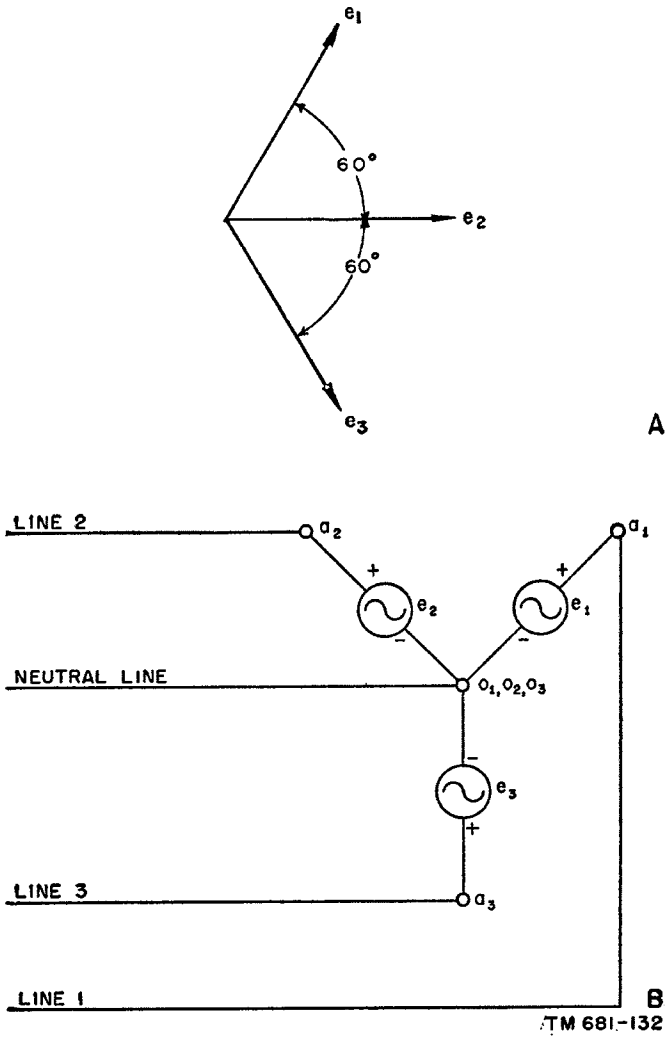


Figure 180. Three-voltage system, 60° phase shift.

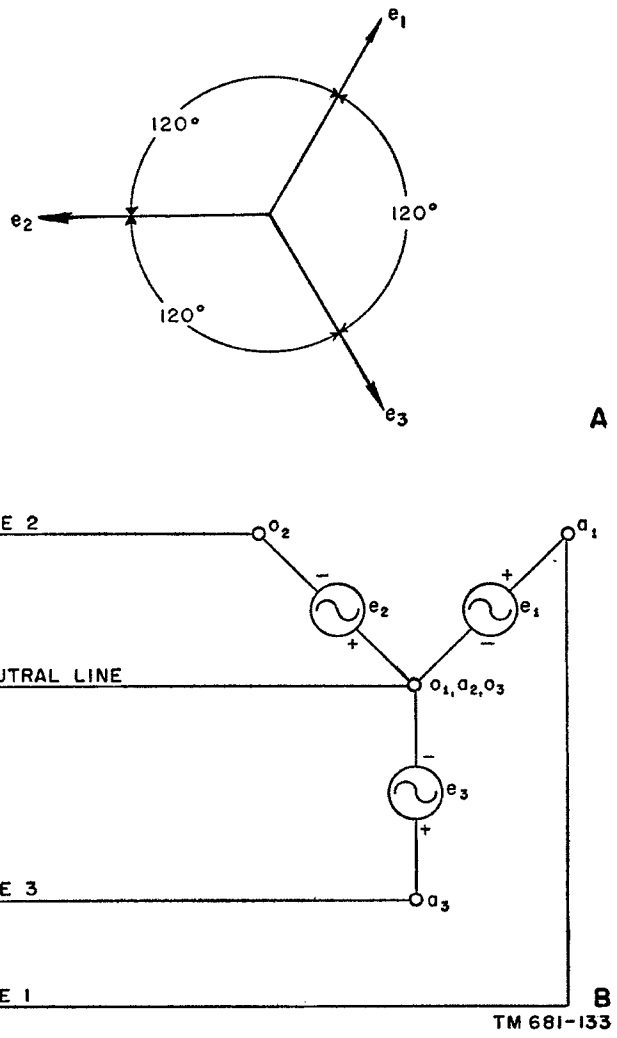


Figure 181. Three-voltage system, 120° phase shift.

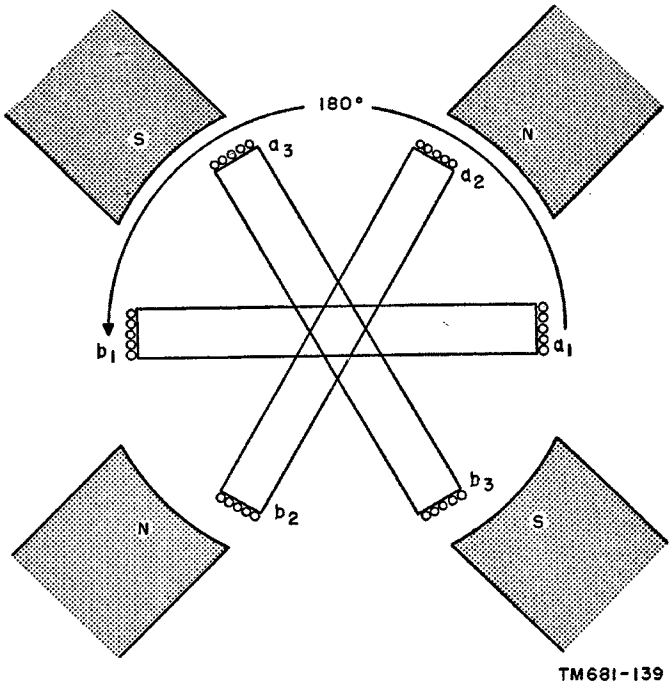


Figure 182. Elementary four-pole generator.

The frequency of the generated voltage:

$$f = \frac{P \times N}{120} \text{ cps}$$

Therefore:

$$N = \frac{120 \times f}{8} = \frac{120 \times 60}{8}$$

$$N = 900 \text{ rpm.}$$

113. Methods of Connecting Three-phase Windings

Three-phase systems are usually connected in either a *delta* or a *wye* configuration. Each of these connections has definite electrical characteristics and the designations delta and wye are derived from the method of connection. The iron ring shown in figure 183 is known as a Gramme ring and has three symmetrical windings. In A of figure 184, the windings are connected in series and form a closed circuit. This is called a delta connection since the shape of the windings and also the connections resemble the Greek letter *d* or delta. In B, the beginning of each winding is connected to a common terminal *o*, and the end of each winding is brought out to form the output. This method of connection, as well as the schematic diagram, shows the similarity to the letter *Y* and is called the *Y*, or wye, connection.

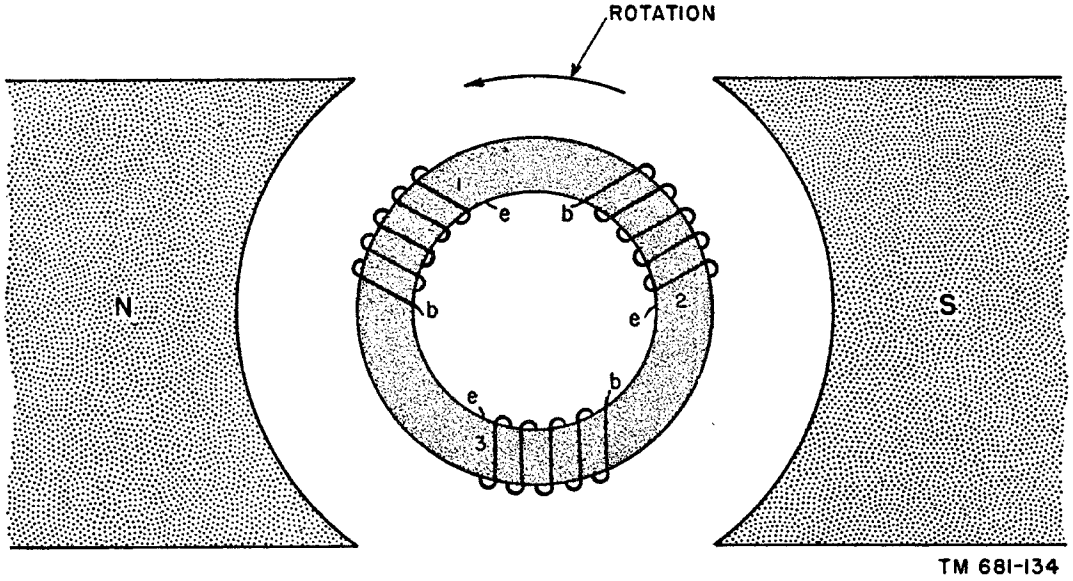


Figure 183. Gramme ring with three-phase winding.

114. Electrical Properties of Delta or Wye Connections

a. In a balanced circuit, when the generators are connected in delta (A of fig. 184), the voltage between any two lines is equal to that of a single phase. The line voltage and the voltage across any winding are in phase, but the line current is 30°, or 150°, out of phase with the current in any of the other windings. The vector diagram of figure 185 shows that the vector sum of e_1 and e_2 is equal to $-e_3$; therefore, the voltage between the two lines is equal to the voltage of the single winding e_3. Although the windings form a closed loop in this circuit, there is no circulating current flow in the windings. The total voltage acting on the closed circuit is the sum of the three-phase voltages, and this sum is zero. For example, the vector representation of the voltages in figure 185 shows that the vector sum of e_1 and e_2 is equal to $-e_3$, and since $(e_1 + e_2)$ plus e_3 is equal to zero, then $-e_3$ plus e_3 equals zero. The net voltage is therefore zero, and no circulating current can exist in the delta-connected three-phase system when the circuit is balanced. Phase current vectors for a balanced load are shown in c of figure 186. The current in the line I_L is out of phase with the currents I_1 and I_2 by 30° since it is the vector sum of these currents. Because the vectors of the two currents are equal, and form a parallelogram with equal sides, the length of the line current vector can be found by multiplying the phase current $I\emptyset$ by 2 times the cosine of the angle. For example, the line current is out of phase by 30°, the cosine of 30° is .8666, and twice the cosine is equal to 1.73 or $\sqrt{3}$. Therefore, the line current I_L is equal to the $\sqrt{3}$ times the phase current—that is, the current in any winding.

b. In the wye connection (A of fig. 186) the

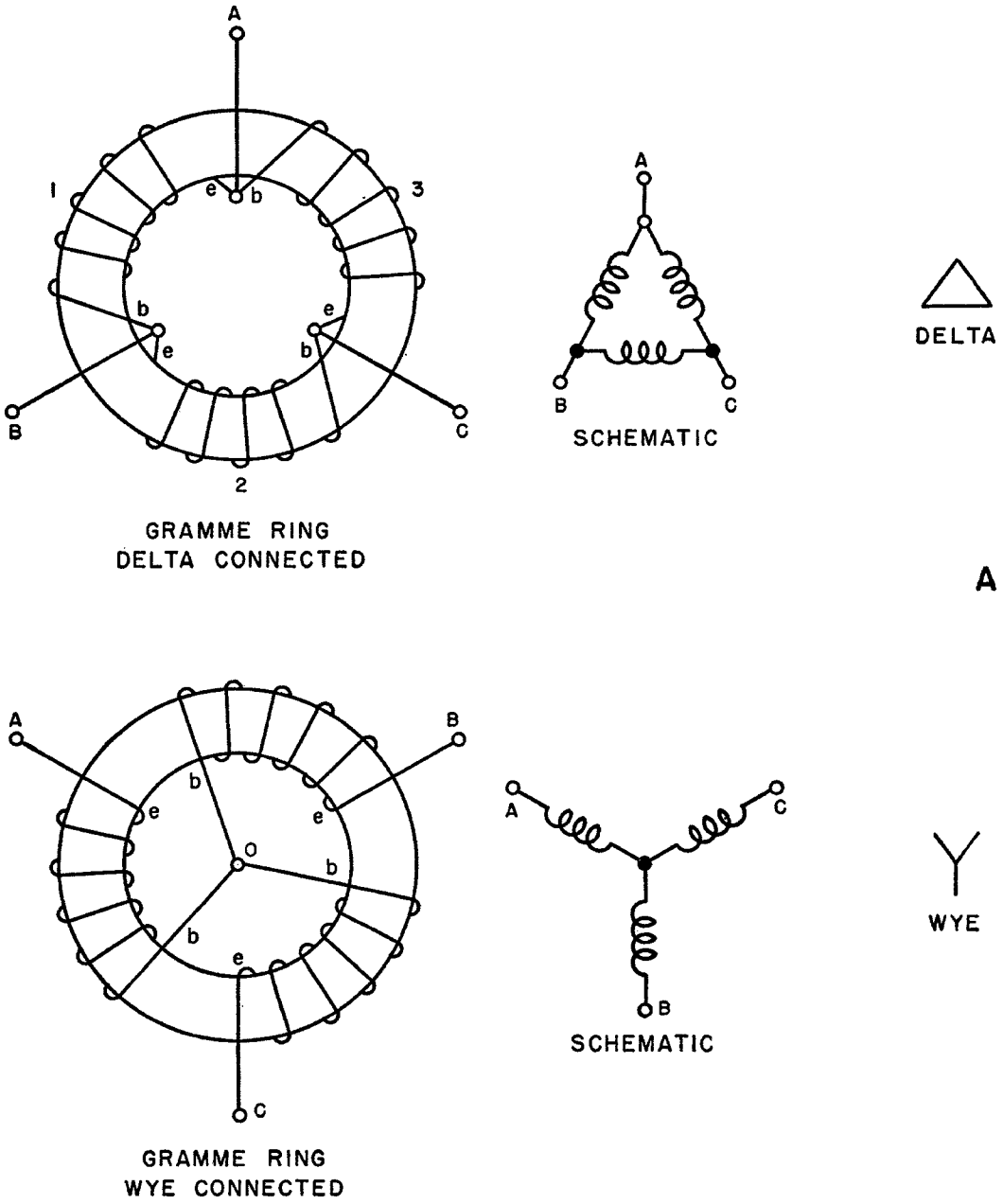


Figure 184. Delta- and wye-connected gramme ring.

current in the line is in phase with the current in the winding. The voltage between any two lines is not equal to the voltage of a single phase, but is equal to the vector sum of the two windings between the lines, as shown in D. The current in line A is the current flowing through the winding L_1, that in line B is the current flowing through the winding I_2, and the current flowing in line C is that of the winding L_3. Therefore the current in any line is in phase with the current in the winding that it feeds. Since the line voltage is the vector sum of the voltages across any two coils, and the

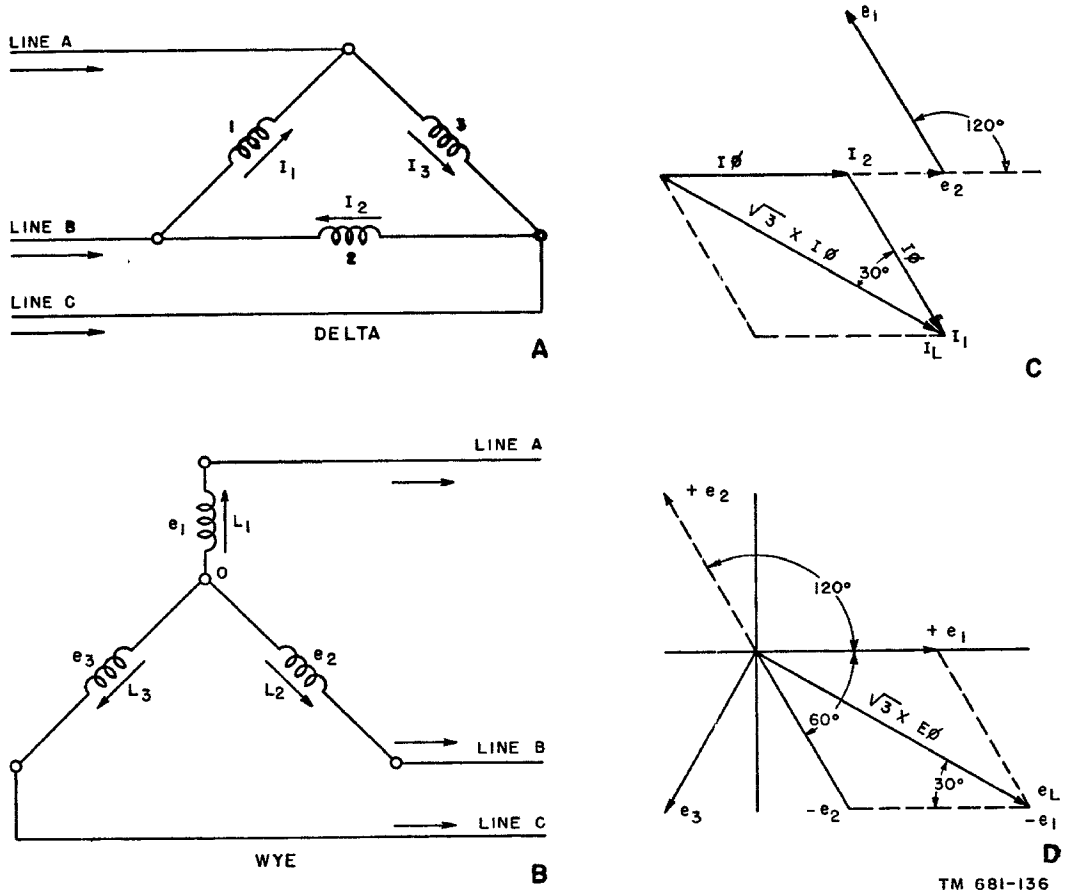


Figure 186. Delta and wye connection, vector diagram.

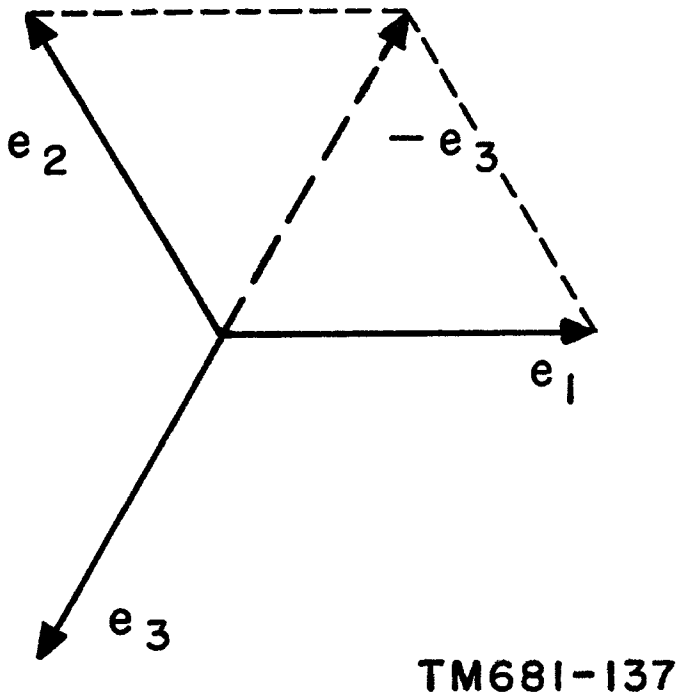


Figure 185. Delta connection, vector diagram.

line voltage E_L and the voltage across the windings $E\emptyset$ are 30° out of phase, the line voltage can be found by multiplying the voltage of any single winding $E\emptyset$ by the $\sqrt{3}$.

c. The properties of the delta connection may be summarized as follows: The three windings of the delta connection form a closed loop. The sum of the three equal voltages 120° out of phase is zero. This means that the circulating current in the closed loop formed by the windings is zero. The magnitude of any line current is equal to the $\sqrt{3}$ times the magnitude of any phase current, or:

$$I_L = \sqrt{3} \times I\emptyset.$$

The properties of the wye connection may be summarized as follows: The three windings of the wye connection do not form a closed loop. The magnitude of the voltage between any two lines equals the magnitude of any phase voltage times the square root of three, or:

$$E_L = \sqrt{3} \times E\emptyset.$$

The current in any winding equals the current in the line.

Problems illustrating the preceding summaries follow:

(1) A three-phase generator is connected in delta. The phase voltage is 300 volts. Find the line voltage.

Solution: In a delta connection, the phase voltage is equal to the line voltage.

$$E_L = 300 \text{ volts.}$$

(2) A three-phase voltage is connected in wye. The phase voltage is 300 volts. Find the line voltage.

Solution: In a wye connection, the line voltage is equal to the $\sqrt{3}$ times the phase voltage; therefore:

$$E_L = \sqrt{3} \times 300 = 1.73 \times 300 = 519 \text{ volts.}$$

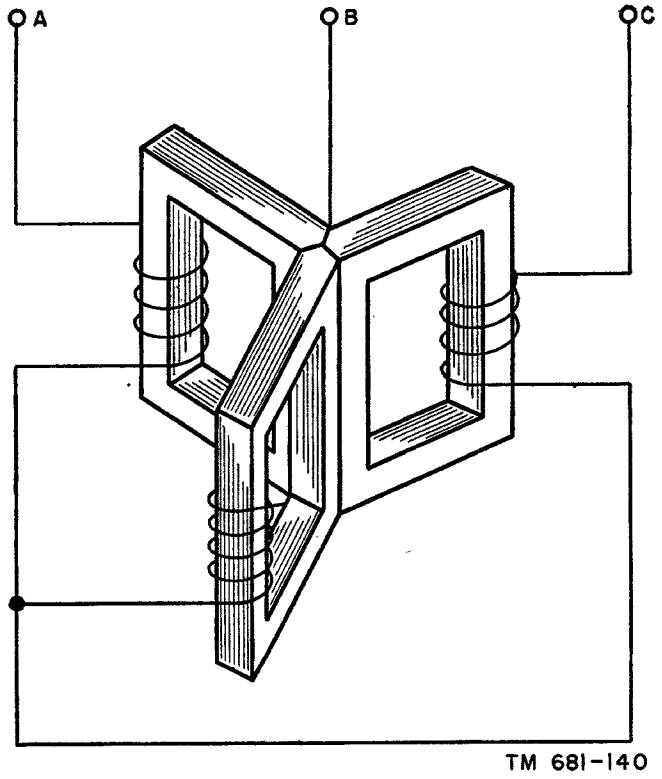


Figure 187. Elementary three-phase transformer.

115. Three-phase Delta and Wye Transformer Connections

a. The three-phase transformer is used when large power outputs are required. It may be a single transformer or three separate transformers and is generally connected in delta or wye. The three-phase transformer core has less iron than that of three single transformers, and therefore occupies less space and is lighter in weight. Figure 187 shows an elementary three-phase transformer that has three single-phase windings connected in wye with their cores 120° apart. The empty legs of each transformer are connected and form a common leg. The flux produced by the current in each of the windings is carried by this leg. The sum of the currents in the three windings is zero; therefore the sum of the fluxes in the common leg must also be zero, and no flux exists in this leg. Two legs of the transformer serve as a return path for the other. This is similar to the three-phase system where two of the windings serve as a return path for the other. The construction of this

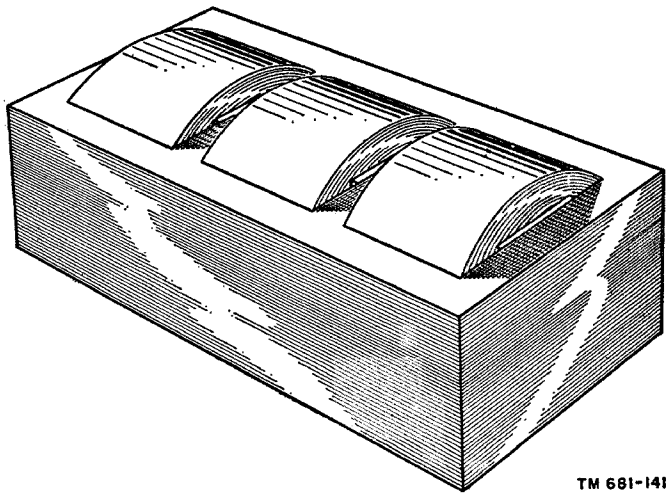


Figure 188. Three-phase shell-type transformer.

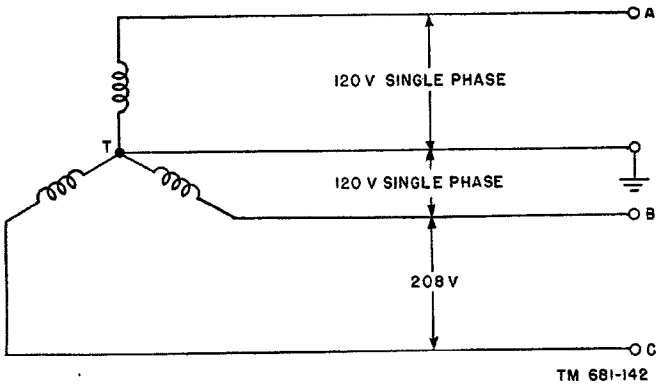


Figure 189. Commercial three-phase voltage from power line.

transformer is not practical but demonstrates the three-phase principle in a transformer.

b. Practical transformer construction uses the standard shell-type mounting shown in figure 188. This transformer is economical to construct and occupies less space than three single transformers; however, if one phase burns out, the whole unit must be replaced.

c. Commercial three-phase voltage from the power lines is generally 208 volts and the standard values of single-phase voltage can be supplied from the line as shown in figure 189. The windings

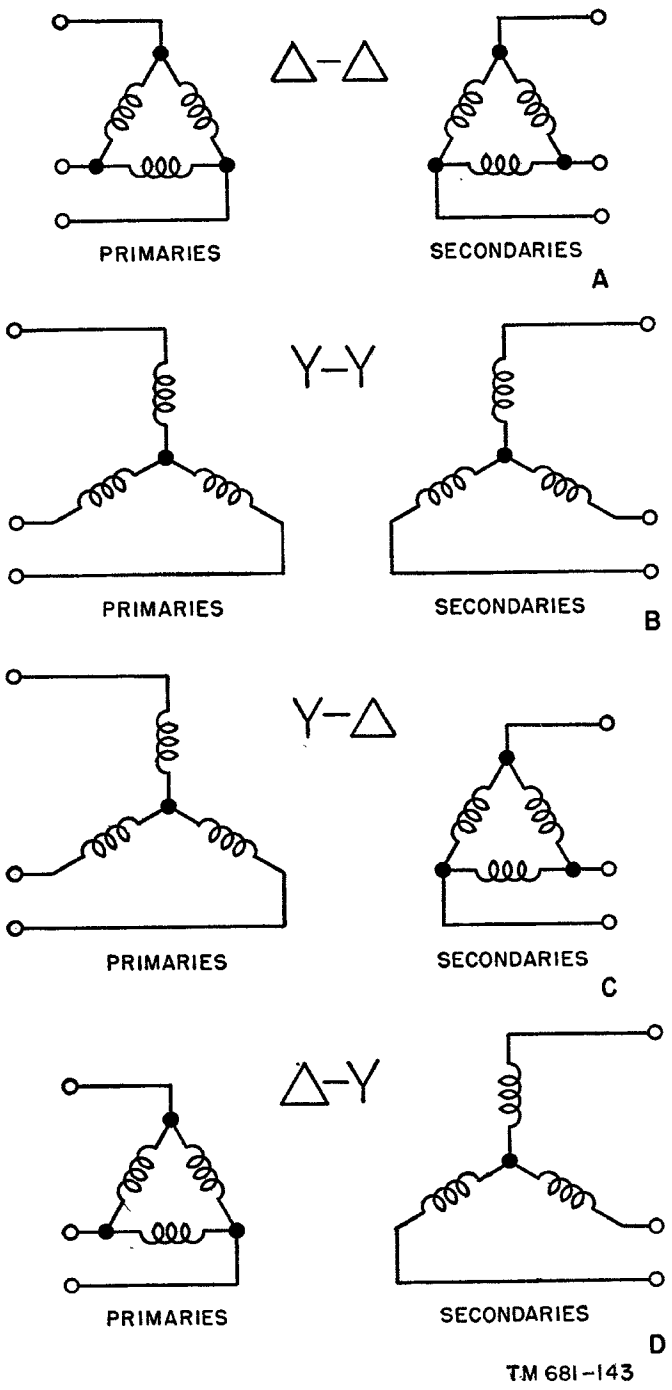


Figure 190. Methods of connecting three-phase transformer.

represent the wye-connected transformer, which is generally an outside installation. The three-phase connections are designated *a*, *b*, and *c*, and the single-phase connections are from *a* or *b* to ground. The three-phase transformer must be properly connected to these lines in order to operate. Four of the most widely used connections (fig. 190) are:

(1) Primary in delta, secondary in delta, or delta-delta (Δ–Δ).

(2) Primary in wye, secondary in wye, or wye-wye (Y–Y).

(3) Primary in wye, secondary in delta, or wye-delta (Y–Δ).

(4) Primary in delta, secondary in wye, or delta-wye (Δ–Y). Of these four combinations, the one used most extensively in military applications is the last one listed, the delta-wye.

d. Regardless of what method of connection is used, the secondary must be connected in the proper phase relationships. To determine these in a wye-connected transformer, the voltage is measured across two windings of the secondary (A of fig. 191). The voltage from *a* to *b* should be equal to the $\sqrt{3}$ times the voltage across either winding. If the voltage reads the same as that across either winding, then one of the windings must be reversed. The third winding, cc_1, is then connected to a_1b_1, as shown in B, and the voltage across any two windings should equal the $\sqrt{3}$ times the voltage of any one winding. If the voltages do not read this value, the winding cc_1 must be reversed.

e. To determine the proper phase relationships in a delta-connected secondary, the voltage is measured across two coils of the secondary (C of fig. 191). The voltage from *a* to b_1 should equal the voltage across either coil. If it does not, either one of the windings must be reversed. The winding cc_1 is then connected to b_1, as shown in *D*. The voltage across the three coils should then measure zero. If the voltage does not read zero, then the winding cc_1 must be reversed. The open ends, *a* and c_1, are then connected and the transformer has the proper phase relationships for delta connection.

f. The wye–wye (Y–Y) connection has the same volts per turn ratio between primary and secondary as that of an individual transformer. When using this connection, unless the primary neutral point is connected to the line neutral point, a "floating neutral" results with an appreciable loss of power in some installations. The secondary also contains large third harmonics. The voltage output of the delta-delta (Δ–Δ) is also dependent on the turns ratio of the primary and secondary. Its greatest advantage is its ability to operate as a V or open delta circuit if one winding becomes inoperative. The delta-wye connection (Δ–Y) is generally used for radio power transformers. This connection, unlike the Y–Y connection, is not affected by a floating neutral point. It also has a

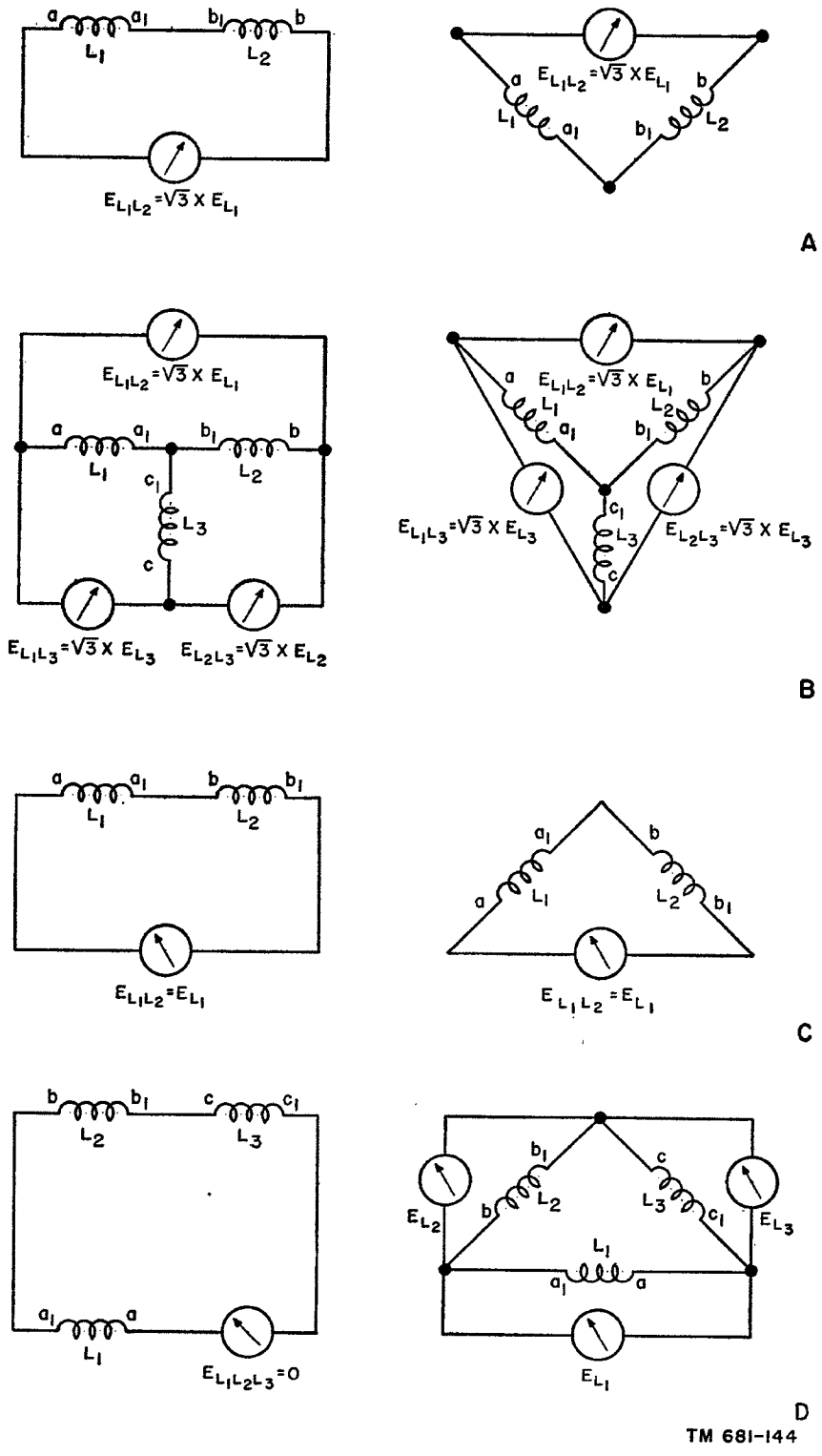


Figure 191. Phasing of transformers for delta or wye connection.

higher voltage ratio between primary and secondary than either the delta-delta or wye-wye connection. This is because the voltage across the two windings of the wye is equal to the $\sqrt{3}$ times E_L. For example, the delta primary is connected across a 208-volt three-phase system (fig. 192) and the voltage across any phase is 120 volts. The voltage across any winding of the secondary will therefore be 120 volts, but the voltage across two of the windings will be:

$$\sqrt{3} \times E_L = E_{sec}$$

$$1.73 \times 120 = 208 \text{ volts} = E_{sec}.$$

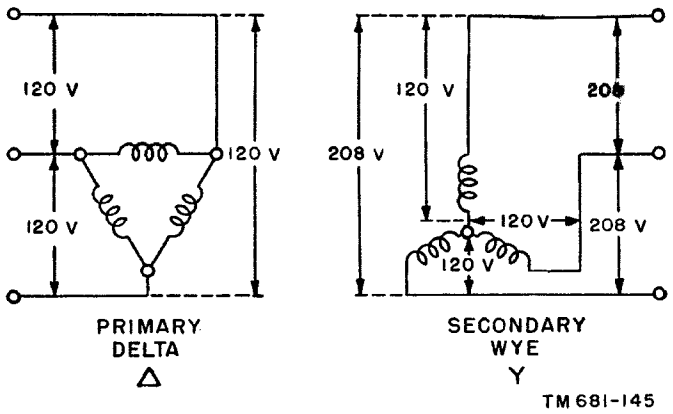


Figure 192. Delta-wye transformer connection.

The voltage across *a* and *b* (fig. 192) is equal to 208 volts. The ratio between line voltage and secondary voltage in the delta-wye transformer connection is not the transformer turns ratio, but the $\sqrt{3}$ times the turns ratio. The output voltage across any two windings is greater than that of a single winding; therefore, the single winding need be insulated for only $E_Y/\sqrt{3}$ volts. For example, if the voltage across two windings of the secondary is 20,000 volts, the voltage across one phase is:

$$\frac{E_Y}{\sqrt{3}} = 20{,}000 \; 1.73 = 11{,}600 \text{ volts (approx)}.$$

Because of this, the delta-wye connection permits cheaper construction of the transformer in high-voltage installations. The wye-delta (Y–Δ) connection is the opposite of the delta-wye connection and often is used for step-down voltages.

Section II. A-C MOTORS

116. Induction Motor

Of all a-c motors, the induction motor is the most widely used. Its design is simple and its construction rugged. The induction motor is particularly well adapted for constant speed applications, and because it does not use a commutator, most of the troubles encountered in the operation of d-c motors are eliminated. An induction motor can be either a single-phase or a polyphase machine. The operating principle is the same in either case, and depends on a revolving, or rotating, magnetic field to produce torque. The key to understanding the induction motor is a thorough comprehension of the rotating magnetic field.

117. Rotating Magnetic Field

Consider the field structure of A of figure 193 where the poles have windings which are energized by three a-c voltages, *a*, *b*, and *c*. These voltages have equal magnitude but differ in phase, as shown in B.

a. At the instant of time shown as 0, the resultant magnetic field produced by the application of the three voltages has its greatest intensity in a direction extending from pole 1 to pole 4. Under this condition, pole 1 can be considered as a north pole and pole 4 as a south pole.

b. At the instant of time shown as 1, the resultant magnetic field will have its greatest intensity in the direction extending from pole 2 to pole 5 and, in this case, pole 2 can be considered as a north pole and pole 5 as a south pole. Thus, be-

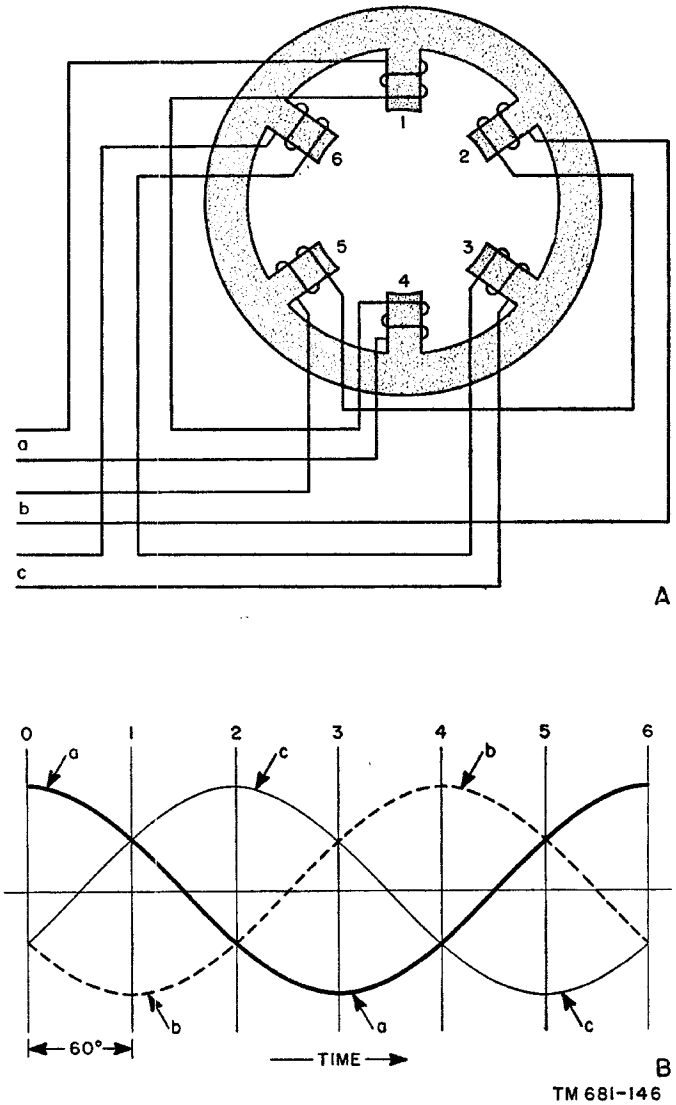


Figure 193. A rotating magnetic field developed by application of three-phased voltages.

tween instant 0 and instant 1, the magnetic field has rotated clockwise.

c. At time 2, the resultant magnetic field has its greatest intensity in the direction from pole 3 to pole 6, and it is apparent that the resultant magnetic field has continued to rotate clockwise.

d. At instant 3, poles 4 and 1 can be considered as north and south poles, respectively, and the field has rotated still farther.

e. At later instants of time, the resultant magnetic field rotates to other positions while traveling in a clockwise direction, a single revolution of the field occurring in 1 cycle. If the exciting voltages have a frequency of 60 cycles per second, the magnetic field makes 60 revolutions per second, or 3,600 revolutions per minute. This speed is known as the *synchronous speed* of the rotating field.

quency by the rated speed or, written as an equation:

$$P = \frac{120 \times f}{N}$$

where: P is the number of poles per phase,
f is the frequency in cycles per second,
N is the rated speed in rpm and 120 is a constant.

The result will be very nearly equal to the number of poles per phase. For example, consider a 60-cycle, three-phase machine with a rated speed of 1,750 rpm. In this case:

$$P = \frac{120 \times 60}{1750} = \frac{7200}{1750} = 4.1.$$

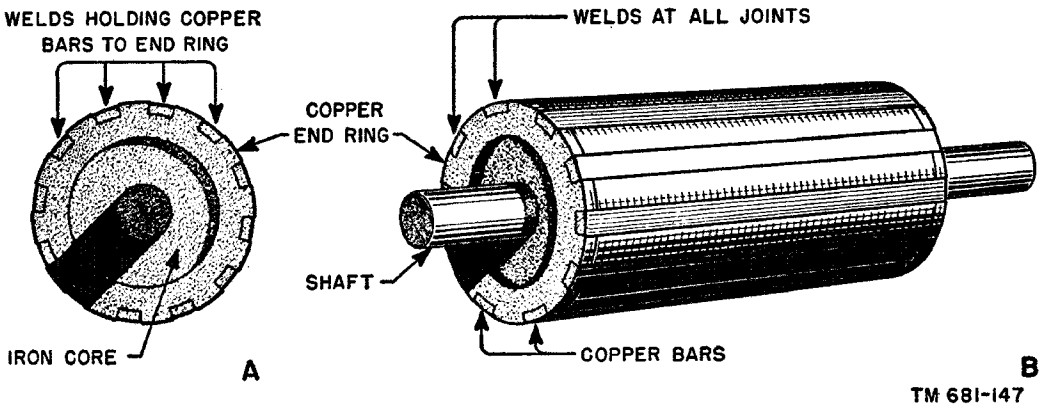


Figure 194. Squirrel-cage rotor.

118. Construction of an Induction Motor

In an induction motor, the stationary portion of the machine is called a stator and the rotating member is called a rotor. Instead of salient poles in the stator, as shown in A of figure 193, distributed windings are used, these windings being placed in slots around the periphery of the stator.

a. It is usually not possible to determine the number of poles on an induction motor by visual inspection but the information can be obtained from the nameplate of the machine. The nameplate usually gives the number of poles and the speed at which the motor is designed to run. This rated, or asynchronous, speed is slightly less than the synchronous speed for reasons explained in paragraph 119. To determine the number of poles per phase on the motor, divide 120 times the frequency by the rated speed or, written as an equation:

Therefore, the machine has four poles per phase. If the number of poles per phase is given on the nameplate the synchronous speed can be determined by dividing 120 times the frequency by the number of poles per phase. In the example used above, the synchronous speed is equal to 7,200 divided by 4, or 1,800 rpm.

b. The rotor of an induction motor consists of an iron core having longitudinal slots around its circumference in which heavy copper or aluminum bars are embedded. These bars are welded to a heavy ring of high conductivity on either end. This composite structure is sometimes called a *squirrel-cage*, and motors containing such a rotor are called squirrel-cage induction motors.

119. Slip

When the rotor of an induction motor is subjected to the revolving magnetic field produced by the stator windings, a voltage is induced in the longitudinal bars. The induced voltage causes a

current to flow through the bars. This current, in turn, produces its own magnetic field which combines with the revolving field in such a way as to cause the rotor to assume a position in which the induced voltage is minimized. As a result, the rotor revolves at very nearly the synchronous speed of the stator field, the difference in speed just being sufficient to induce enough current in the rotor to overcome the mechanical and electrical losses in the rotor. If the rotor were to turn at the same speed as the rotating field, the rotor conductors would not be cut by any magnetic lines of force, no emf would be induced in them, no current could flow, and there would be no torque. The rotor would then slow down. For this reason, there must always be a difference in speed between the rotor and the rotating field. This difference in speed is called *slip* and is expressed as a percentage of the synchronous speed. For example, if the rotor turns at 1,750 rpm and the synchronous speed is 1,800 rpm, the difference in speed is 50 rpm. The slip is then equal to 50/1,800 or 2.78 percent.

120. Single-phase Machines

The previous discussion has applied only to polyphase machines. In a single-phase machine, the field, instead of rotating, merely pulsates, and no rotation of the rotor takes place. However, a single-phase pulsating field may be visualized as two rotating fields revolving at the same speed but in opposite directions. It follows, therefore, that the rotor will revolve in either direction at nearly synchronous speed provided it is given an initial impetus in either one direction or the other. The exact value of this initial rotational velocity varies widely with different machines but a velocity higher than 15 percent of the synchronous speed is usually sufficient to cause the rotor to accelerate to rated speed. A single-phase motor can be made self-starting if means can be provided to give the effect of a rotating field.

a. The first effort in the development of a self-starting, single-phase motor was the *shaded-pole* induction motor (fig. 195). This machine has salient poles, a portion of each pole being encircled by a heavy copper ring. The presence of the ring causes the magnetic field through the ringed portion of the pole face, in B, to lag appreciably behind that through the other part of the pole face, in A. The net effect is the production of a slight component of rotation of the field, sufficient to cause the rotor to revolve. As the rotor accelerates, the torque increases until the rated speed is obtained. Such motor have low starting torque and find their greatest application in small fan motors where the initial torque required is low.

b. Various types of self-starting motors, known as *split-phase* motors, have been made. Such motors have a starting winding displaced 90 electrical degrees from the main or running winding. In some types, the starting winding has a fairly high resistance, which causes the current in this winding to be out of phase with the current in the running winding. This condition produces, in effect, a rotating field and the rotor revolves. A centrifugal switch is used to disconnect the starting winding automatically after the rotor has attained approximately 25 percent of its rated speed.

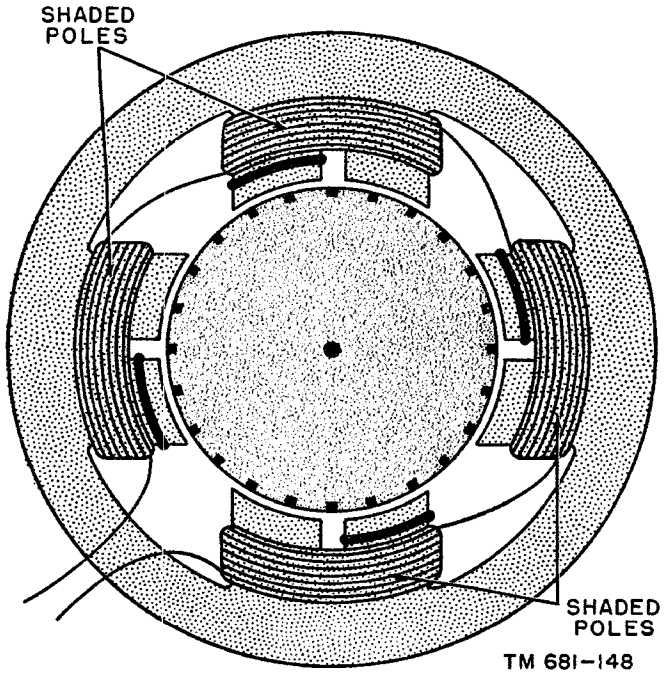


Figure 195. Shaded pole.

c. With the development of high-capacity electrolytic capacitors, a variation of the split-phase motor, known as the *capacitor-start* motor, has been made, and nearly all fractional horsepower motors in use today on refrigerators, oil burners, and other similar appliances are of this type. In this adaptation, the starting winding and running winding have the same size and resistance value, the phase shift between currents of the two windings being obtained by means of capacitors connected in series with the starting winding. Capa-

citor-start motors have a starting torque comparable to their torque at rated speed and can be used in applications where the initial load is heavy. Again, a centrifugal switch is required for disconnecting the starting winding when the rotor speed is approximately 25 percent of the rated speed.

d. Although some single-phase induction motors are rated as high as 2 horsepower, the major field of application is 1 horsepower, or less, at a voltage rating of 115 volts for the smaller sizes and 110 to 220 volts for ¼ horsepower and up. For still larger power ratings, polyphase motors are generally used, the latter types having excellent starting torque.

121. Direction of Rotation and Emergency Operation

a. The direction of rotation of a three-phase induction motor can be changed by simply reversing two of the leads to the motor. The same effect can be obtained in a two-phase motor by reversing connections to one phase. In a single-phase motor, reversing connections to the starting winding will reverse the direction of rotation. Most single-phase motors designed for general application have provision for readily reversing connections to the starting winding. Nothing can be done to a shaded-pole motor to reverse the direction of rotation because the direction is determined by the physical location of the copper shading ring.

b. If, after starting, one connection to a three-phase motor is broken, the motor will continue to run but will deliver only one-third the rated power. Also, a two-phase motor will run at one-half its rated power if one phase is disconnected. Neither motor will start under these abnormal conditions.

122. Synchronous Motor

The synchronous motor is one of the principal types of a-c motors. Like the induction motor, the synchronous motor makes use of a rotating magnetic field. Unlike the induction motor, however, the torque developed does not depend on the induction of currents in the rotor. Briefly, the principle of operation of the synchronous motor is as follows: A multiphase source of ac is applied to the stator windings and a rotating magnetic field is produced. A direct current is applied to the rotor windings and another magnetic field is produced. The synchronous motor is so designed and constructed that these two fields react upon each other in such a manner that the rotor is dragged along and rotates at the *same* speed as the rotating magnetic field produced by the stator windings.

123. Motor Theory

An understanding of the operation of the synchronous motor can be obtained by considering the simple motor of figure 196. Assume that poles A and B are being rotated clockwise by some mechanical means in order to produce a rotating magnetic field. The rotating poles induce poles of opposite polarity, as shown in the illustration of the soft-iron rotor, and forces of attraction exist

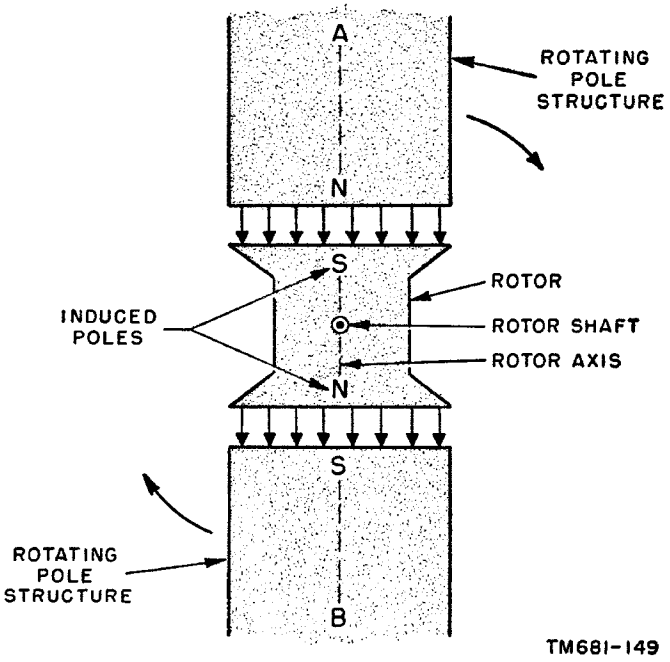


Figure 196. Simple synchronous motor.

between corresponding north and south poles. Consequently, as poles A and B rotate, the rotor is dragged along at the same speed. However, if a load is applied to the rotor shaft, the rotor axis will momentarily fall behind that of the rotating field but will thereafter continue to rotate with the field at the same speed as long as the load remains constant. If the load is made too large, the rotor will pull out of synchronism with the rotating field and, as a result, will no longer rotate with the field at the same speed. The motor is then said to be overloaded.

a. Such a simple motor as that shown in figure 196 is never used.

(1) The idea of using some mechanical means

of rotating the poles is impractical because another motor would be required to perform this work. Also, such an arrangement is unnecessary because a rotating magnetic field can be produced electrically by using phased a-c voltages (fig. 197). In this respect, the synchronous motor is similar to the induction motor.

(2) The magnitude of the induced poles in the rotor shown in figure 196 is so small that sufficient torque cannot be developed for most practical loads. To avoid such a limitation on motor operation, a winding is placed on the rotor and this winding is energized with dc. A rheostat placed in series with the d-c source provides the operator of the machine with a means of varying the strength of the rotor poles, thus placing the motor under control for varying loads.

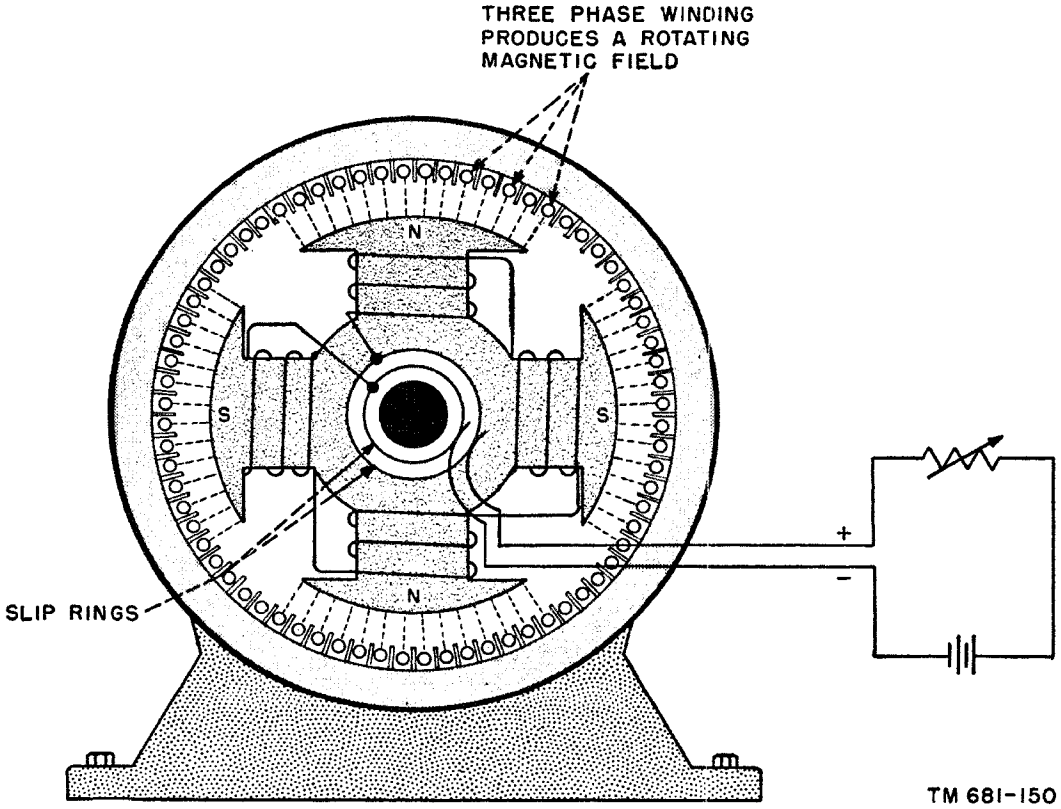


Figure 197. Synchronous motor.

b. If a synchronous motor is driven by an external power source and the excitation, or voltage applied to the rotor, is adjusted to a certain value called 100 percent excitation, no current will flow from or to the stator winding. In this case, the voltage generated in the stator windings by the rotor, or cemf, exactly balances the applied voltage. However, if the excitation is reduced below the 100-percent value, the difference between the cemf and the applied voltage produces a react ve component of current which lags the applied voltage. The machine then acts as an *inductance.* Similarly, if the excitation is increased above the 100-percent value, the react ve component leads the applied voltage, and the machine acts as a *capacitor.* This feature of the synchronous motor permits use of the machine as a power-factor correction device. When so used, it is called a synchronous capacitor.

(1) When used as a synchronous capacitor, the motor is connected on the a-c line in parallel with the other motors on the line, and run either without load or with a very light load. The rotor field is overexcited just enough to produce a leading current which offsets the lagging current of the line. Unity power factor results.

(2) The synchronous motor can be made to produce up to 80-percent leading power factor. However, because leading power factor on a line is just as detrimental as

a lagging power factor, the synchronous motor is regulated to produce just enough leading current to compensate for lagging current in the line.

c. The synchronous motor is not a self-starting motor. The rotor is heavy and, from a dead stop, it is impossible to bring the rotor into magnetic lock with the rotating magnetic field. For this reason, all synchronous motors have some kind of starting device. A simple starter is another motor, either ac or dc, which brings the rotor up to approximately 90 percent of its synchronous speed. The starting motor is then disconnected and the rotor locks in step with the rotating field. Another starting method is a second winding of the squirrel-cage type on the rotor. This induction winding brings the rotor almost to synchronous speed and, when the dc is connected to the rotor windings, the rotor pulls into step with the field. The latter method is the more commonly used.

124. Summary

a. A single-phase alternator differs from a d-c generator in that it must be separately excited and also in using slip rings instead of a commutator for the collection of the energy produced.

b. A two-phase voltage can be produced by placing two separate windings 90 mechanical degrees apart in a two-pole machine.

c. A three-phase voltage can be produced by placing three separate windings 120 mechanical degrees apart in a two-pole machine.

d. In any machine, the necessary mechanical separation is 240/P mechanical degrees.

e. A three-phase system of voltages has the following properties:

(1) The three voltages have equal magnitudes.

(2) They are out of phase by 120 electrical degrees.

f. Before a three-phase system of voltages is completely specified, the phase sequence must be known. If e_1 leads e_2 which leads e_3, the phase sequence is said to be positive. If e_1 lags e_2 which lags e_3, the phase sequence is said to be negative.

g. A three-phase winding can be connected either in wye or in delta.

h. The wye connection has the following electrical properties:

(1) Phase current is equal to line current.

(2) The line voltage is equal to the $\sqrt{3}$ times the phase voltage.

i. The delta connection has the following electrical properties:

(1) The line current is equal to the $\sqrt{3}$ times the phase current.

(2) The line voltage is equal to the phase voltage.

j. Three-phase transformers may be a single unit or three separate transformers.

k. The most extensively used transformer connection is the delta-wye.

l. Transformer secondaries must be connected in the proper phase relationships.

m. The output voltage of a delta-delta or a wye-wye transformer connection is determined by the volts per turn ratio of the transformer.

n. The output voltage of a delta-wye transformer connection is not determined by the voltage per turn ratio of the transformer, but by the $\sqrt{3}$ times the turns ratio.

o. The output voltage of a wye-delta transformer connection acts as a step-down voltage.

p. A rotating magnetic field is required for the operation of induction and synchronous motors.

q. A rotating magnetic field is obtained by using a system of phased voltages.

r. Distributed windings are used in the stator of an induction motor.

s. Squirrel-cage is a term often used to describe the rotor of an induction motor.

t. The torque developed by an induction motor depends on the induction of currents in the rotor windings.

u. Single-phase induction motors, including the split-phase motor and the capacitor-start motor, find wide application.

v. The direction of rotation of a three-phase induction motor can be changed by simply reversing two of the leads to the motor.

w. The torque developed by a synchronous motor is not dependent upon the induction of currents in the rotor windings.

x. A synchronous motor runs at the speed of its rotating magnetic field.

y. Power factor in a synchronous motor can be varied by means of a rheostat in series with the d-c rotor windings.

z. Synchronous motors are often used as synchronous capacitors.

125. Review Questions

a. Describe the important characteristics of a single-phase alternator.

b. Explain how a two-phase voltage can be obtained. A three-phase voltage.

c. What is the complete definition of a three-phase system?

d. How is the mechanical separation of the coils related to the number of poles?

e. What is meant by phase sequence?

f. What are the properties of a wye-connected winding?

g. What are the properties of a delta-connected winding?

h. What is a neutral connection?

i. Give four methods of connecting a three-phase transformer.

j. How are the phase relationships of a delta-secondary in a three-phase transformer determined?

k. How are the phase relationships of a wye-secondary in a three-phase transformer determined?

l. A wye-connected transformer secondary delivers 75,000 volts. For what voltage must the secondary winding be insulated?

m. Will a single-phase induction motor start under load?

n. What two methods can be used to start a synchronous motor?

o. Does an induction motor use a commutator?

p. What emf is applied to the stator of an induction motor? A synchronous motor?

q. What is the outstanding characteristic of synchronous motors?

r. What is the basic difference between an induction motor and a synchronous motor?

s. How are polyphase induction motors started?

APPENDIX I

GRAPHS, ANGLES, TRIGONOMETRY, AND VECTORS

Section I. GRAPHS

1. Elementary Principles

A graph is a pictorial representation of the relationship between two or more quantities. Thus, A of figure 198 is a representation of the relationship between temperature and time; B of figure 198 is a representation of the relationship between the voltage and current in a circuit as the voltage is varied.

a. DEVELOPMENT OF SIMPLE GRAPH. In A of figure 199 the lines $X'OX$ and $Y'OY$ are reference or axis lines. Line $X'OX$ is called the X axis or *abscissa* and line $Y'OY$ is called the Y axis or *ordinate.* These axes divide the plane into four quadrants which, by agreement, are always numbered as indicated. The intersection of the axes (marked O) is called the *origin.* On the X axis, values of X *greater* than zero are *positive* and to the right of the origin; values of X *less* than zero are *negative* and to the *left* of the origin. On the Y axis, values of Y *greater* than zero are *positive* and *above* the X axis; values of Y *less* than zero are *negative* and *below* the X axis.

b. COORDINATES. Consider point A in B of figure 199. This point can be located accurately in terms of the X and Y axes. A line through A and perpendicular to the X axis passes through $+3$ and a line through A and perpendicular to the Y axis passes through $+2$. This means that when X equals 3 and Y equals 2, A is the corresponding point on the graph. Suppose that X equals -4 and Y equals $+3$. Locate on the graph the point identified by these X and Y values. Since X is *negative,* it is located on $X'OX$ four unit distances to the *left* of the origin O. Since Y is *positive* it is located on $Y'OY$ three unit distances *above* the X axis. If we now erect a perpendicular from -4 on the X axis and another perpendicular from $+3$ on the Y axis, the perpendiculars will intersect at point B. In other words, point B is completely identified by $X = -4$ and $Y = +3$. The identifying values of X and Y are called the Cartesian

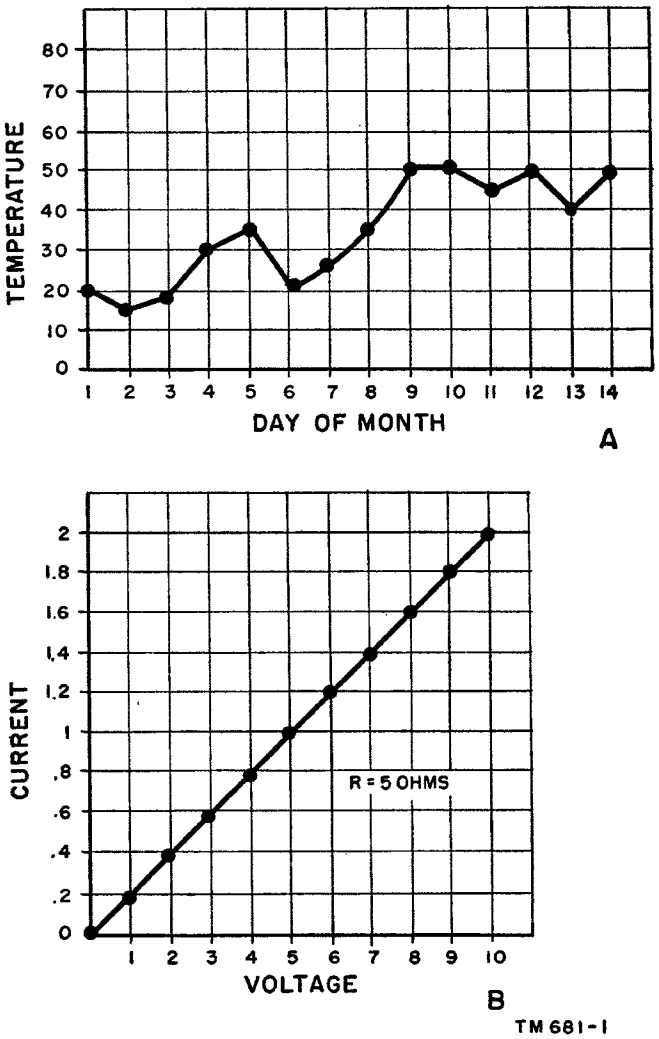
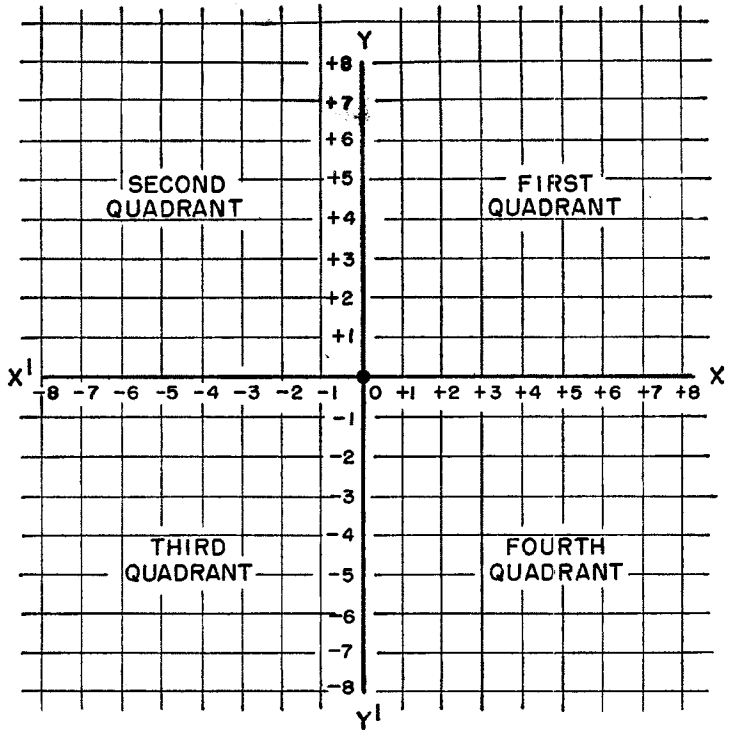
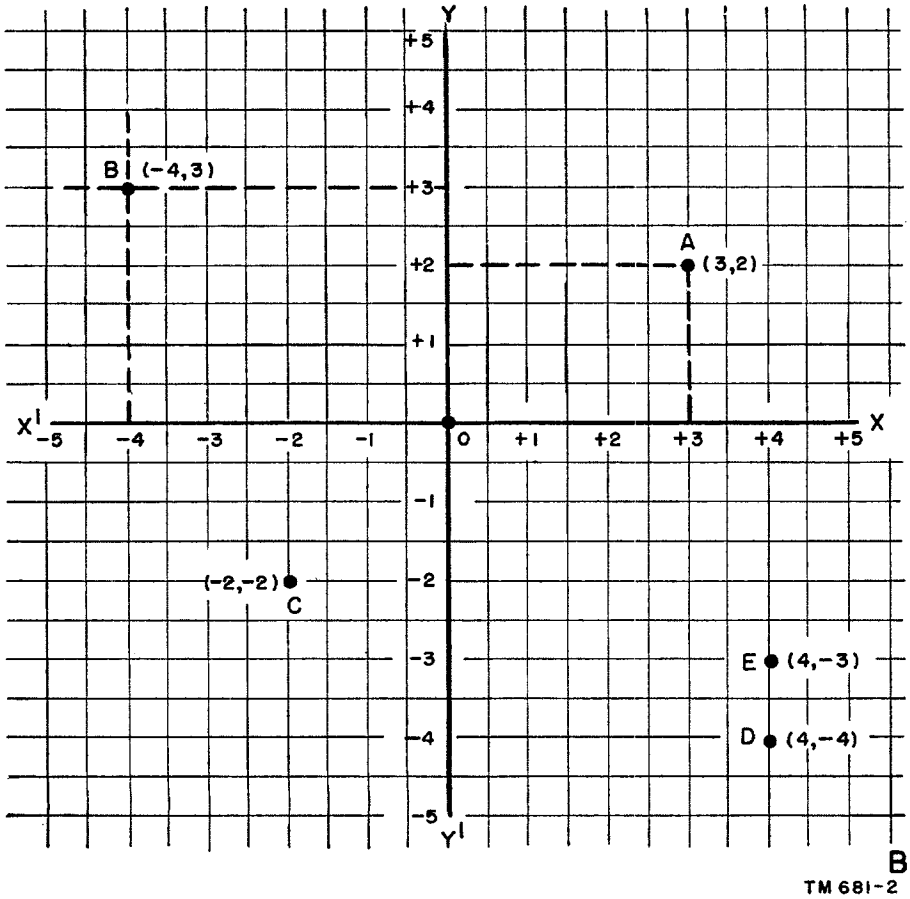


Figure 198. A. Graph of temperature variation. B. Graph of relation of voltage and current.

or rectangular coordinates and they are written (X, Y). Thus, in B of figure 199, the $(-2, -2)$ at point C means that $X = -2$ and $Y = -2$.



A



B

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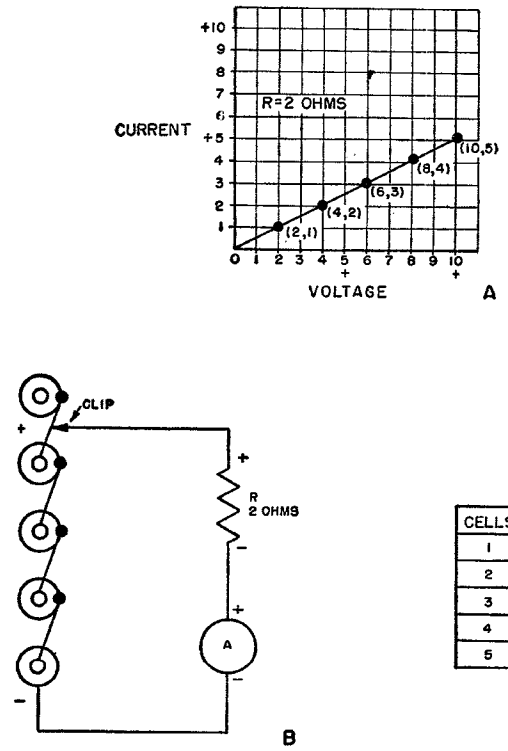
Figure 199. X and Y, quadrants, and coordinates.

The X value or abscissa is always written first; for example, point D is identified by (4, —4). Note that only one point on the graph can have the same *pair* of coordinates. Points D and E have the same abscissa, $X = -4$, but the ordinates differ; for point E, $Y = -3$, and for point D, $Y = -4$.

c. GRAPH OF EXPERIMENTAL DATA. Connect a number of 2-volt cells, an ammeter A, and a 2-ohm resistor R in series (B of fig. 200). Vary the voltage E by changing the number of cells and tabulate the circuit conditions as the voltage is changed (C of fig. 200). Use the X axis for voltage, E, values, and use the Y axis for current, I,



	AXIS	
	X	Y
CELLS	E	I
1	2	1
2	4	2
3	6	3
4	8	4
5	10	5

C

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Figure 200. Graph of experimental data.

values. Locate the graph points for each pair of coordinates in the table and complete the graph by drawing a line through successive points (A of fig. 200). Since only positive values of X and Y appear in the table, only the first quadrant is required for the curve. Note that the *curve* is a straight line, because in this experiment equal changes of E produced correspondingly equal changes of I.

2. Graphs of Equations

In electrical work many of the graphs are *graphs of equations.* For example, B of figure 198 and A of figure 200 are graphs of the Ohm's Law equation $I = E/R$, when R is held constant.

a. USEFUL TERMS. Before considering graphs of equations, it is necessary to understand the following terms:

(1) *Variable and constant.* In a given problem a *variable* is a number symbol which may take *different* values during the discussion. A constant is a number symbol whose value does not change.

Example: The area of a circle is given by the equation $A = \pi r^2$. In this equation A is the area, r is the radius, and π is a constant. In considering different circles (fig. 201), π is a *constant,* always 3.14 (approx), but A and r are *variables* which take different values.

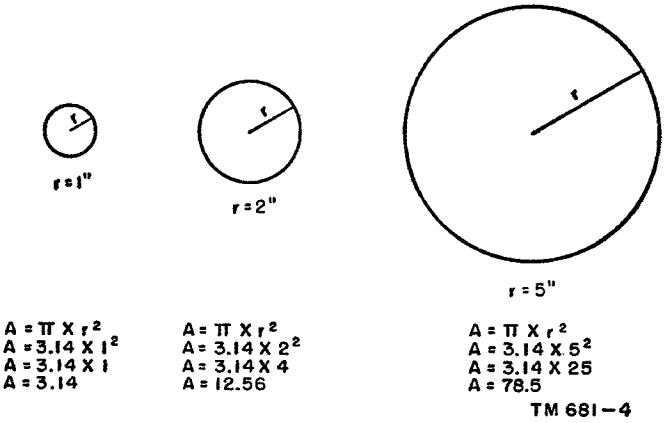

Figure 201. Area of a circle, π as a constant, A and Y as variables.

(2) *Function.* One variable, y, is a *function* of another variable, x, if a corresponding value of y can be determined whenever a value is given to x.

Example: Because of the relation $A = \pi r^2$, the area of a circle is said to be a *function* of the radius r. If a value is given to r (fig. 201), the corresponding area A can be determined. Conversely, if a value is given to A, the corresponding radius r can be determined; r is then said to be a function of A.

b. DEPENDENT AND INDEPENDENT VARIABLES. In the equation $A = \pi r^2$, if values are *assigned* to r, r is said to be the *independent* variable. As values are assigned to r, the value of A will *depend* on the value given to r; therefore, A is called the *dependent* variable. Of course, if values are assigned at will to A, it becomes the independent

variable and r becomes the dependent variable. In plotting graphs, points along the X axis are used to represent the independent variable; the varying height of the curve with respect to the X axis represents values of the dependent variable, y, for each corresponding value of x.

c. EQUATIONS. An equation or equality is a statement that two algebraic statements are equal. There are two kinds of equations: identical and conditional.

(1) *Identical equations.* If the two members of an equation are equal for *all* values of the symbols, the equation is called an identity.

Example: $(a + b)(a - b) = a^2 - b^2$.

When a equals 4 and b equals 3:

$(4 + 3)(4 - 3) = 16 - 9 = 7$.

When a equals 5 and b equals 2:

$(5 + 2)(5 - 2) = 25 - 4 = 21$.

(2) *Conditional equations.* If the two members of an equation are equal for *certain particular* values of the symbols involved, *but not for all values,* the equation is a conditional equation or, simply, an equation.

Example: $X - 3 = 2$ is an equality *only if X equals 5*

and $Y + 3 = 8$ is an equality *only if Y equals 5.*

d. LINEAR EQUATIONS IN TWO UNKNOWNS, X AND Y. An equation in the form:

$$Ax + by + C = 0$$

where A, B, and C are constants is called the *general* equation of the first degree in X and Y. *An equation of the first degree contains no terms with an exponent other than 1.* Thus, $E = IR$ is an equation in the first degree, but $P = I^2R$ is an equation of the second degree in I.

Example: Form: $Ax + by + C = 0$.

Equation: $3x + 4y + 12 = 0$.

In this equation, A equals 3, B equals 4, and C equals 12. In determining the form of an equation, it must be remembered that A, B, and C may equal 0. Thus, the equation $2x + 8y = 0$ is in the form $Ax + \text{by} + \text{C} = 0$.

Form: $Ax + by + C = 0$.

Equation: $2x + 8y + 0 = 0$.

e. GRAPHS OF LINEAR EQUATIONS. The relation between two unknowns, x and y, may be represented by means of an equation or by means of a graph. In analytic geometry it is proved that the graph of any linear equation in x and y is a straight line. Therefore only two points are needed to determine the graph of a linear equation in x and y. In practice, the two points should not be chosen too close together. The *intercepts* on the coordinate axes (i.e., the points at which the line crosses the axes) are usually convenient points. To determine the intercepts let x equal 0 and find y; then let y equal 0 and find x.

Example: Plot the graph of $3x + 4y + 12 = 0$.

First let x equal 0 and solve for y:

$$4y = -12$$
$$y = -3.$$

Then let y equal 0 and solve for x:

$$3x = -12$$
$$x = -4.$$

It is well to establish arbitrarily a third pair of coordinates as a check. Thus, letting x equal 6 and solving for y:

$$3x + 4y + 12 = 0$$
$$(3 \times 6) + 4y + 12 = 0$$
$$18 + 4y + 12 = 0$$
$$4y = -30$$
$$y = -7.5.$$

We now have three pairs of coordinates:

X	0	−4	6
Y	−3	0	−7.5

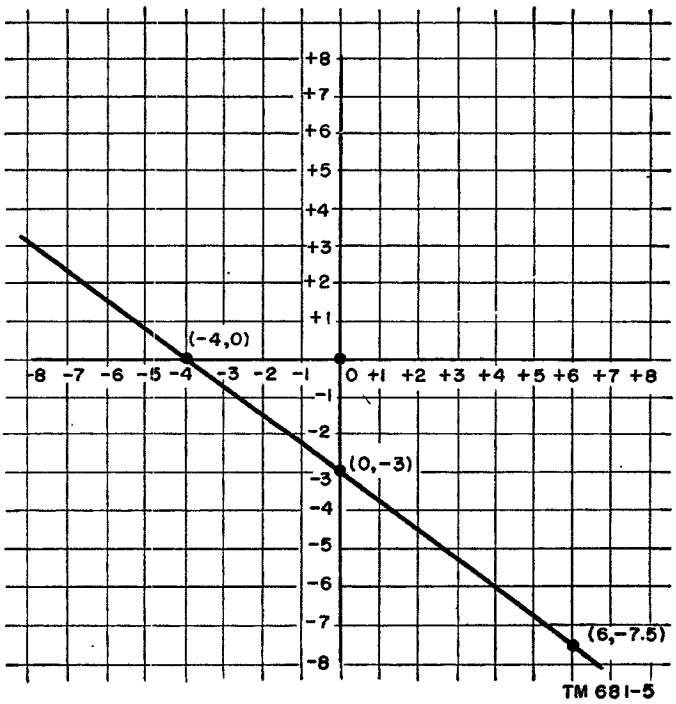

Figure 202. Graph of $3x + 4y + 12 - 0$.

Plotting these on graph paper we obtain a graph (fig. 202) which shows pictorially the relation between x and y in the equation $3x + 4y + 12 = 0$.

f. NONLINEAR EQUATIONS.

(1) *Equation in the second degree.* As previously mentioned, the power equation $P = I^2R$ is an equation of the second degree in I. Let us first assume a resistance of 2 ohms and plot a graph to show the relation between I and P as I is varied. Then reduce R to ½ ohm and plot a second graph. To plot these graphs, assume values of I and calculate for P. Since I is varied *at will*, it is the independent variable. Plot the independent variable along the abscissa (X) and plot the dependent variable along the ordinates (Y). Following this procedure, we obtain the curves of figure 203. *These curves are not straight lines;* portions to the left of the Y axis indicate that the *direction* of current is opposite to that for portions to the right of the Y axis. Note that the *power* is positive, regardless of the direction of current I through the resistance R. (It is beyond the scope of this text to discuss the many varieties of nonlinear equations. Students should learn to recognize linear equations.)

(2) *Equation with fraction.* At first glance it might seem that the equation $I = E/R$ is a linear equation in both E and R. However, let us first hold R constant, vary E, and plot a graph to show the relation between E and I. The resultant curve (C of fig. 200), plotted by assuming R equal to 2 ohms and calculating values of I for assigned values of E, is a straight line. This indicates that the equation is linear in E. Let us now hold E constant and vary R. The resultant curve (fig. 204), plotted by assuming E equal to 10 volts and calculating values of I for assigned values of R, is not a straight line. This shows that the equation $I = E/R$ is not linear in R. In algebra the denominator of a fraction may be written in the numerator, if the sign of its exponent is changed:

$$\frac{1}{a} = \frac{1}{a^1} = a^{-1}.$$

On writing $I = E/R$ as $I = ER^{-1}$, we see that the exponent of R is *not* 1 and, therefore, $I = E/R$ is not a linear equation in R. However, if the equation is written $I = E \times 1/R$, and the value of $1/R$ is varied, the equation is linear in $1/R$ and its graph is a straight line (fig. 205). For ease of plotting, assume fractional values

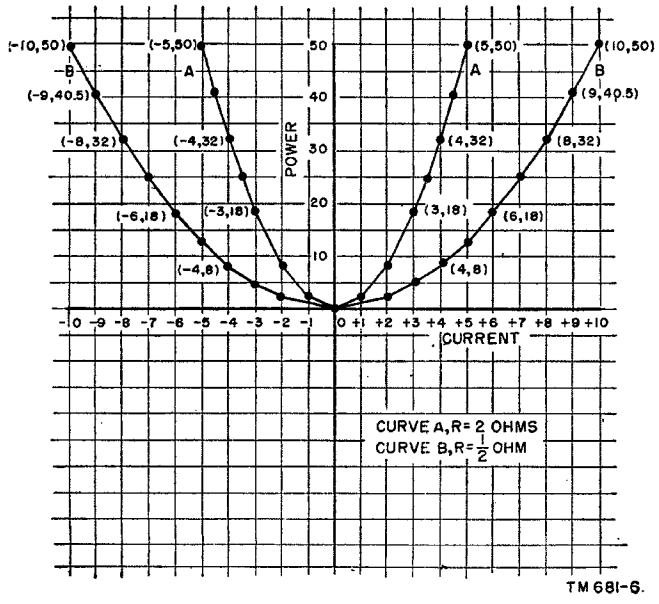

Figure 203. Graph of $P = I^2R$.

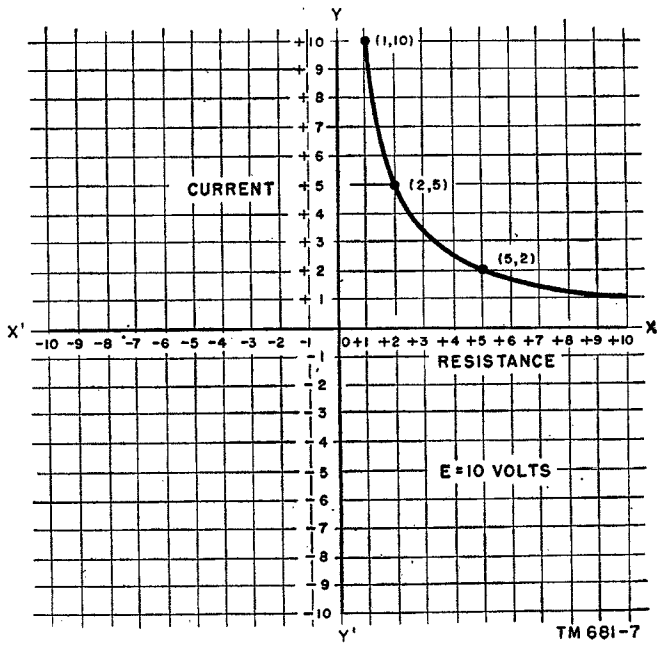

Figure 204. Graph of $I = E/R$ as R is varied.

TABLE I

	x	y
R	$\frac{1}{R}$	I
$\frac{1}{2}$	2	20
$\frac{1}{4}$	4	40
$\frac{1}{6}$	6	60
$\frac{1}{8}$	8	80
$\frac{1}{10}$	10	100
$\frac{1}{16}$	16	160

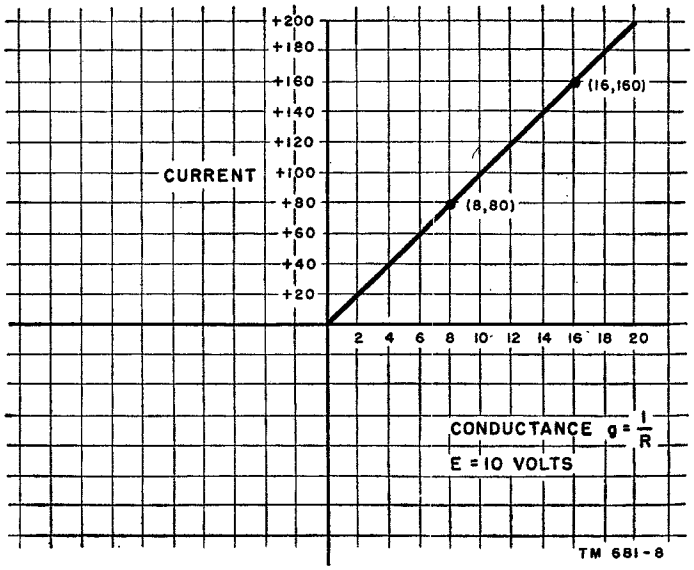

Figure 205. Graph of $I = E \times \frac{1}{R}$ as $\frac{1}{R}$ is varied.

of R, so that 1/R will be a whole number. With E equal to 10 volts, calculate and tabulate values of $1/R$ and corresponding values of I. Use values of $1/R$ as abscissas (X-axis values) and values of I as ordinates (Y-axis values). Compare figures 204 and 205. Note that in figure 204 values of R are assigned to the X axis. Obviously, R and $1/R$ *are not the same thing;* R is *resistance* and $1/R$ is *conductance* (symbol G). Thus, the graph in figure 204 shows that the relation between current and resistance is not linear; the graph in figure 205 shows that the relation between current and conductance is linear.

3. Summary

a. A graph is a pictorial presentation of the relations between two or more quantities.

b. Only one point on a graph can have the same pair of coordinates.

c. The X value or abscissa is always written first: (X, Y).

d. Experimental data may be plotted on a graph.

e. The relationship between quantities may be expressed as an equation or by a graph of the equation.

f. A variable is a number symbol which may take different values.

g. A constant is a number symbol whose value does not change.

h. An equation of the first degree contains no terms with an exponent other than +1.

i. The graph of a linear equation in x and y is a straight line.

Section II. ANGLES

4. Basic Ideas

In elementary geometry an angle is thought of as a figure consisting of two lines radiating from a common point O (A of fig. 206). The common point O is called the vertex and the lines OA and OB are the sides of the angle. The angle may be referred to as the angle O, using the vertex letter; or it may be referred to as the angle θ, using any convenient symbol. The angle may also be called angle AOB, using the end letter of one side, the vertex, and the end letter of the other side. The symbol /_____ is used to represent the word angle; thus, /_____ AOB.

a. TYPES OF ANGLES.

(1) *Right angles.* If a perpendicular is drawn to any line CD, two right angles ABD and ABC are formed (B of fig. 206).

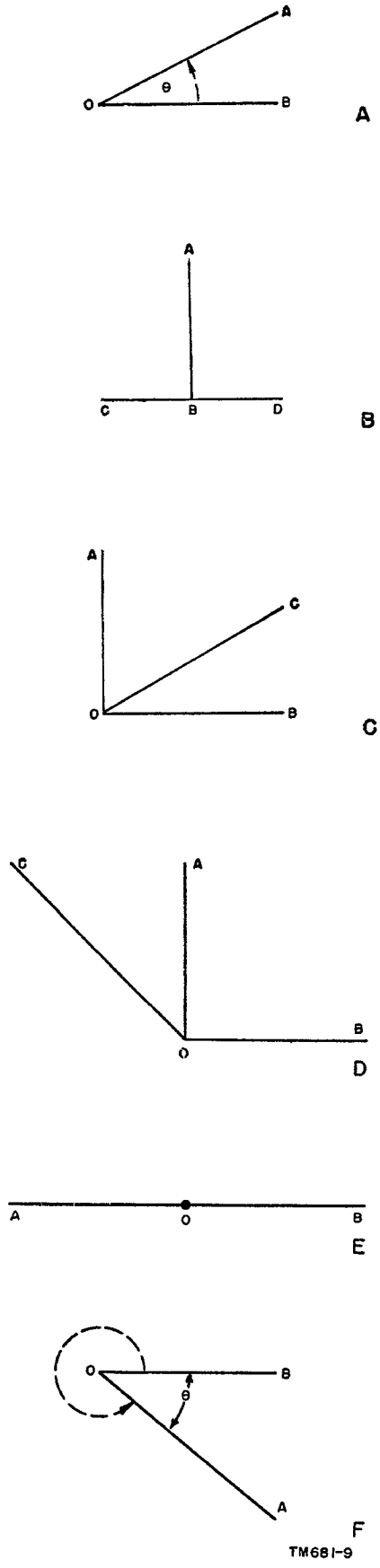

Figure 206. Types of angles.

(2) *Acute angle.* An angle smaller than a right angle is called an acute angle. In C of figure 206, angle *AOB* is a right angle; angles *AOC* and *COB* are acute angles.

(3) *Obtuse angle.* An angle which is greater than a right angle, but less than two right angles (a straight line) is called an obtuse angle. In D of figure 206 angle *AOB* is a right angle and angle *COB* is an obtuse angle.

(4) *Straight angle.* A straight line forms an angle equal to the sum of two right angles. Compare straight angle *AOB* of E of figure 206 with angle *CBD* in B of figure 206.

(5) *Reflex angle.* An angle greater than two right angles, but less than four right angles, is called a reflex angle. In F of figure 206, angle θ is an acute angle, but angle *BOA* (indicated by arrow) is a reflex angle.

(6) *Adjacent angles.* Angles which have a common side and a common vertex, and which are on opposite sides of the common side are called adjacent angles. The right angles in B of figure 206 are adjacent angles. (When one straight line meets another straight line to form two *equal* adjacent angles, each of the angles is a right angle, and the lines are mutually perpendicular.)

(7) *Complementary angles.* When the sum of two angles is equal to a right angle, the angles are complementary angles. In C of figure 206, angles *AOC* and *COB* are complementary angles.

(8) *Supplementary angles.* When the sum of two angles is equal to two right angles, the angles are called supplementary angles. Thus, in B of figure 206, angles *ABC* and *ABD* are supplementary angles.

b. MEASUREMENT OF ANGLES.

(1) *Degrees.* The *degree* is the unit of angular measurement in the sexagesimal system; it is 1/360th part of a circle. The *minute* and the *second* are subdivisions of the degree; there are sixty minutes in 1 degree, and 60 seconds in 1 minute.

(2) *Radian.* The *radian* is the unit of the *circular* system for measuring angles. A radian is the angle formed by rotating

the radius of a circle until the arc described or traced out by the end of the radius is equal in length to the radius. Thus, in figure 210, the arc BC is equal in length to the radius AB and, therefore, angle CAB is one radian (approximately 57.3°).

5. General Concept of an Angle

The concept of an angle as a figure is satisfactory for a study of elementary geometry. The study of a-c theory requires a more general concept of an angle.

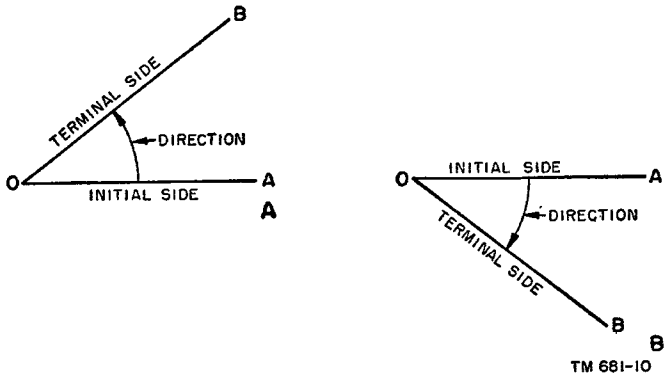

Figure 207. Positive and negative angles.

a. GENERATION OF AN ANGLE. Suppose that a line OA, radiating from point O (A of fig. 207), is rotated counterclockwise from its original or *initial* position OA to a final or *terminal* position, OB. This rotation *generates* an angle AOB with an initial side OA and a terminal side OB. The value of angle AOB is the amount *of rotation* used in generating the angle.

b. SIGN OF ANGLE. By agreement, *an angle generated by counterclockwise rotation of the initial side is considered as a positive angle; clockwise rotation produces a negative angle.* An angle is not completely described unless the *direction and amount* of rotation are given (fig. 207).

c. MEASURE OF ROTATION. As a measure of rotation, 1° is 1/360th of a revolution. Thus, if line OA is rotated through one revolution, the terminal side rests on the initial side and the angle is 360°. Consider figure 208. In *A*, side OA has been rotated 45° *counterclockwise* from its original position to form the *positive* 45° angle AOB. In B, side OA has been rotated 360° (one complete revolution) *plus* an additional 45° to form a positive 405° angle. In C, side OA has been rotated *clockwise* 45° *less* than 360°; therefore, the amount of rotation is 360° — 45° = 315°. Since the rotation is *clockwise*, angle AOB is a *negative* angle of —315°. In D, side OB is rotated 45° clockwise from its initial position; therefore, angle BOA is a negative angle of —45°. Note that any number of complete revolutions may be made. An angle generated by rotating the initial side through three complete revolutions plus 60° is an angle of (3 x 360°) + 60° = 1,140°.

6. Standard Position

An angle is said to be in standard position on a rectangular system (par. 1*b*), if the vertex is at the origin and the initial side lies along the *positive part of the horizontal* (X) *axis*. In D of figure 206, angles AOB and COB are in standard position; angle COA is not in standard position.

a. QUADRANT. An angle is in a certain quadrant if its terminal side is *inside* the quadrant when the angle is in standard position. Thus, in D of figure 206, angle COB is in the second quadrant.

b. COTERMINAL. Angles are coterminal if their terminal sides coincide when the angles are placed in standard positions. Figure 208 shows that an angle of 405° and one of 45° are coterminal.

7. Circular Measure

In figure 209, the radius line AB is rotated counterclockwise about point A. When this line reaches position AC it is perpendicular to AB and angle CAB is a 90° or right angle. As line AB continues to rotate counterclockwise, it successively passes through AC and AE and returns to

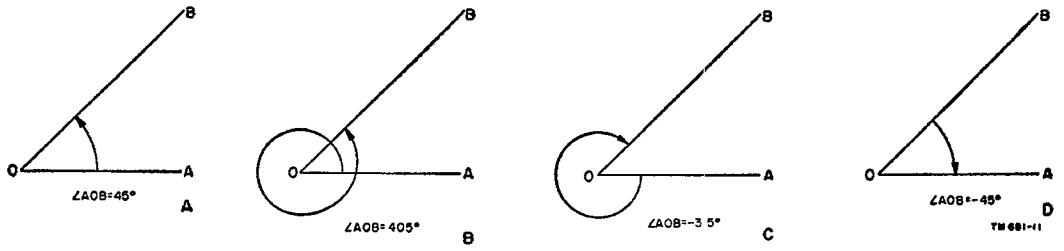

Figure 208. Various angles represented by the same geometric figure.

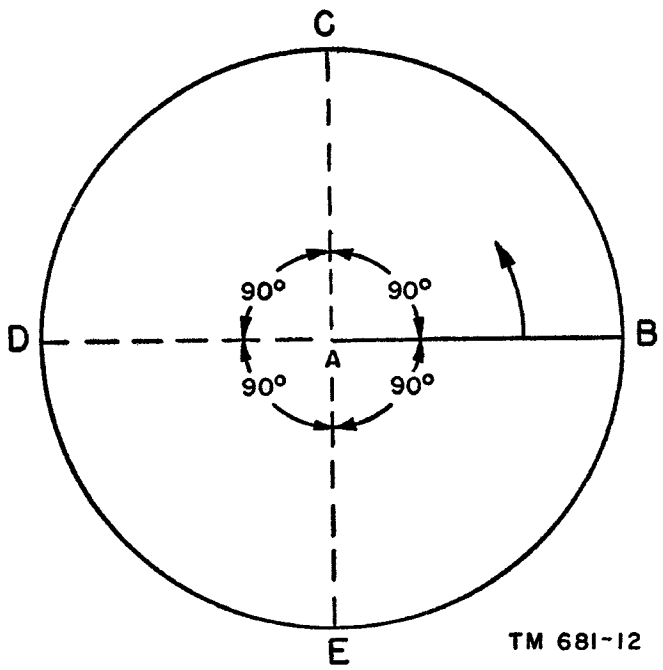

Figure 209. Generating an angle by revolving a line.

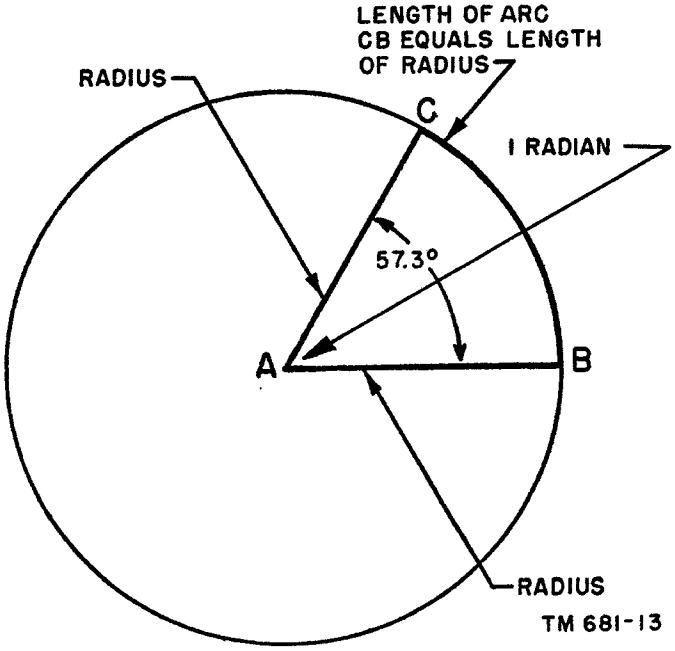

Figure 210. Relation of radius, radian, and arc of a circle.

AB. Thus, in making one revolution, line AB generates an angle equal to four right angles or 360°. In other words, there are 360° in a circle.

a. THE RADIAN. In discussions of a-c theory, the radian is the commonly used unit of angular measure. In figure 210, point A is the center of the circle, AC and AB are radii, and *the length of the arc CB equals the length of the radius.* By definition (par. 4*b*(2)), angle CAB is one radian. Figure 211 shows that there are approximately 6.28 radians in a circle. The Greek letter π represents a constant equal to 3.14 (approximately); therefore, $2\pi = 6.28$. Thus, one revolution of side AB generates an angle of 2π radians.

b. CONVERSION FORMULAS. As explained above, if AB makes a complete revolution it generates an angle of 360°, or 2π radians; therefore,

$$2\pi \text{ radians} = 360°$$

and, $$1 \text{ radian} = \frac{360°}{2\pi} = \frac{180°}{\pi} = 57.3° \text{ (approx)}.$$

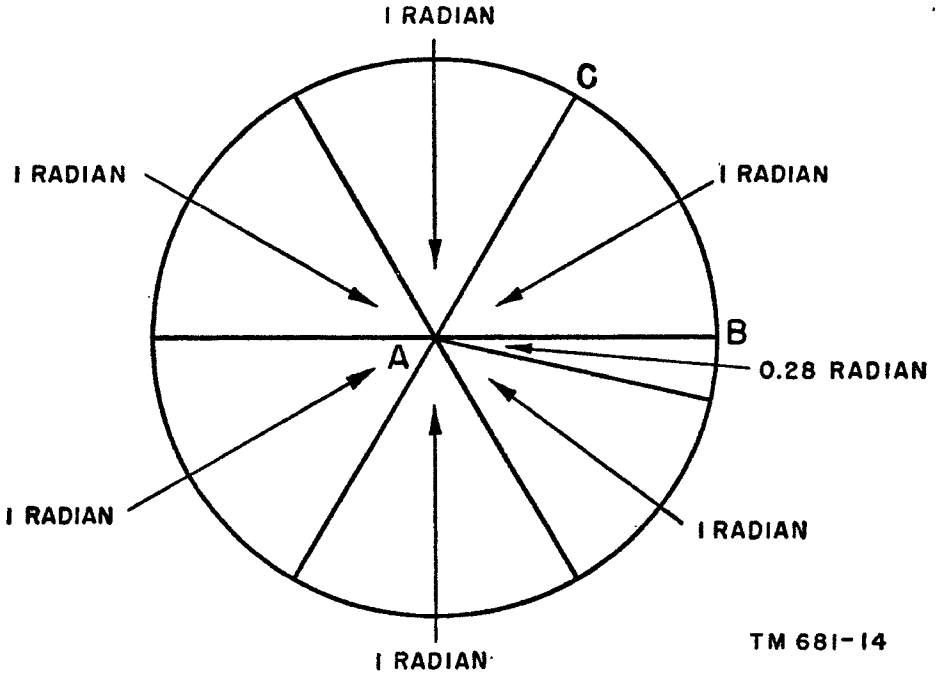

Figure 211. Number of radians in a circle.

The above formula is used to convert radians to degrees, and vice versa. To convert degrees to radians, multiply the number of degrees by $\pi/180$; to convert radians to degrees, multiply the number of radians by $\frac{180.}{\pi}$

c. SUMMARY.

(1) Angles can be measured in degrees or in radians.

(2) There are 90° in a right angle.

(3) There are four right angles or 360° in a circle.

(4) 360° equals 2π radians.

(5) It is useful to memorize the following relations:

$90° = \pi/2$ radians
$180° = \pi$ radians
$270° = 3\pi/2$ radians
$360° = 2\pi$ radians.

8. Summary

a. An angle is generated by rotating a line from its original or *initial* position to a final or terminal position.

b. An angle generated by counterclockwise rotation of the initial side is a positive angle.

c. An angle generated by clockwise rotation of its initial side is a negative angle.

d. An angle is described when the *direction and amount* of rotation are given.

e. One degree is 1/360th part of a revolution.

f. An angle is in a certain quadrant if its *terminal* side is inside the quadrant.

g. One radian equals 573° (approx).

Section III. SIMPLE TRIGONOMETRY

9. Trigonometry and A-C Theory

Trigonometry, as the term implies, is a method of measuring triangles by numerical computation. With the development of alternating-current generators, it became impossible to analyze or understand the behavior of even the simplest circuit, without considering angles and their computation.

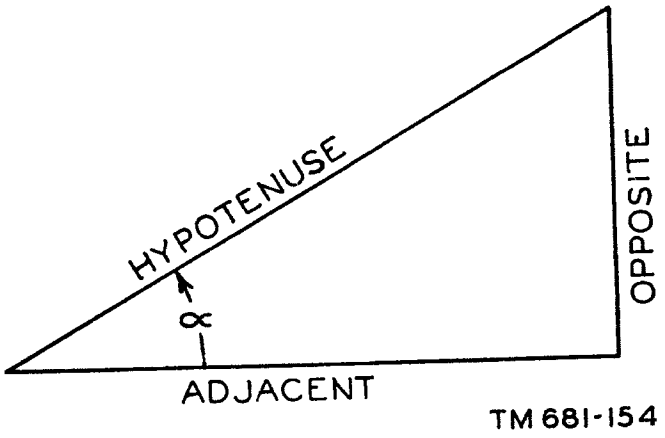

Figure 212. Property of right triangles.

Thus, in other sections of this manual, students study such a-c concepts as phase angle, phase difference, angles of lead and lag, and so on. A better understanding of this a-c theory is possible if the student is familiar with elements of plane trigonometry given in this appendix.

10. Property of Right Triangles

In any right triangle (fig. 212) the side opposite the right angle is called the *hypotenuse;* the side opposite the acute angle being considered ($\propto$ in figure 212) is called the *opposite* side; the side adjacent to the acute angle is called the *adjacent* side.

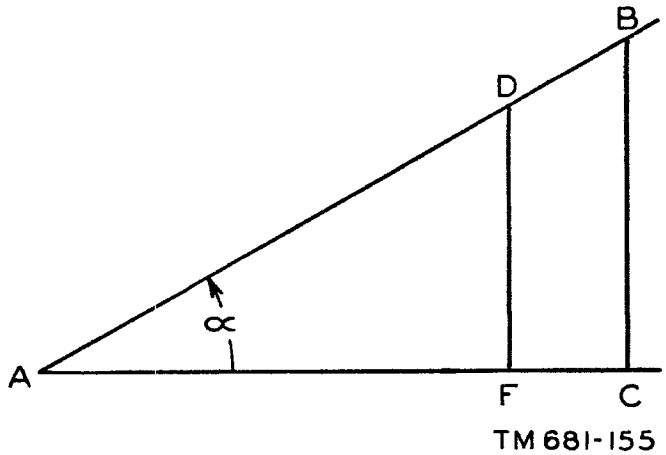

Figure 213. Ratios of sides of right triangles.

a. In figure 213, let angle CAB represent any acute angle $\propto$. If DF and BC are drawn perpendicular to side AC, right triangles ABC and ADF are similar. In geometry it is proved that the ratio of side AB to side AC equals the ratio of side AD to side AF; that is:

$$\frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{AB}{AC} = \frac{AD}{AF}.$$

b. This shows that the difference in size between the right triangles has no effect on the *ratio* of the hypotenuse to the adjacent side; *this ratio has the same value for all right triangles with a given acute angle* $\propto$ *as the adjacent angle.* By extending the same reasoning to any two sides of a right triangle, we derive the following law: THE RATIO OF ANY TWO SIDES OF A RIGHT TRIANGLE, WITH $\propto$ AS ONE ACUTE ANGLE, DEPENDS ONLY ON THE SIZE OF THE ANGLE $\propto$ AND NOT ON THE SIZE OF THE TRIANGLE.

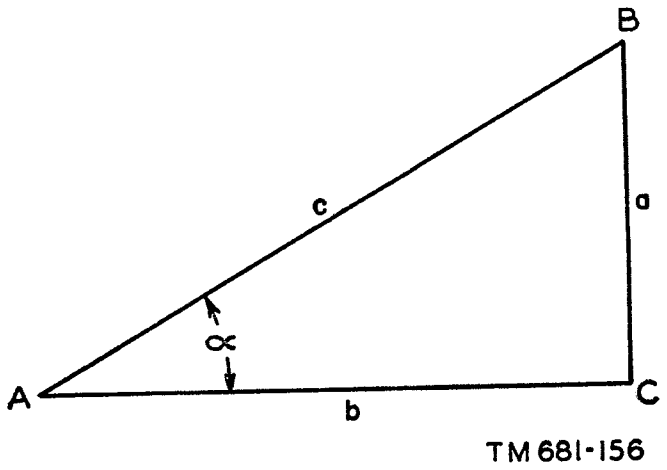

Figure 214. Right triangle, functions.

11. Trigonometric Functions

The above law states that the ratio of any pair of sides of a right triangle is a *function* of the acute angle $\propto$. The six possible ratios are called the *trigonometric functions* of the acute angle $\propto$.

a. NAMES OF FUNCTIONS. In figure 214, $\propto$ represents any acute angle and BC is a line drawn from point B perpendicular to C to form a right triangle ABC. The small letters, a, b, and c, represent the *lengths* of the sides respectively opposite the vertices A, B, and C. Referring to this illustration, we can now list the names and abbreviations given to the six ratios or trigonometric functions:

Ratio	*Name*	*Abbreviation*
a/b	tangent of $\propto$	tan $\propto$
b/a	cotangent of $\propto$	cot $\propto$
a/c	sine of $\propto$	sin $\propto$
b/c	cosine of $\propto$	cos $\propto$
c/b	secant of $\propto$	sec $\propto$
c/a	cosecant of $\propto$	csc $\propto$

b. FUNCTIONS USED IN THIS MANUAL. Only three of the six trigonometric functions are used for solving a-c problems in this manual—namely, the since, cosine, and tangent. In the right triangle of figure 214, *a* is the leg *opposite* to $\propto$, *b* is the leg *adjacent* to $\propto$, and *c* is the *hypotenuse*, then,

$$\sin \propto = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \propto = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \propto = \frac{a}{b} = \frac{\text{opposite.}}{\text{adjacent}}$$

Example: $\propto$ and β are the acute angles in the right triangle of figure 215; then:

$\sin \propto = 3/5 = 0.6$ $\quad\quad$ $\sin \beta = 4/5 = 0.8$

$\cos \propto = 4/5 = 0.8$ $\quad\quad$ $\cos \beta = 3/5 = 0.6$

$\tan \propto = 3/4 = 0.75$ $\quad\quad$ $\tan \beta = 4/3 = 1.33$

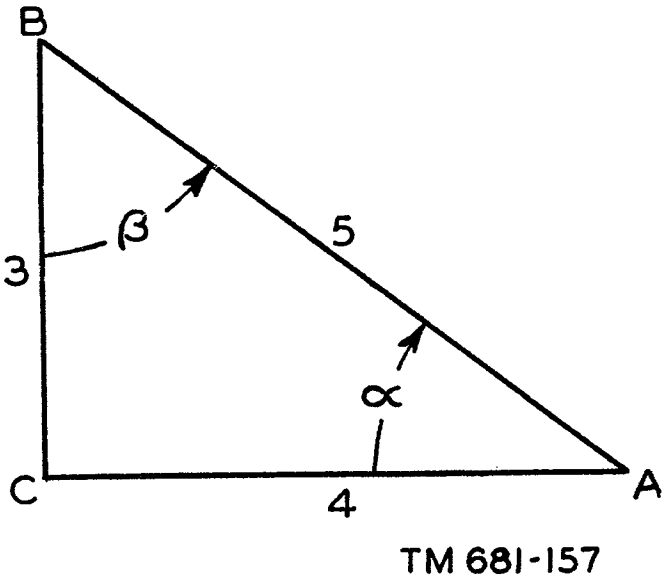

Figure 215. Acute angles.

12. Solutions by Pythagorean Theorem

The Pythagorean theorem states that in a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides. Referring to figure 216,

$$c^2 = a^2 + b^2.$$

Example 1: In figure 216, $c = 6$ and $b = 5$. Find *a*.

Solution: $c^2 = a^2 + b^2$

$6^2 = a^2 + 5^2$

$a^2 = 36 - 25 = 11$

$a = \sqrt{11} = 3.317.$

Example 2: Find the trigonometric functions of the acute angle $\propto$ in figure 216.

Solution: From figure 216, read:

$$\sin \propto = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{11}}{6}$$

$$\cos \propto = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{6}$$

$$\tan \propto = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{11}}{5}.$$

Example 3: If $\sin \propto = 3/4$, find the cosine and tangent of $\propto$.

Solution: Refer to the equations of paragraph 11. Since $\sin \propto = 3/4 = a/c$, construct a right triangle with $a = 3$ and $c = 4$. (See figure 217.)

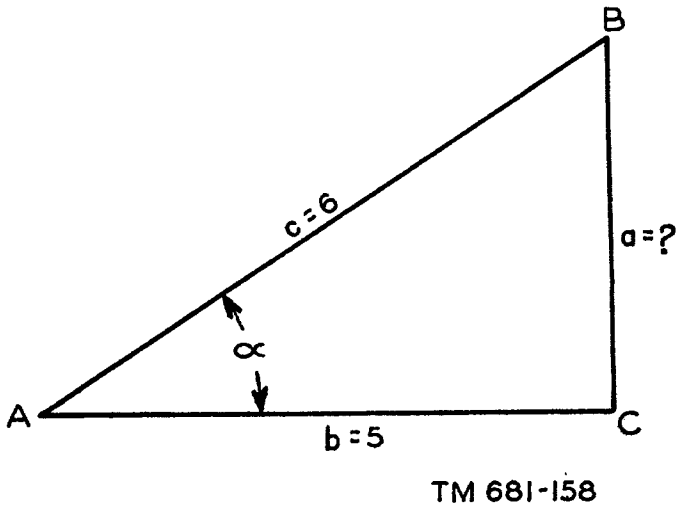

Figure 216. Trigonometric functions of acute angle.

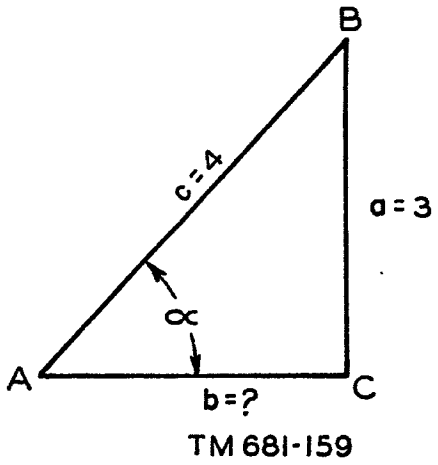

Figure 217. Acute angle, trigonometric functions.

From the Pythagorean theorem,

$$c^2 = a^2 + b^2$$

and,

$$b^2 = c^2 - a^2$$

$$b^2 = 16 - 9 = 7$$

$$b = \sqrt{7} = 2.64 \text{ (approx)}.$$

From figure 217 read:

$$\cos \propto = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c} = \frac{\sqrt{7}}{4}$$

$$\tan \propto = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

Note. Example 3 shows that if the value of one function of an unknown acute angle is given, the angle can be constructed and all of its functions obtained.

13. Trigonometric Tables

By use of advanced mathematics, the functions of an angle can be computed to any number of decimal places. In such computations, the error is no greater than 1/2 of a unit in the last place. This accuracy is considerably better than can be obtained by graphical methods (drawing angles and their sides to scale and measuring lengths). A table of natural trigonometric functions for decimal fractions of a degree is given in appendix II. This table includes angles from 0° to 90° in steps of 1/10 of 1° with not less than four-decimal-place accuracy for sines and cosines. The instructions and examples are given in appendix II.

14. Trigonometric Equations

Partly because Greek letters are used as symbols and partly because new terms are introduced, students sometimes feel that trigonometry is a difficult subject. This is not true. Trigonometric functions are merely *ratios*, the quotient obtained by dividing the length of one side by the length of another side of a right triangle. A direct comparison shows that the function equations are as simple as those expressing Ohm's Law:

Ohm's Law	*Trigonometric Function*
$I = E/R$	$\sin \theta = a/c$
$E = I \times R$	$a = c \times \sin \theta$
$R = E/I$	$c = a/\sin \theta$

The Ohm's Law equation $I = E/R$ indicates that, if E and R are known, I can be found; the sine function equation, $\sin \theta = a/c$, indicates that, if the length of the hypotenuse, c, and the length of the opposite side, a, are known, the sine can be found. Similarly, $a = c \times \sin \theta$ shows that, if the hypotenuse, c, and the sine of θ are known, the opposite side, a, can be determined; the equation $c = a/\sin \theta$ shows that, if the opposite side, a, and the sine of θ are known, the hypotenuse, c, can be determined. In other words, these equations for trigonometric functions are as simple and as readily solved as the familiar Ohm's Law equations.

Example 1: In triangle ABC (fig. 214), find the unknown side by use of the Phythagorean theorem and obtain all the functions of $\propto$ when $b = 5$ and $a = 12$.

Solution: $c^2 = a^2 + b^2$

$c^2 = 25 + 144$

$c^2 = 169$

$c = 13.$

Since the three sides are now known, the functions may be read directly: sin $\propto$ $= a/c = 12/32$; cos $\propto$ $= b/c = 5/13$; tan $\propto$ $= a/b = 12/5$.

Example 2: Cos $\propto$ $= 15/17$; construct the angle and find its other functions.

Solution: Referring to figure 214,

Cos $\propto$ $= b/c = 15/17$

$a^2 = c^2 - b^2$

$a^2 = 17^2 - 15^2 = 64$

$a = \sqrt{64} = 8.$

Then, sin $\propto$ $= a/c = 8/17$; cos $\propto$ $= b/c = 15/17$; tan $\propto$ $= a/b = 8/15$. The angle $\propto$ can be found from any one of its functions in the table of Natural Trigonometric functions in appendix II.

Sin $\propto$ $= 8/17 = 0.4705$ (approx)

Sin^{-1} $0.4705 = 28°$.

Note. Sin^{-1} $0.4705 = 28°$ may also be written as arcsin $0.4705 = 28°$. Both are read: The angle whose sine is 0.4705 is 28°. Consider the statement: The tangent of 37.2° is 0.7590. Stated inversely, this is written,

$37.2° = \tan^{-1} 0.7590$

or, $37.2° = \arctan 0.7590.$

Both are read: 37.2° is the angle whose tangent is 0.7590.

Example 3: A painter wishes to reach a window 40 feet above the ground. Find the length of the shortest ladder he can use, if it must not incline more than 78° from the ground.

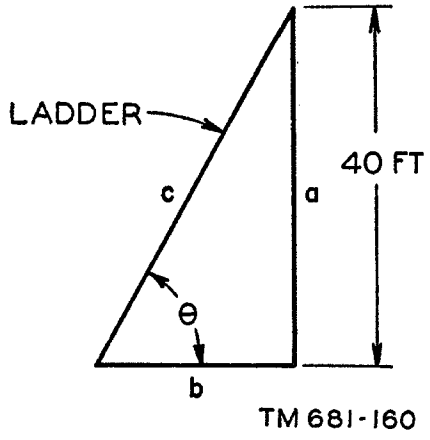

Figure 218. Ladder problem.

Solution: Draw a rough sketch as shown in figure 218. From the sketch we see that we are given the length of the opposite side, $a = 40$ feet; and the angle $\theta = 78°$. The length of the ladder must be the length of the hypotenuse c. Using the trigonometric function containing both c and a, we get,

$\sin \theta = a/c$

and, $c = a/\sin \theta.$

From the tables we find that sin 78° = 0.9781; therefore, $c = 40/0.9781 = 40.88$ feet.

15. Trigonometry and A-C Voltages

Figure 219 is a graph of the amplitude variations of a certain type of a-c voltage (or current) with time. Time is plotted along the x-axis and amplitude variations are indicated by the varying height of the curve relative to the x-axis. Actually, as is explained in this section, figure 219 is essentially a graph of the equation

$a = c \sin \theta$

when c is held constant and θ is varied. Voltages (or currents) which vary, as shown in figure 219, are called sinusoidal voltages (or currents). The generation of sinusoidal voltages (or currents) is discussed in chapter 1. These voltages and currents form a basis for all a-c theory; therefore, an understanding of the trigonometry involved is of considerable importance.

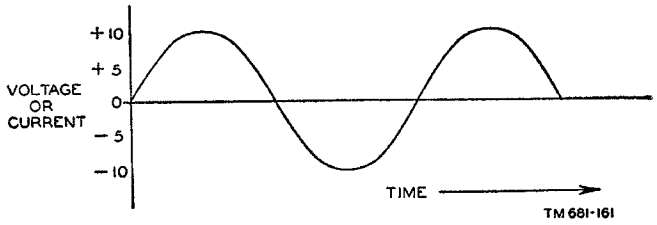

Figure 219. Sine wave.

16. Trigonometric Functions are Periodic

Let θ be any varying angle (A of fig. 220). This angle θ varies from one value to another by rotation of its terminal side OP. Thus, if side OP makes three complete revolutions counterclockwise from its initial position OX, it generates an angle of 1,080°, or 3 x 2π radians. *The positions of the terminal side OP repeat themselves at intervals of 360°, or 2π radians; consequently, the trigonometric functions also repeat themselves.*

Example: Consider figure 220. In A the side OP has rotated counterclockwise from its initial position OX to generate angle θ of 45°. From paragraph 9 we get

$\sin 45° = a/c.$

Now consider that OP is rotated an additional 360° (fig. 220–B) to generate an angle of (θ + 360°), or

$\propto = (\theta + 360°) = 45° + 360° = 405°.$

We see that OP′ coincides with OP; that is, the 45° angle θ and the 405° angle $\propto$ are coterminal. Because of this, a and c have the same numerical values for both angles, thus:

$$\sin \propto = \sin (405° - 360°) = \sin 45° = a/c.$$

This shows that the trigonometric functions repeat and we say that *any function of θ is periodic, with 360°, or 2π radians, as the period.*

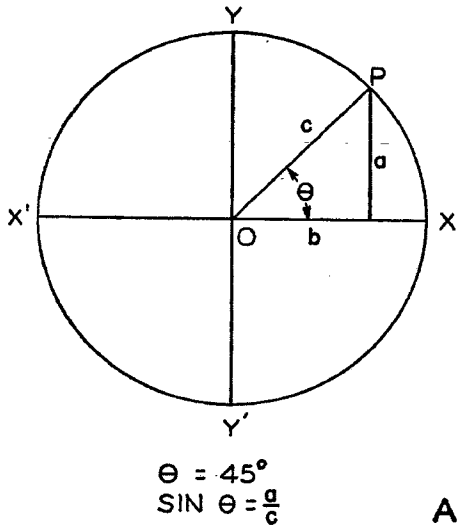

Figure 220. Periodic trigonometric functions: A, 45°, B, 405°.

17. Variation of Trigonometric Functions

a. SIN θ. In triangle AOP (fig. 221), the 45° angle θ is in standard position; OP is the hypotenuse c and equal to the radius r; AP is the opposite side a; and OA is the adjacent side b. If the hypotenuse c is taken as a unit length, $OP = r = c = 1$, then,

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} = \frac{a}{1} = a.$$

The dashed line from P to the Y-axis indicates that the opposite side a is equal to y (the ordinate of point P). Similarly, the adjacent side b of triangle AOP is equal to x (the abscissa of point P); then, in terms of the coordinates of a point P on side c of angle θ,

$$\sin \theta = a/c = y/1 = y \text{ or } y = \sin \theta.$$

In other words, *when the hypotenuse c is taken as a unit length, the length of a is numerically equal to the sine of θ.*

(1) *Minimum value of sin θ.* In figure 222 the angle XOP′ is a small angle with c′, a′, and b′, respectively, as the hypotenuse, opposite side, and adjacent side. If angle XOP′ is made smaller and smaller,

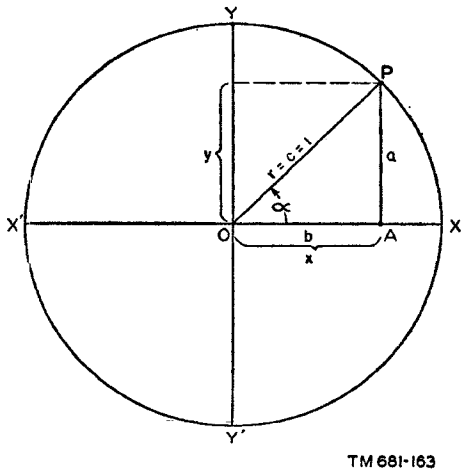

Figure 221. Triangle, variation of trigonometric functions.

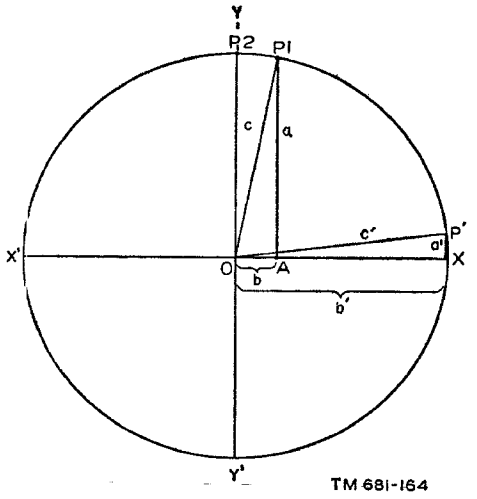

Figure 222. Minimum value of sine θ.

the opposite side a′ must also become smaller and smaller. For an angle of 0°, OP′, and OX coincide and a′ (or y) vanishes. In other words a′ and, therefore, sin θ are reduced to 0 as angle XOP′ is reduced to 0.

(2) *Maximum value of sin θ.* As angle XOP′ (fig. 222) is increased to angle XOP_1 (angle AQP_1) the length of a increases. If angle XOP_1 is increased to 90°, P_1 moves to P_2, so that sides a and c coincide on the y-axis. This means that $a = c$ and, therefore,

$$\sin \theta = a/c = 1/1 = 1.$$

We can now state that *the sine of θ varies from 0 to 1 as the angle θ varies from 0° to 90°.*

b. Cos θ. Again referring to figure 221 and again considering the hypotenuse c as a unit length, we see that

$$\cos \theta = b/c = b/1 = b.$$

Therefore, when c is considered as a unit length, the length of b is numerically equal to the cosine of θ. Also, in terms of the coordinates of point P, b equals x, the abscissa. Then,

$$\cos \theta = x/c = x/1 = x$$
$$x = \cos \theta.$$

(1) *Maximum value of cosine.* As the angle AOP in figure 221 is reduced, the length of the adjacent side b increases. When angle AOP is reduced to zero, c and b coincide and

$$\cos \theta = b/c = 1/1 = 1.$$

(2) *Mimimum value of cosine.* As angle AOP is increased, point A moves toward the origin O and the length of b decreases to 0. Thus, *as the angle θ increases from 0° to 90°, cos θ varies from 1 to 0.*

c. TAN θ. In geometry, a line drawn through a point P on the circumference of a circle and perpendicular to the radius at point P is said to be tangent to the circle. Thus, in A of figure 223, line AB is perpendicular to radius OA and, therefore, it is the tangent line. The radius r is extended to meet the tangent line at P. For angles less than 90°, point P is so chosen that the abscissa ($x = OA$) is a unit length. Then, in terms of the rectangular coordinates (x, y) of point P,

$$\tan \theta = y/x = y/1 = y.$$

(1) *Maximum value of tan θ.* As angle AOP is increased to angle AOP_1 (B of fig. 223), point P moves up from P to P_1. If θ approaches 90° (C of fig. 223), point P

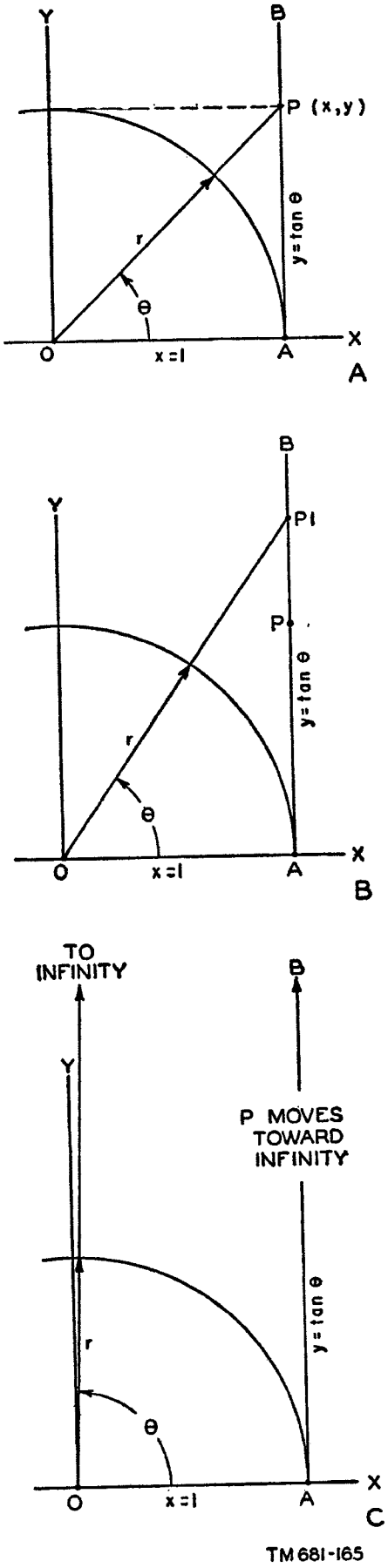

Figure 223. Tangent of θ as θ varies.

moves up beyond all limits on the tangent line AB. This means that the length of AP increases without limit, becoming greater than any specified number however large. It is said that tan θ becomes positively infinite as θ approaches 90°.

moves downward. When θ is reduced to 0, OP coincides with OA and the length AP is reduced to 0; then

$$\tan 0° = 0.$$

We can now state that *tan θ varies from 0 to infinity as θ is increased from 0° to 90°.*

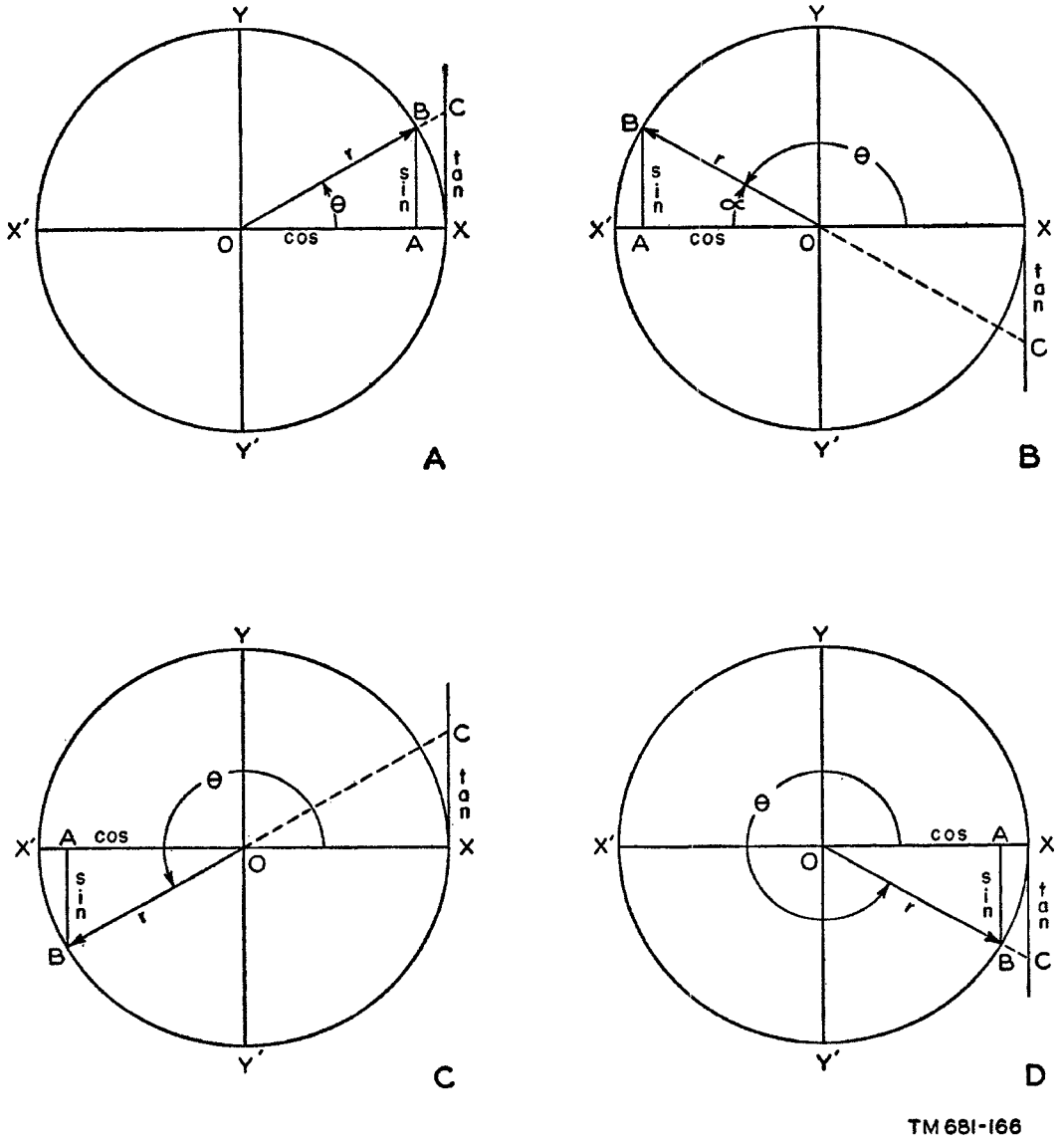

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Figure 224. Sine θ, cos θ, and tan θ in the four quadrants.

Example: Tan 89° = 57; tan 89° 59′ = 3,438; tan 89° 59′ 59″ = 206,265. When θ is 90°, P is on the y-axis and $x = 0$. The tangent cannot be defined when $x = 0$, because y/x is meaningless when $x = 0$,

$$\tan \theta = y/x = y/0 = \infty.$$

That is,

tan 90° is infinite.

(2) *Minimum value of tan θ.* As θ (angle AOP in A of figure 223) is decreased, point P

18. Functions of Any Angle

a. So far, only the trigonometric functions of acute angles have been considered in this text. Limiting ourselves to acute angles would make it impossible to solve or understand even the simpler a-c problems. For example, as is explained in chapter 1, a sinusoidal voltage is developed by continual rotation of an armature. At any instant the generated voltage is a *function of the angle* be-

tween the rotating armature and certain fixed components in the machine.

b. In figure 224, the radius r is called the *radius vector.* (Vectors are explained in section IV of this appendix.) This radius vector r revolves about the origin O to generate an angle θ. When r is a unit length ($r = 1$) A, B, C, D of figure 224 shows that in each quadrant

$$\sin \theta = AB/r = AB/1 = AB$$

and $$\cos \theta = OA/r = OA/1 = OA.$$

(1) It is now apparent that the sine of an angle is numerically equal to the ordinate (y value) of any point where the radius vector coincides with the circumference; that is, when r equals 1,

$$y = \sin \theta.$$

(2) The cosine of an angle is numerically equal to the abscissa (x value) of any point where the radius vector coincides with the circumference of the circle; that is, when r equals 1,

$$x = \cos \theta.$$

(3) The tangent of an angle is numerically equal to the ordinate (y value) of any point at which the *extended* radius vector intersects the tangent line; that is, when x equals 1, y (the length between the x-axis and point C on the tangent line) is numerically equal to the tangent of θ,

$$y = \tan \theta.$$

19. Signs of Functions

In figure 224, the ordinate, y, of point B gives both the sign (+ or —) and the numerical value of the sine in each quadrant; the abscissa, x, of point A gives both the sign and numerical value of the cosine; and the ordinate, y, of point C gives both the sign and numerical value of the tangent.

a. FIRST QUADRANT. In the first quadrant (A of fig. 224), as the angle θ increases from 0° to 90°, we note the following relations:

(1) x is positive and *decreases* from $x = r$ to $x = 0$.

(2) y is positive and *increases* from $y = 0$ to $y = r$. Then

(*a*) Sin $\theta = +y/+r$ and the sine is *positive,* increasing from 0 to 1.

(*b*) Cos $\theta = +x/+r$ and the cosine is *positive,* decreasing from 1 to 0.

(*c*) Tan $\theta = +y/+x$ and the tangent is *positive,* increasing from 0 to $+\infty$.

b. SECOND QUADRANT. In this quadrant, θ increases from 90° to 180°.

(1) x is *negative,* decreasing from $x = 0$ to $x = -r$.

(2) y is *positive,* decreasing from $y = r$ to $y = 0$; therefore,

(*a*) $\sin \theta = +y/+r$ and the sine is *positive,* decreasing from 1 to 0.

(*b*) $\cos \theta = -x/r$ and the cosine is *negative,* decreasing from 0 to —1.

(*c*) $\tan \theta = y/-x$ and the tangent is *negative,* increasing from $-\infty$ to 0.

c. THIRD QUADRANT. In this quadrant (C of fig. 224), θ increases from 180° to 270°.

(1) x is negative and increases from $x = -r$ to $x = 0$.

(2) y is negative and decreases from $y = 0$ to $y = -r$.

(*a*) $\sin \theta = -y/+r$ and the sine is negative, decreasing from 0 to —1.

(*b*) $\cos \theta = -x/r$ and the cosine is negative, increasing from —1 to 0.

(*c*) $\tan \theta = -y/-x$ and the tangent is positive, increasing from 0 to $+\infty$.

d. FOURTH QUADRANT. As θ increases from 270° to 360° we observe the following relations.

(1) x is positive, increasing from $x = 0$ to $x = r$.

(2) y is negative, increasing from $y = -1$ to $y = 0$.

(*a*) $\sin \theta = -y/+r$ and the sine is negative, increasing from —1 to 0.

(*b*) $\cos \theta = +x/+r$ and the cosine is positive, increasing from 0 to 1.

(*c*) $\tan \theta = -y/+x$ and the tangent is negative, increasing from $-\infty$ to 0.

Note 1. The line representing x or y may be getting *longer,* when the above tabulation states that x or y is decreasing. This will cause no confusion when it is remembered that, if the increasing length represents a more *negative* value of x or y, the actual value is decreasing. (A negative number is always *less* than 0.)

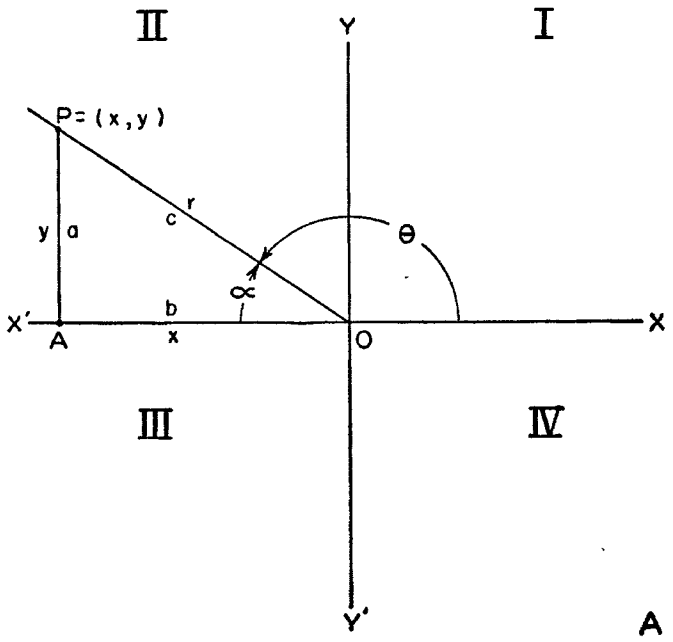
Note 2. In the preceding discussion emphasis is placed on using the rectangular coordinates and the radius vector r. In effect, y, x, and r have been substituted respectively for the original *a*, *b*, and *c* of figure 223. This substitution is an introduction to the use of trigonometry in solving a-c probems.

20. Reduction of Angles Greater than 90°

In general, trigonometric problems cannot be solved conveniently or accurately by constructing the angles to scale. Algebraic methods and accurate tables of trigonometric functions generally are

used. Since the tables provide functions for angles between 0° and 90° only, angles greater than 90° must be *reduced* to equivalent angles between 0° and 90°. Figure 224 shows that as the radius vector rotates,

$$\sin \theta = AB/OB = y/r$$
$$\cos \theta = OA/OB = x/r$$
$$\tan \theta = AB/OA = y/x.$$



SECOND QUADRANT	FIRST QUADRANT
SIN +	ALL FUNCTIONS +
COS -	
TAN -	
SIN -	SIN -
COS-	COS+
TAN+	TAN-
THIRD QUADRANT	FOURTH QUADRANT

B

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Figure 225. Sign of functions in the four quadrants.

From this we can derive the following rule:
TO DETERMINE ANY FUNCTION OF ANY ANGLE θ, DETERMINE THE SAME FUNCTION OF THE ACUTE ANGLE FORMED BY THE TERMINAL SIDE (radius vector r) AND THE HORIZONTAL AXIS AND PREFIX THE CORRECT ALGEBRAIC SIGN FOR THAT QUADRANT.

The correct algebraic signs are given in B of figure 225.

a. REFERENCE ANGLE. In A of figure 225, angle X'OX is a straight or 180° angle (it is the sum of two right angles). From this, angle POA ($\propto$) must be equal to 180° *minus* the value of angle XOP (θ); then

$$\propto = 180° - \theta.$$

Since it is used to determine the functions of θ, $\propto$ is called the reference angle for θ.

b. REDUCTION FORMULAS. If the reasoning used to develop the formula $\propto = 180° - \theta$ is applied to the other quadrants, we obtain the following formulas:

(1) For θ between 0° and 180°, $\propto = 180° - \theta$.
(2) For θ between 180° and 270°, $\propto = \theta - 180°$.
(3) For θ between 270° and 360°, $\propto = 360° - \theta$.

If θ is unknown and $\propto$ is known, then,

for (1) above, $\theta = 180° - \propto$.
for (2) above, $\theta = 180° + \propto$.
for (3) above, $\theta = 360° - \propto$.

Note. If the numerical value of θ is greater than 360°, change the angle by adding or subtracting an integral multiple of 360°. This does not change the functions, because the new angle is co-terminal with the original angle.

21. Graph of Sine Curve, y = sin x

From A of figure 224, we read $\sin \theta = y/r$ and, if r is a unit length, $\sin \theta = y/1 = y$. Suppose that we now use the symbol x to represent any angle θ, so that the equation becomes $y = \sin x$. The graph of this equation is plotted just as the graphs of algebraic equations are plotted in paragraph 2; that is, values are assigned to x and the corresponding values of y are calculated, points are plotted from the coordinates obtained, and a smooth curve is drawn through the points.

Example: In a-c problems, *radians* are usually used as the unit of angular measure. The sine curve is plotted with the angle x and its function y having the same scale, one unit on the y-axis being equal to the length representing one radian on the x-axis. Thus, in setting up the tabulation (B of fig. 226), the first column lists angles in degrees; the second gives x in radians; the third gives the number of unit lengths representing x; the fourth gives the number of unit lengths representing y; and the fifth column lists the coordinates of a number of points on the curve. Thus, assuming that a unit length represents one radian along the

x-axis in A of figure 226, the calculation for changing 30° to radians is as follows:

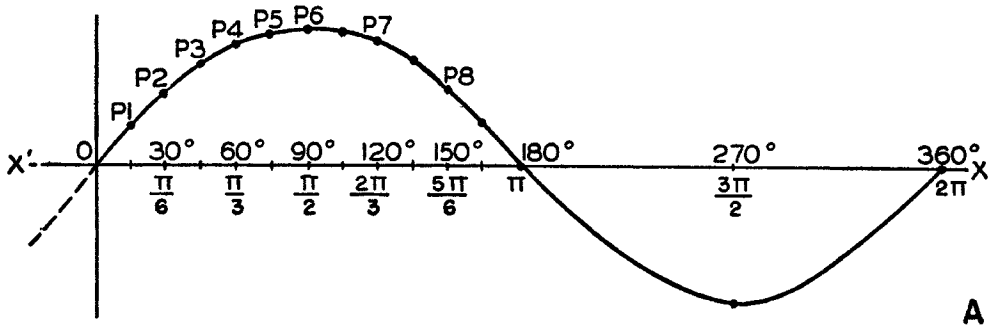
$$360° = 2\pi \text{ radians}$$

$$30 \times \frac{\pi}{180} = \frac{30\pi}{180} = \frac{\pi}{6} \text{ radians.}$$

This is entered in column 2. To obtain the number of unit *lengths* for column 3, complete the division indicated by $\pi/6$, getting

$$\frac{3.14}{6} = 0.52 \text{ unit length.}$$

From the sine table (appendix II) we find that sin 30° = 0.5. This means that y is equal to one-half the unit length chosen to represent one radian along the x-axis. The values in columns 3 and 4 are entered in column 5 as coordinates for point P2. For positive angles assign only positive values to x, plot the computed coordinates, and draw a smooth curve to obtain the solid line curve in A of figure 226. For negative angles assign negative values to x, and the curve extends to the left of the y-axis as indicated by the broken line curve. Values of sine x between 180° and 360° are negative, but equal in magnitude to the sines of angles between 0° and 180°. Because the trigonometric functions are periodic, the curve extends indefinitely to the right and left of the y-axis.



1	2	3	4	5
X DEGREES	X RADIANS	X UNIT MEASURE	Y (SIN X)	POINT
0	0	0	0	$P_0=(0,0)$
* 15	$\frac{\pi}{12}$	0.261	0.258	$P_1=(0.261,0.258)$
30	$\frac{\pi}{6}$	0.52	0.50	$P_2=(0.52,0.50)$
* 45	$\frac{\pi}{4}$	0.785	0.707	$P_3=(0.785,0.707)$
60	$\frac{\pi}{3}$	1.05	0.87	$P_4=(1.05,0.87)$
* 75	$\frac{5\pi}{12}$	1.30	0.965	$P_5=(1.30,0.965)$
90	$\frac{\pi}{2}$	1.57	1.00	$P_6=(1.57,1.00)$
120	$\frac{2\pi}{3}$	2.09	0.87	$P_7=(2.09,0.87)$
150	$\frac{5\pi}{6}$	2.62	0.50	$P_8=(2.62,0.50)$
180	π	3.14	0.00	$P_9=(3.14,0)$

* NOT INDICATED ON GRAPH.

B

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Figure 226. Graph of sine curve, $4 = \sin x$.

22. Graph of Cosine Curve, $y = \cos x$

The cosine curve of figure 227 is obtained by plotting the graph of the equation $y = \cos x$. The graphs of $y = \sin x$ and $y = \cos x$ have the same shape, but differ in their positions with relation to the axes. The cosine of any angle x is equal to the sine of an angle which is 90° greater. Thus, if the graph of $y = \sin x$ is shifted 90° to the *left*, we obtain the graph of $y = \cos x$.

23. Graph of Tangent Curve, y = tan x

As discussed in paragraph 15, there is no finite tangent for 90°; that is, tan 90° = ∞. This causes breaks in the curve, and the graph of $y = \tan x$ (fig. 228) consists of an infinite number of separated branches. The broken lines, called asymptotes, do not touch the curve because there is no tangent (the tangent is infinite) and, hence no point on the graph corresponding to $x = \pi/2$ and $x = 3\pi/2$.

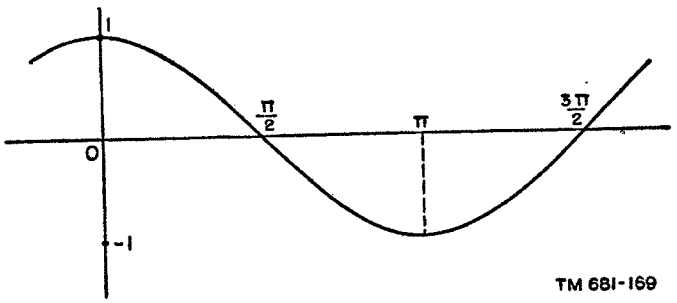


Figure 227. Graph of y = cos x.

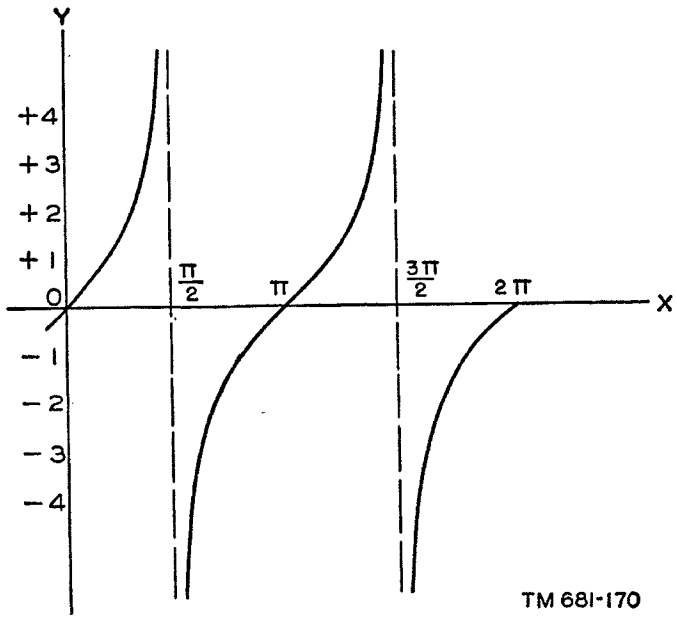


Figure 228. Graph of y = tan x.

24. Velocity

In a-c theory problems, we are concerned not only with the fact that rotation of the initial side generates an angle, but also with the *rate* at which the angle is generated. To develop the principles involved, let us consider *linear* and *angular* velocity and the relation between them.

a. LINEAR VELOCITY. The linear velocity of a point or object traveling in a particular direction is defined as the rate at which distance is traveled by the point or object. The unit of velocity v is the distance traveled in unit time: miles per hour, feet per second, and so on. Linear velocity is expressed by the following simple equation in which s is distance and t is time.

$$v = s/t.$$

In A of figure 229, the car travels a distance of 30 miles from A to B. If this distance is traveled in 0.4 hour (24 minutes) the linear velocity is then,

$$v = s/t = 30/.4 = 75 \text{ miles per hour.}$$

If the minute is used as the unit of time,

$$v = s/t = 30/24 = 1.25 \text{ miles per minute.}$$

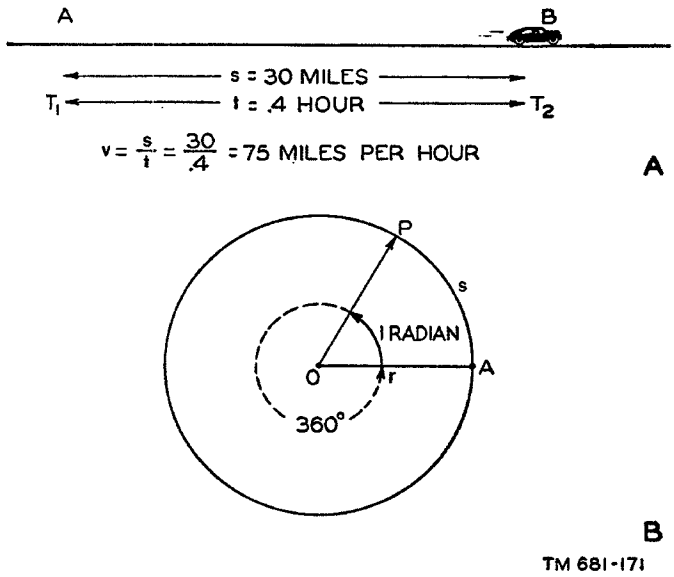


Figure 229. A, linear and B, angular velocity.

b. ANGULAR VELOCITY. When a wheel rotates on its axis, its velocity or rate of turning usually is expressed as so many *revolutions per minute.* However, a revolution can be measured in *angles.* The radian is a unit of angular measure (par. 7), and angular velocity, symbol ω, is measured in *radians per second.*

(1) *The radian.* From geometry we know that the circumference of a circle is given by the following equation

$$C = 2\pi r$$

where C is the circumference, r is the radius, and 2π is 6.28. Dividing both sides of the equation $C = 2\pi r$ by r, we get

$$\frac{C}{r} = 2\pi.$$

This equation states that the ratio of the circumference to the radius is 2π; that is, the circumference is 6.28 times longer than the radius. From paragraph 7 we know that a radian is an angle which,

when placed with its vertex at the center of a circle, intercepts an arc s equal in length to the radius r. Thus, in figure 229, arc s is equal in length to the radius r, and angle AOP is one radian. Since the circumference C is equal to 2π times the radius, there are 2π s-arcs around the circumference and, therefore, there are 2π s-arcs around the circumference and, therefore, there are 2π radians in the circle. In other words, as the radius vector r makes one revolution from OA back to OA, it generates an angle of 2π radians or 360°.

(2) *Angular velocity.* In electrical problems the *second* is commonly used as the unit of time, and the number of revolutions per second is called the frequency, symbol f. If the radius vector makes ten revolutions per second, f is 10; then,
radians per second $= 2\pi f$
radians per second $= 6.28 \times 10 = 62.8$.
The number of radians per second is the *rate* at which the angle increases; that is, *angular velocity is the number of radians per second.* The Greek letter omega, ω, is used to represent angular velocity or radians per second; therefore,

$$\omega = 2\pi f.$$

c. RELATION BETWEEN LINEAR AND ANGULAR VELOCITY. Refer to figure 230 and let the symbol θ represent an angle in radians generated in t seconds. If θ (in radians) is divided by t, we get the angular velocity:

$$\omega = \frac{\theta}{t}$$

However, circular measure is the ratio of the arc s to the radius r; therefore:

$$\theta = \frac{s}{r}$$

In the equation $\omega = \theta/t$, substitute for θ its equal s/r to get

$$\omega = \frac{s}{rt}.$$

This equation is still an equation for angular velocity, but it now contains s/t which is defined as linear velocity. Multiplying by r we get

$$\omega r = \frac{s}{t}$$

and, on substituting for s/t its equal v, we get

$$v = \omega r.$$

This is the linear velocity of a point P distant r from the axis.

If we divide $v = \omega r$ by r, we get

$$\omega = \frac{v}{r}$$

and this is the *angular velocity.*

Example: A flywheel is rotating at an angular velocity of 200 revolutions per second.

(1) What is the angular velocity in radians per second?

Solution: $\omega = 2\pi f$
$\omega = 6.28 \times 200$
$\omega = 1{,}256$ radians per second.

(2) What is the angle generated in 0.75 second?

Solution: Since $\omega = \frac{\theta}{t}$,
$\theta = \omega t$
$\theta = 1{,}256 \times 0.75$
$\theta = 942$ radians.

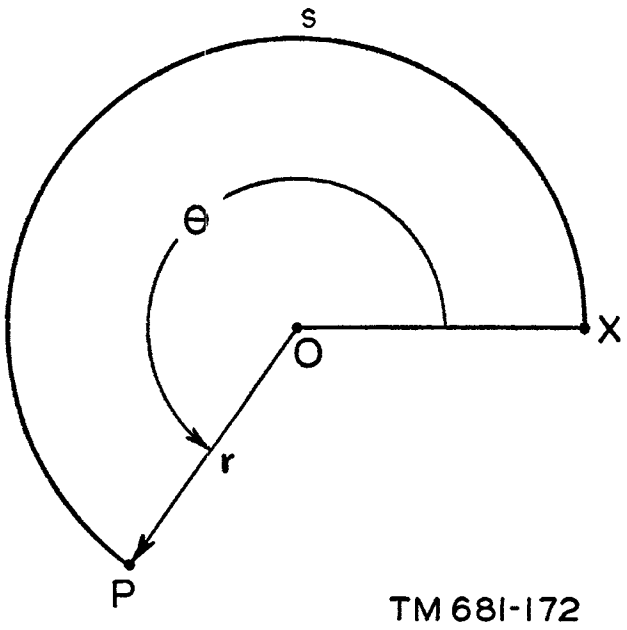


Figure 230. Linear and angular velocity.

(3) How much time is required for the wheel to generate 4,500 revolutions?

Solution: Since $\omega = \frac{\theta}{t}$
$t = \frac{\theta}{\omega}$
$t = 4{,}500 \times \frac{2\pi}{1{,}256}$
$t = 22.5$ seconds

(4) What is the linear velocity of a point P which is 2 feet from the axis?

Solution: $v = \omega r$

$v = 2\pi fr$

$v = 6.28 \times 200 \times 2$

$v = 2{,}512$ feet per second.

(5) The number of revolutions per second is changed so that the linear velocity of point P (2 feet from the axis) increases to 3,500 feet per second. What is the angular velocity in radians per second? In revolutions per second?

Solution: Since $\omega = \frac{v}{r}$

$\omega = \frac{3{,}500}{2}$

$\omega = 1{,}750$ radians per second.

Then, since $\omega = 2\pi f = 1{,}750$,

$f = \frac{1{,}750}{2\pi}$

$f = \frac{1{,}750}{6.28}$ revolutions per minute.

sents 1/12 second, or 30° of rotation. If the radius vector starts at 0°, at the end of 1/12 second it will have rotated 30° or 0.523 radian to A; at the end of 1/6 second, it will have rotated to B and the angle will be 60° or 1.05 radians, and so on. To plot the curve, draw *vertical* lines through the division lines on the x-axis. Through points corresponding to the end points of the radius vector at the time of each division on the x-axis, draw lines *parallel* to the x-axis. Complete the curve by drawing a smooth curve through the points of intersection. Compare the resultant curve with that obtained by plotting a graph of the equation $y = \sin x$ (fig. 226).

a. EQUATION OF CURVE. The y value of the curve is proportional to the sine of the angle θ and the value of the radius vector r,

$$y = r \sin \theta.$$

The radius vector generates an angle of 2π radians in 1 second (assumed for figure 231); therefore, the y value at any time t

is $y = r \sin 2\pi t$

or, $y = r \sin 6.28t$.

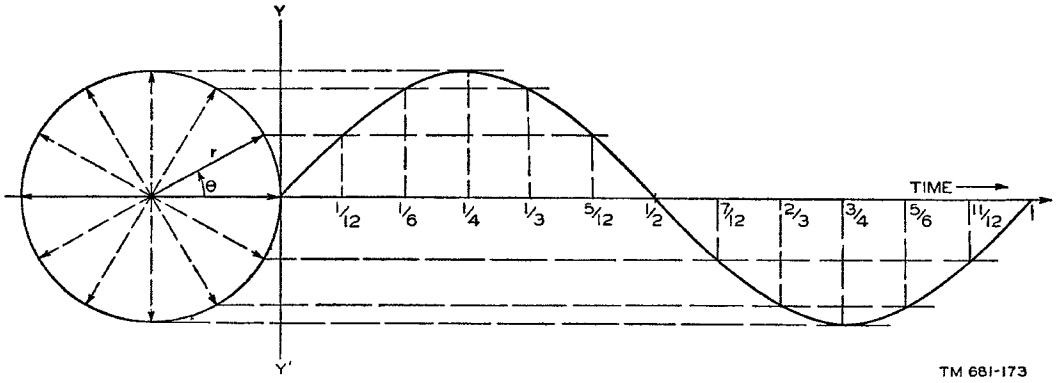


Figure 231. Projection of point having uniform circular motion.

25. Projection of Point Having Uniform Circular Motion

The projection, or y value, at any instant, of the end point of a radius vector may be plotted as a curve. In figure 231, the horizontal diameter of the circle is extended to the right to form a *time* axis. The x-axis is divided into intervals corresponding to the angles to be plotted. To avoid confusion in the illustration (fig. 231), angles are plotted at 30° intervals; therefore, the x-axis is divided into 12 equal divisions (360/30 = 12). Also, for convenience, the radius vector is assumed to be rotating counterclockwise with a uniform angular velocity of one revolution per second. Consequently, each division on the x-axis represents 1/12 second, or 30° of rotation. If the radius vector starts at 0°, at the end of 1/12 second it will have rotated 30° or 0.523 radian to A; at the end of 1/6 second, it will have rotated to B and the angle will be 60° or 1.05 radians, and so on. To plot the curve, draw *vertical* lines through the division lines on the x-axis. Through points corresponding to the end points of the radius vector at the time of each division on the x-axis, draw lines *parallel* to the x-axis. Complete the curve by drawing a smooth curve through the points of intersection. Compare the resultant curve with that obtained by plotting a graph of the equation $y = \sin x$ (fig. 226).

a. EQUATION OF CURVE. The y value of the curve is proportional to the sine of the angle θ and the value of the radius vector r,

$$y = r \sin \theta.$$

The radius vector generates an angle of 2π radians in 1 second (assumed for figure 231); therefore, the y value at any time t

is $y = r \sin 2\pi t$

or, $y = r \sin 6.28t$.

The equation $y = r \sin 6.28t$ is the equation of the curve in figure 231.

b. FREQUENCY. In figure 231, f is 1 and, therefore, it does not appear in the equation. In a-c circuits, f may represent less than one revolution per second or millions of revolutions per second. The angular velocity in radians per second is

$$\omega = 2\pi f.$$

Consequently, the point at the end of the radius vector will have a circular motion represented by the following equation,

$$y = r \sin 2\pi ft$$

or, $y = r \sin \omega t$.

This equation describes the motion of any quantity or object that varies with simple harmonic motion. Any motion described by this equation is said to be

sinusoidal and can be represented by a sine curve.

Example: A crank 12 inches long, starting from 0°, is turned counterclockwise at the rate of one revolution in 5 seconds. What is the equation for the projection of the crank handle at any instant? What is the *vertical* distance of the handle from the shaft at the end of 4 seconds?

Solution: The projection or y value is expressed by the following equation:

$$y = r \sin \omega t$$

where, r = length of crank
ω = angular velocity in radians per second
t = time in seconds.

In figure 232, AB is the vertical distance of the handle from the shaft. This distance is equal to the y value of the end point of the crank at $t = 4$ seconds. Then, since the crank makes one revolution, or 2π radians, in 5 seconds, the angular velocity in radians per seconds is,

$$\omega = 2\pi/5 = \frac{6.28}{5} = 1.256 \text{ radians per second.}$$

Substituting values for r, ω, and t,

$$y = r \sin \omega t$$
$$y = 12 \sin (1.256 \times 4)$$
$$y = 12 \sin 5.024 \text{ radians.}$$

Converting radians to degrees,

$$y = 12 \sin (5.024 \times 57.3)$$
$$y = 12 \sin 287.88°$$

Reducing to acute angle,

$$y = 12 \sin (360° - 287.88°)$$
$$y = 12 \sin 72.12°$$

Using correct sign for quadrant,

$$y = 12 \times -.9516$$
$$y = -11.41 \text{ inches.}$$

The negative sign indicates that the handle is below the shaft.

c. AMPLITUDE OF CURVE. The amplitude of the curve representing the equation $y = r \sin \omega t$ is a function of the value, or length, of the radius vector r. In figure 233, curve A represents the equation $y = r \sin \omega t$ when $r = 1$; curve B represents the equation when $r = 2$.

d. CYCLES. In completing one revolution, the radius vector completes one sine wave, and the sine wave has passed through 1 *cycle.* The number of cycles completed in unit time is called the frequency of the curve. Curves for different frequencies are drawn in figure 233. In the equation $y = r \sin t$, $\omega t = t$. The angular velocity is then one radian per second and, therefore, the curve completes 1 cycle in 2π seconds. In the equation $y = r \sin 2t$, $\omega = 2$ radians per second; therefore, at the end of 2π seconds the sine wave has completed 2 cycles, or $2 \times 2\pi = 4\pi$ radians.

Since $\omega = 2\pi f$,

$$f = \frac{\omega}{2\pi}.$$

The time T required to complete 1 cycle is called the period. Since $f = \frac{\omega}{2\pi}$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi f} = \frac{1.}{f}$$

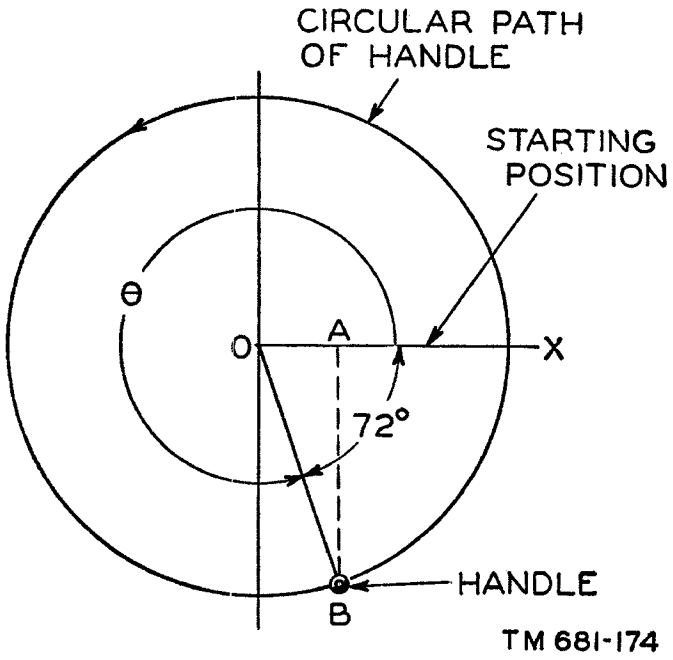


Figure 232. Rotating crank, projection of handle.

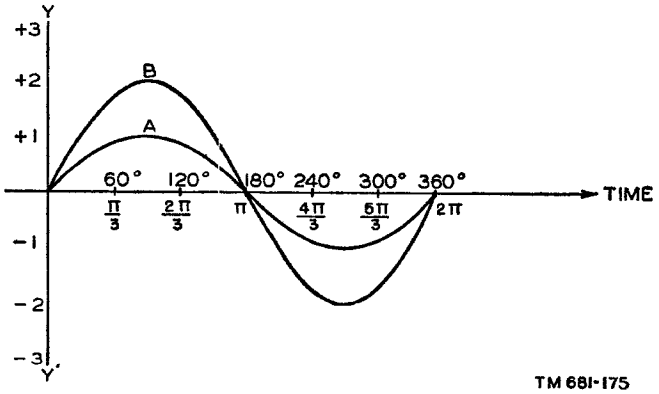


Figure 233. Graph of $y = r \sin \omega t$.

26. Phase

When a series of events repeats itself with regularity, we speak of a particular event as a *phase* of the series. Thus, we speak of the moon as being in its first, second, third, or fourth quarter. Each

quarter is a particular *phase*. Similarly, in a sine curve, each point on the curve is a *phase* in a cycle; the point at which y is maximum and positive is one phase, the point of which y is maximum and negative is another phase, and so on. It must be remembered that *every* point represents a different phase in a cycle.

a. PHASE DIFFERENCE. Assume that radius vectors r_1 and r are in the positions shown in figure 235. Start counterclockwise rotation at the same angular velocity, ω, and plot the respective sine curves A and B. Note that corresponding phases do not occur at the same time; that is, there is a *phase difference.*

b. PHASE ANGLE. Points on A occur earlier than the corresponding points on B; consequently,

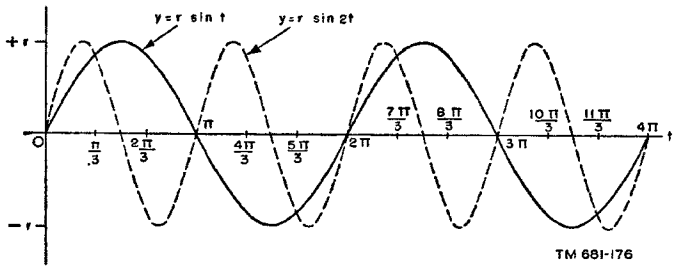


Figure 234. Graph of $y = r \sin t$ and $y = r \sin 2t$.

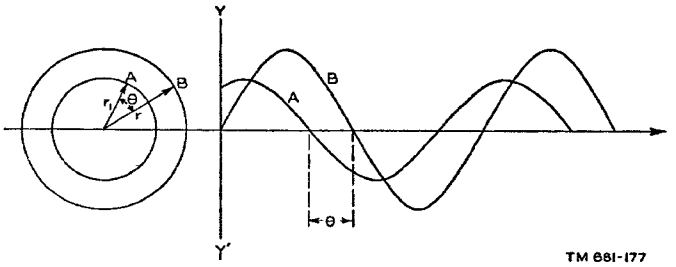


Figure 235. Rotation of two radii, showing phase difference.

A is said to *lead* B. When $t = 0$, r starts to generate the curve B, $y = r \sin \omega t$. At the same time, r_1 is *ahead* of r by an angle θ. Consequently, r_1 generates curve A, $y_1 = r_1 \sin (\omega t + \theta)$. This moves the y_1 curve along the time axis by an angle θ. The angle θ is the *angular difference* between the curves and it is called the *phase angle.* Since it is ahead of y, y_1 *leads* y; therefore, in the equation $y_1 = r_1 \sin (\omega t + \theta)$, the angle θ is called the *angle of lead.* If the positions of r_1 and r are interchanged, r_1 will be *behind* r. The phase angle θ will be the same, but y_1 will now *lag* y. Consequently, the curve generated by r_1 is $y_1 = r_1 \sin (\omega t - \theta)$ and θ is the *angle of lag.*

27. Summary

a. Trigonometry is a method of measuring triangles.

b. Trigonometry is used to solve a-c circuit problems.

c. In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

d. The ratio of any pair of sides in a right-angled triangle is a function of the acute angle being considered. The six possible rations are called trigonometric functions.

e. Trigonometric functions repeat themselves and are said to be periodic.

f. The sine of an angle θ varies from 0 to 1 as θ varies from 0° to 90°.

g. The cosine of an angle θ varies from 1 to 0 as θ varies from 0° to 90°. The tangent of an angle θ varies from 0 to infinity as θ is increased from 0° to 90°.

h. The functions of angles greater than 90° are found by reducing the larger angle to an equivalent angle of less than 90°.

i. In a-c problems the radian is the commonly used unit of angular measure.

j. The velocity of a point traveling in a particular direction is the rate at which distance is traveled in unit time: $v = s/t$.

k. Angular velocity is the rate of *turning* of a rotating body such as a wheel.

l. Angular velocity may be measured in revolutions per unit time. In electrical problems, angular velocity is measured in radians per second.

m. In completing one revolution the radius vector completes one sine wave and the sine wave has passed through 1 cycle.

n. The number of cycles per unit time is called the frequency, symbol f. The time T of 1 cycle is called the period:

$$T = \frac{1.}{f}$$

o. When corresponding phases of one or more waves do not occur at the same time, there is a phase difference between the waves.

p. Phase differences are measured in degrees or in radians; the phase difference is called the phase angle.

Section IV. ELEMENTARY PLANE VECTORS

28. Quantities

a. SCALAR. Consider the statement: A ship sailed 4 miles. This statement does not describe the movement of the ship completely, because nothing is said about direction; whether the ship sailed to the north, south, east, or west is not mentioned. A quantity that is described by a number, but does not include the concept of direction, is called a *scalar* quantity.

b. VECTOR. Consider the statement: A ship sailed 4 miles to the east. This statement involves *magnitude and direction. Quantities that have both magnitude and direction are called vector quantities.*

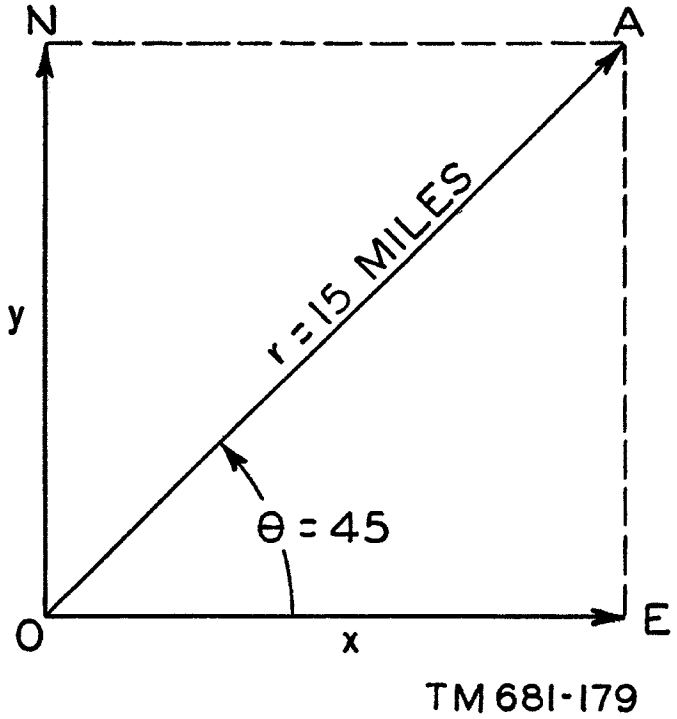


Figure 236. Addition of vectors.

A vector quantity is represented by a *directed* straight line (————→). The length of the directed line is proportional to the magnitude and the head points in the direction of the vector quantity. The directed straight line is called a *vector.*

29. Addition and Subtraction

a. SCALAR QUANTITIES. Scalar quantities are combined by algebraic addition or subtraction. Algebraic addition and subtraction take account of the signs + (positive) and — (negative):

150 miles + 50 miles = 200 miles.
150 miles — 50 miles = 100 miles.
150 miles + (—50) miles = 100 miles.
150 miles — (—50) miles = 200 miles.

b. VECTOR QUANTITIES. Vector quantities must be added or subtracted geometrically.

(1) *Vector addition.* In figure 236, point A is 15 miles northeast of the origin O and the angle θ is 45°. From O we can reach A by traveling directly from O to A along OA, by traveling from O to N and then from N to A along AN, or by traveling from O to E and then from E to A. If we travel exactly north from O to N we get no farther east or west; if we travel exactly east from O to E we get no farther north or south. From this we conclude that vector OA is the sum or resultant of two components, a northerly component ON and an easterly component OE. In terms of rectangular coordinates (fig. 199), the northerly component is a *vertical* component equal to y, the easterly component is a *horizontal* component equal to x, and the vector $\overline{OA}$ is a *radius vector* r (the bar over OA indicates that OA is a vector quantity). From trigonometry we know that $y = r \sin \theta$; then

$$\overline{ON} = \overline{OA} \sin \theta$$
$$\overline{ON} = 15 \sin 45°$$
$$\overline{ON} = 15 \times .707 = 10.6 \text{ miles.}$$

Similarly, since $x = r \cos \theta$,

$$\overline{OE} = \overline{OA} \cos \theta$$
$$\overline{OE} = 15 \times .707 = 10.6 \text{ miles.}$$

If we now add $\overline{OE}$ and $\overline{ON}$ *algebraically,*

$$\overline{OA} = \overline{OE} + \overline{ON}$$
$$\overline{OA} = 10.6 + 10.6 = 21.2 \text{ miles.}$$

Obviously, this answer is in error, because the magnitude of $\overline{OA}$ is given as 15 miles. Recalling the Pythagorean theorem (par. 12) $C^2 = a^2 + b^2$, then

$$\overline{OA}^2 = \overline{EO}^2 + \overline{ON}^2$$
$$\overline{OA} = \sqrt{\overline{OE}^2 + \overline{ON}^2}$$
$$\overline{OA} = \sqrt{10.6^2 + 10.6^2}$$
$$\overline{OA} = \sqrt{225}$$
$$\overline{OA} = 15 \text{ miles.}$$

This *geometric* or *vectorial* solution not only gives the correct length of $\overline{OA}$, but also shows that $\overline{OA}$ is the sum, or *resultant,* of vertical and horizontal components.

(2) *Subtraction of vectors.* A vector *difference*

is obtained by subtracting one vector from another *vectorially*. To subtract one vector from another, *reverse* the vector to be subtracted and then solve for the resultant by geometric addition. (This is the same as thinking of $a - b$ as the sum of $a + (-b)$.) In figure 237 we are given the vectors $\overline{ON}$ and $\overline{OE}$. To *subtract* $\overline{ON}$ from $\overline{OE}$, *reverse* $\overline{ON}$, bringing it to $\overline{ON'}$; then

$$\overline{OA}^2 = \overline{OE}^2 + (-ON)^2$$
$$\overline{OA} = OE^2 + (-ON)^2$$
$$\overline{OA} = 10.6^2 + (-10.6)^2$$
$$\overline{OA} = 122.36 + (122.36)$$
$$\overline{OA} = 225$$
$$\overline{OA} = 12 \text{ miles.}$$

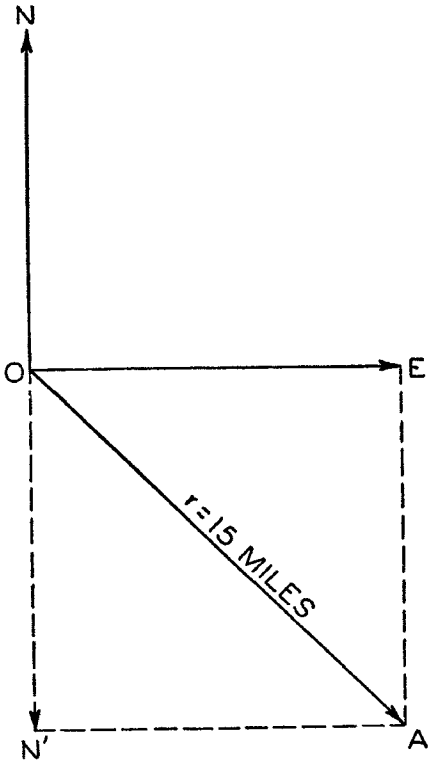


Figure 237. Subtraction of vectors.

30. Methods of Solution

There are two basic methods of solving plane vector problems, the parallelogram and the triangle method.

a. PARALLELOGRAM METHOD. Refer to figure 238. We are given the vectors $\overline{OA}$ and $\overline{OB}$ and wish to determine graphically the resultant R. By drawing AC parallel to and equal in length to OB, and BC parallel to and equal in length to OA, we obtain the parallelogram OACB. By definition, the diagonal OC is the resultant, or sum, of vectors $\overline{OA}$ and $\overline{OB}$ and the direction is given by the angle α. When there are more than two vectors, we may first add two of them by the parallelogram method, then add this resultant to the third, then the sum of the three to the fourth, and so on. *Notice that when the parallelogram method is used, the vectors must be placed origin to origin.*

Example: In A of figure 240, forces $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ are acting on point O (the origin). The resultant of these vectors can be found graphically by the parallelogram method as follows (B of fig. 239):

(1) Draw the vectors to scale.
(2) Find the resultant of any two, say $\overline{OA}$ and $\overline{OC}$, by constructing a parallelogram OADC.
(3) Draw the diagonal of OADC. This diagonal $\overline{OD}$ is the *resultant* of $\overline{OA}$ and $\overline{OC}$.
(4) Find the resultant of $\overline{OD}$ and $\overline{OB}$ by constructing parallelogram OBED. The diagonal $\overline{OE}$ then represents the resultant force on point O.

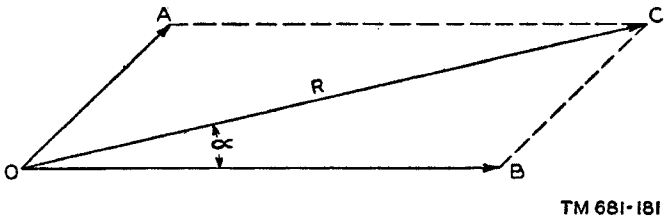
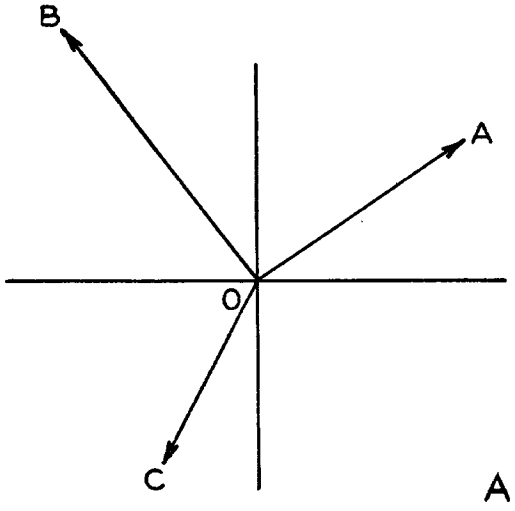
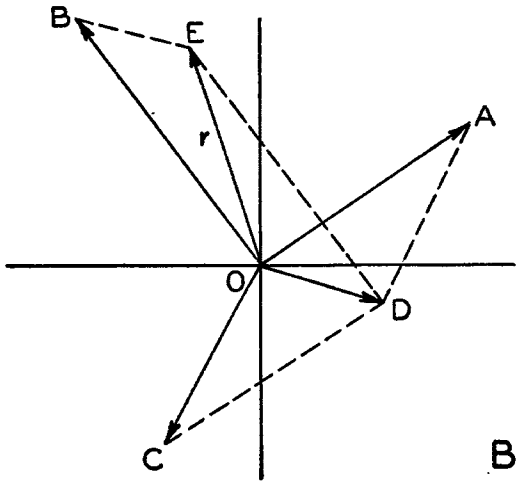


Figure 238. Parallelogram method of adding vectors.

(5) Measure $\overline{OE}$ with a scale and protractor to determine its magnitude and direction.

b. TRIANGLE METHOD. In A of figure 240, vectors $\overline{OA}$ and $\overline{OB}$ are acting on point O. $\overline{BA'}$ is drawn equal in direction and magnitude to $\overline{OA}$ and, therefore, it represents the *same* vector quantity. Similarly, in B of figure 240, $\overline{AB'}$ is drawn equal in direction and magnitude to $\overline{OB}$: therefore, $\overline{AB'}$ and $\overline{OB}$ represent the *same* vector quantity. In figures 240–A and 240–B, the given vectors are placed *head to tail* and the line which *closes* the figure is the sum or resultant r. Since the closing line forms a triangle, this method of adding vectors is called the triangular method. When there are more than two vectors (A of fig. 239), the triangular method may be used by placing the vectors *head to tail* in a continuous sequence (C of figure 239). The line $\overline{OE}$ closes the figure and represents the resultant r.





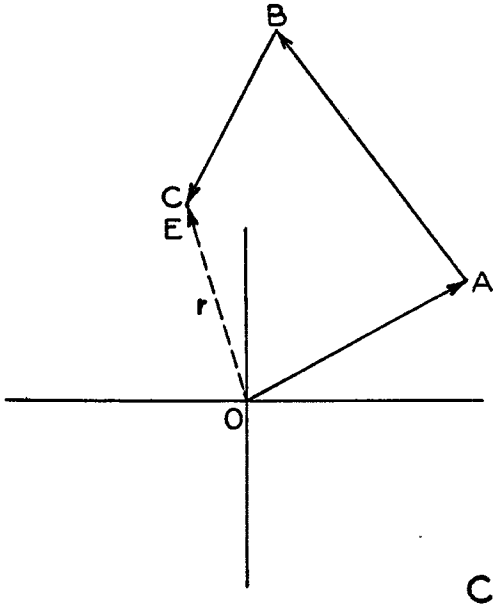


Figure 239. Graphical solution of vectors.

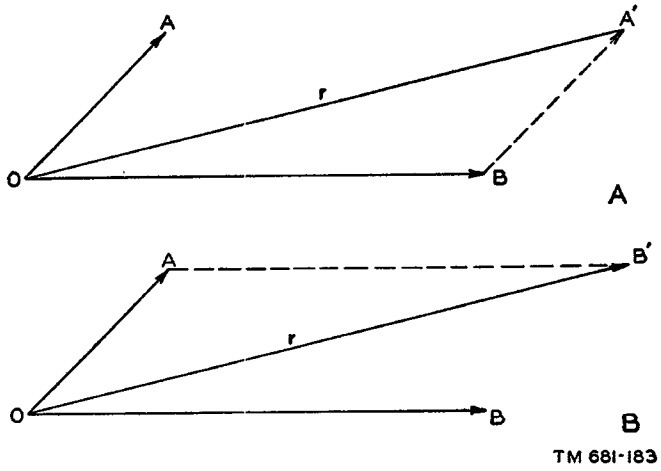
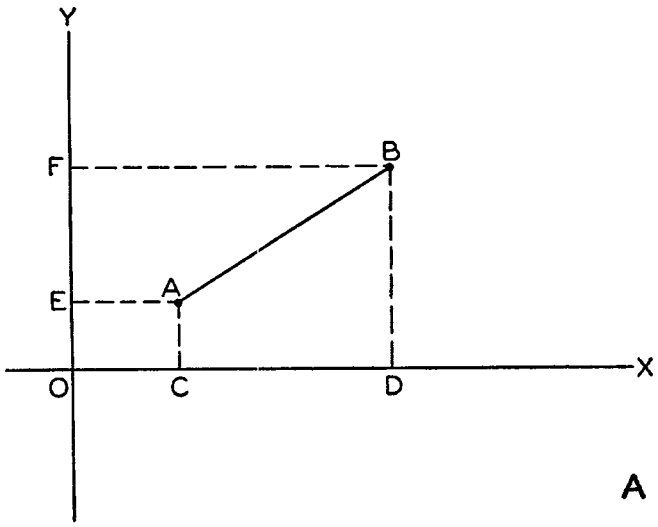


Figure 240. Solving vectors by triangle method.



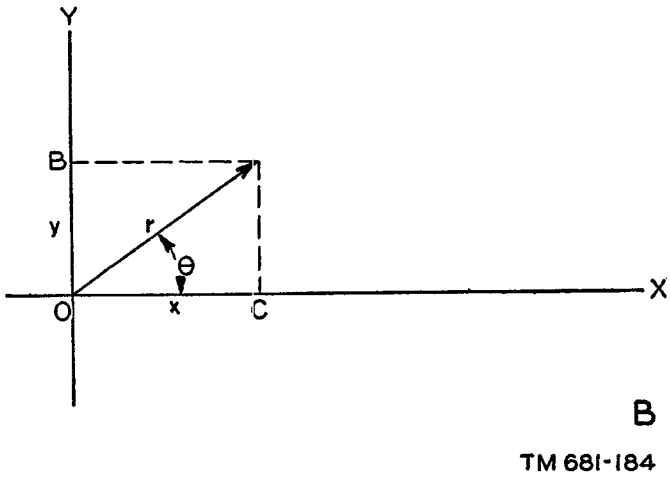


Figure 241. A. Projections of line section. B. Projetcion of vectors.

31. Projections

Consider line AB in A of figure 241. If AC and BD are drawn perpendicular to OX, CD is the *projection* of AB on OX. If BF and AE are drawn perpendicular to OY, EF is the *projection* of AB on OY. In B of figure 241, OB is the projection y of the radius vector r on OY and, from trigonometry,

$$y = r \sin \theta$$

OC is the projection x of the radius vector r on OX, and

$$x = r \cos \theta.$$

32. Resolution of Vectors

A given vector can be broken down or *resolved* into two (or any number) of components in two (or more) selected directions. In this manual,

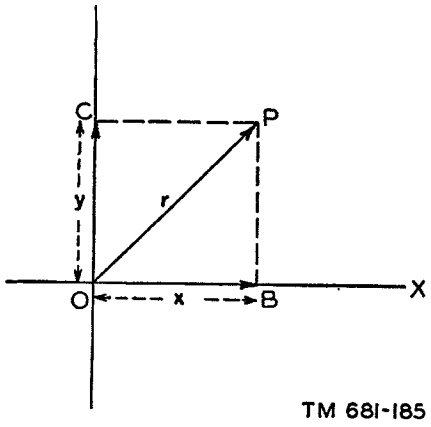


Figure 242. Projection of vector.

vector problems are solved only by resolution into two components along two rectangular axes.

a. RECTANGULAR COORDINATES.

(1) As discussed in section I, the *horizontal coordinate,* or the *abscissa* of a point P is the perpendicular distance x from OY to P. This distance is positive when P is to the *right* of OY, negative when P is to the *left* of OY.

(2) The vertical *coordinate,* or the *ordinate* of a point P, is the perpendicular distance y from OX to P. This distance is positive when P is *above* OX, negative when P is *below* OX.

(3) The distance of a point P from the origin is the radius vector of P. *The radius vector (r) is always positive.*

b. COMPONENTS OF VECTOR. Consider vector OP in figure 242. The rectangular coordinates of point P are (x, y). By projecting OP to the vertical or y axis, OY, and to the horizontal axis OX, we form parallelogram OCPB. If we now consider projections OC and OB as vectors, their sum or resultant must be $\overline{OP}$. In other words $\overline{OC}$ and $\overline{OB}$ are *vertical and horizontal components* of $\overline{OP}$. Since $\overline{OC}$ is equal to y and $\overline{OB}$ is equal to x, it follows that the coordinates of point P, (x, y), are the x and y components of $\overline{OP}$. From this,

$$\overline{OP} = \sqrt{x^2 + y^2}$$

or

$$r = \sqrt{x^2 + y^2}$$

then, in figure 243, $x = 8$ and $y = 4$

therefore,

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{8^2 + 4^2}$$

$$r = \sqrt{64 + 16}$$

$$= \sqrt{80}$$

$$r = 8.9$$

and,

$$\tan \theta = \frac{y}{x} = \frac{4}{8} = .5$$

and arc

$$\tan .5 = 26.6°$$

therefore,

$$\theta = 26.6°$$

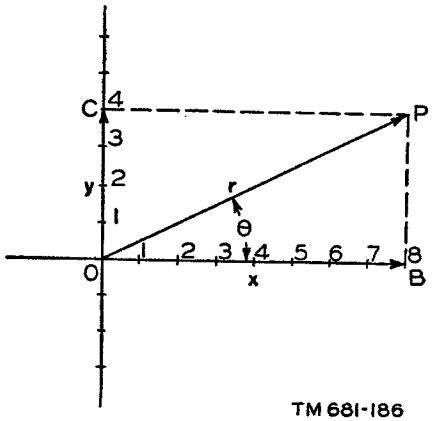


Figure 243. Resolution of vector into components.

This means that the length and direction of the resultant can be computed easily by the familiar properties of a right angled triangle. We may now state the following rules:

(1) The horizontal component of a vector is the projection of the vector upon a horizontal line. In terms of rectangular coordinates the magnitude of the horizontal component is determined from the equation

$$x = r \cos \theta.$$

(2) The vertical component of a vector is the projection of the vector upon a vertical line. In terms of rectangular coordinates the magnitude of the vertical component is determined from the equation

$$y = r \sin \theta.$$

(3) When the vertical and horizontal components are known the angle θ is determined from the equation

$$\tan \theta = \frac{y}{x} .$$

(4) When the vertical component, y, and the angle, θ, are known, the resultant, r, is determined from the equation

$$r = \frac{y}{\sin \theta} .$$

(5) When the horizontal component, x, and the angle, θ, are known, the resultant is determined from the equation

$$r = \frac{x}{\cos \theta} .$$

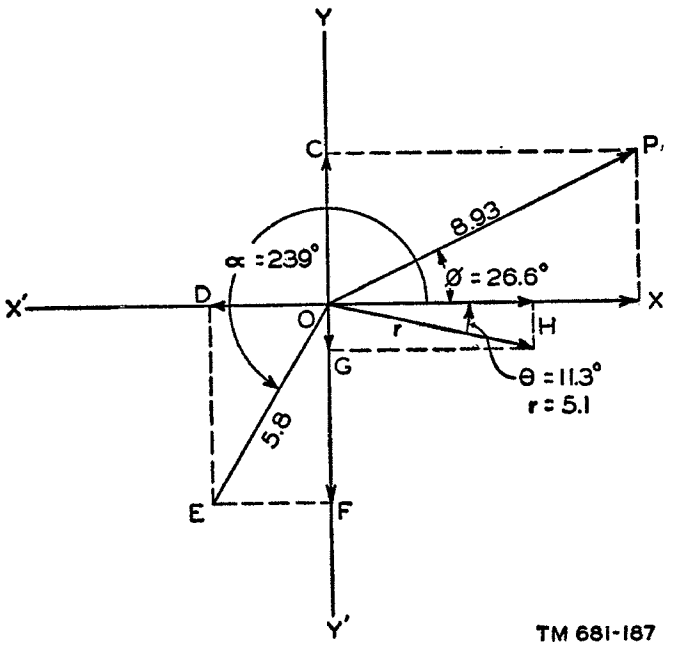


Figure 244. Solution of vectors not at right angles.

c. SOLUTION OF VECTORS NOT AT RIGHT ANGLES. In figure 244, vectors $\overline{OP}$ and $\overline{OE}$ are not at right angles. We could, of course, determine the resultant graphically (fig. 239). However, by resolving each vector into its vertical and horizontal components, we can determine the resultant easily and accurately. The procedure is as follows:

(1) Resolve each vector into its vertical, y, and horizontal, x, components.

(2) Add the vertical components to obtain the vertical component of the resultant.

(3) Add the horizontal components to obtain the horizontal component of the resultant.

(4) Find the angle between the resultant and the reference line (usually the positive portion of the x-axis) by substituting the vertical component (step 2 above) and the horizontal component (step 3 above) for y and x respectively, in the equation

$$\tan \theta = \frac{y}{x} .$$

(5) Obtain the angle θ from the function table.

(6) Find the magnitude of the resultant r by substituting known values into either of the following equations

$$r = \frac{y}{\sin \theta} .$$

or,

$$r = \frac{x}{\cos \theta} .$$

Example: In figure 244 we are given vectors $\overline{OP}$ and $\overline{OE}$ acting on point O. $\overline{OP}$ represents a magnitude of 8.93 and it is at an angle $\emptyset$ of 26.6° from the reference line OX. $\overline{OE}$ represents a magnitude of 5.81 and it is at an angle of 239° from the reference OX. What is the magnitude and direction of the resultant?

Solution:

Step 1. Components of $\overline{OP}$ and $\overline{OE}$.

a. Components of OP. Since the magnitude of $\overline{OP}$ is 8.93 and the angle $\emptyset$ is 26.6°, the horizontal component is given by

$$x = \overline{OX} = r \cos \emptyset$$
$$x = \overline{OX} = 8.93 \times .8942$$
$$x = \overline{OX} = 8.$$

The vertical component is given by

$$y = \overline{OY} = r \sin \emptyset$$
$$y = OY = 8.93 \times .4478$$
$$y = OY = 4.$$

b. Components of OE. Since the magnitude of OE is given as 5.81 and the angle $\propto$ is 239°, the horizontal component is given by the equation

$$x = r \cos (\propto - 180)°, \text{ then}$$
$$x = r \cos 59°$$
$$x = 5.81 \times .5150$$
$$x = -3.$$

The vertical component is given by the equation

$$y = r \sin (\propto - 180)°, \text{ then}$$
$$y = r \sin 59°$$
$$y = 5.81 \times -.8581$$
$$y = -5.$$

Step 2. Adding vertical components, to obtain the vertical component OG of the resultant,

$$y = \overline{OG} = \overline{OC} + \overline{OF}$$
$$y = \overline{OG} = 4 + (-5) = -1.$$

Step 3. Adding horizontal components, to obtain the horizontal component $\overline{OH}$ of the resultant,

$$x = \overline{OH} = \overline{OX} + \overline{OD}$$
$$x = OH = 8 + (-3) = 5.$$

Step 4. By substitution in the equation

$$\tan \theta = \frac{y}{x}$$
$$\tan \theta = \frac{-1}{5} = -.2.$$

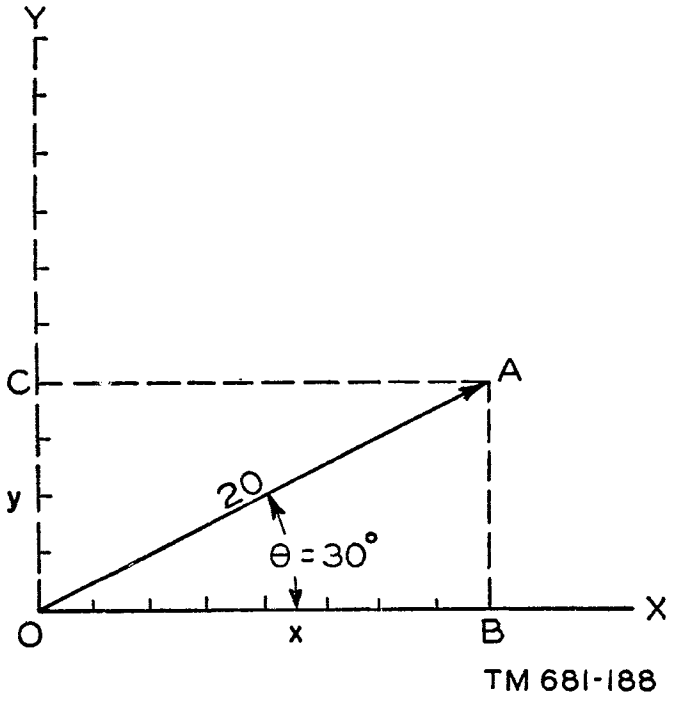


Figure 245. Component velocities.

Step 5. From the function tables arctan $-.2 = -11.3°$. This is the angle between the resultant r and the reference OX.

Step 6. Using the equation

$$r = \frac{y}{\sin \theta}$$
$$r = \frac{-1}{\sin -11.3°}$$
$$r = \frac{-1}{-.1959} = 5.1$$

or using the equation

$$r = \frac{x}{\cos \theta}, \quad r = \frac{.5}{.9806} = 5.1.$$

Example: A ship sails 30° north of East at a speed of 20 miles per hour. What are the component velocities north and east?

Solution: Draw a diagram to repredent the problem. In figure 245, $\overline{OA}$ represents the *velocity* of the ship. $\overline{OC}$ and $\overline{OB}$ are the *northerly* (vertical) and *easterly* (horizontal) components of $\overline{OA}$, then

$$\overline{OC} = \overline{OA} \sin 30°$$
$$\overline{OC} = 20 \times .5$$
$$\overline{OC} = 10 \text{ miles north}$$

and

$$\overline{OB} = OA \cos 30°$$
$$\overline{OB} = 20 \cos .866$$
$$\overline{OB} = 17.3 \text{ miles east.}$$

33. Summary

a. A scalar quantity does not include the concept of direction.

b. A vector quantity involves both magnitude and direction.

c. Vector quantities must be added geometrically (vectorial addition).

d. A vector quantity may be resolved into horizontal (x) and vertical (y) components.

e. Vectors may be solved graphically by either the parallelogram or the triangle methods.

f. When the parallelogram method is used, the vectors must be placed *origin to origin.*

g. When the triangle method is used, the vectors must be placed *head to tail.*

h. A vertical projection is the projection of a line segment to the vertical axis.

i. A horizontal projection is the projection of a line segment to the horizontal axis.

j. Vectors may be solved by resolution into two components along two rectangular axes.

k. Use of rectangular coordinates enables solution of vectors by familiar properties of right angle triangles.

l. The vertical component of a vector is the projection of the vector on the vertical or y-axis. It is given by the equation $y = r \sin \theta$.

m. The horizontal component of a vector is the projection of the vector on the horizontal or x-axis. It is given by the equation $x = r \cos \theta$.

n. The radius vector is always positive.

o. The angle θ is determined from the equation $\tan \theta = \frac{y}{x}$.

p. The vertical and horizontal components of any number of vectors are added algebraically to determine the vertical and horizontal components of the resultant.

APPENDIX II

TRIGONOMETRIC GRAPHS AND TABLES

1. Graphical Method for Determining Sine and Cosine Functions of Angles

The graphs (figs. 246 and 247) can be used when the sine or cosine function of any angle between 0° and 360° must be known. As explained in appendix I, the terms *sine* and *cosine* indicate ratios of sides in a right triangle. For example, the term *sine 30°* represents the ratio of the side opposite the 30° angle to the hypotenuse or side opposite the 90° angle. It was explained that this ratio is always the same for any given angle regardless of the size of the right angle. Thus, the ratio or function of sine 30° is always 1/2 or .5. Likewise, the function of any other sine or cosine angle is a constant and can be determined from the graphs.

a. SINE FUNCTIONS. The function of the sine of any angle between 0° and 360° may be found by the following procedure:

(1) Select the angle whose function must be determined and locate this angle on the X-axis of figure 246.

(2) Locate the point on the sine curve which is directly above (or below) the selected angle.

(3) Read the sine function which is shown directly to the left of the point located on the sine curve.

b. EXAMPLES.

(1) To find the function for an angle of 30° (sin 30°), use the procedure given above. That is, first locate the angle of 30° on the X-axis of figure 246. Then follow the vertical line which passes through 30° to the point where the line intersects the sine curve. From this intersection, follow the horizontal line to the left and read the function, or .5.

(2) To find the function of 240° (sin 240°), locate the point where the vertical line from 240° intersects the sine curve. Then read the function shown directly to the left of this intersection, or —.86.

c. COSINE FUNCTIONS. To find a cosine function of any angle between 0° and 360°, a similar method to that described above is used. The procedure is as follows:

(1) Select the angle whose function must be determined and locate this angle on the X-axis of the graph shown in figure 247.

(2) Locate the point on the cosine curve which is directly above (or below) the selected angle.

(3) Read the cosine function which is shown directly to the left of the point located on the cosine curve.

d. EXAMPLES.

(1) To find the cosine function of an angle of 45° (cos 45°), use the procedure given above. That is, locate the angle of 45° on the X-axis of the graph (fig. 247). From this angle, follow the vertical line up to the intersection on the cosine curve and then follow the horizontal line to the left from the intersection to find the function, or .7.

(2) To find the function of cosine 260°, locate the 260° angle on the graph. Then follow the vertical line down to the intersection on the cosine curve. The function can then be read directly to the left of this intersection on the Y-axis, or —.17.

e. SINE AND COSINE ANGLES. The data given above explain how to use the graphs in figures 246 and 247 when determining sine and cosine functions of angles. By reversing the processes used for determining these functions when the angle is known, it is possible to find the angle when the function is known. Thus, to find the angle whose sine or cosine function is known, proceed as follows:

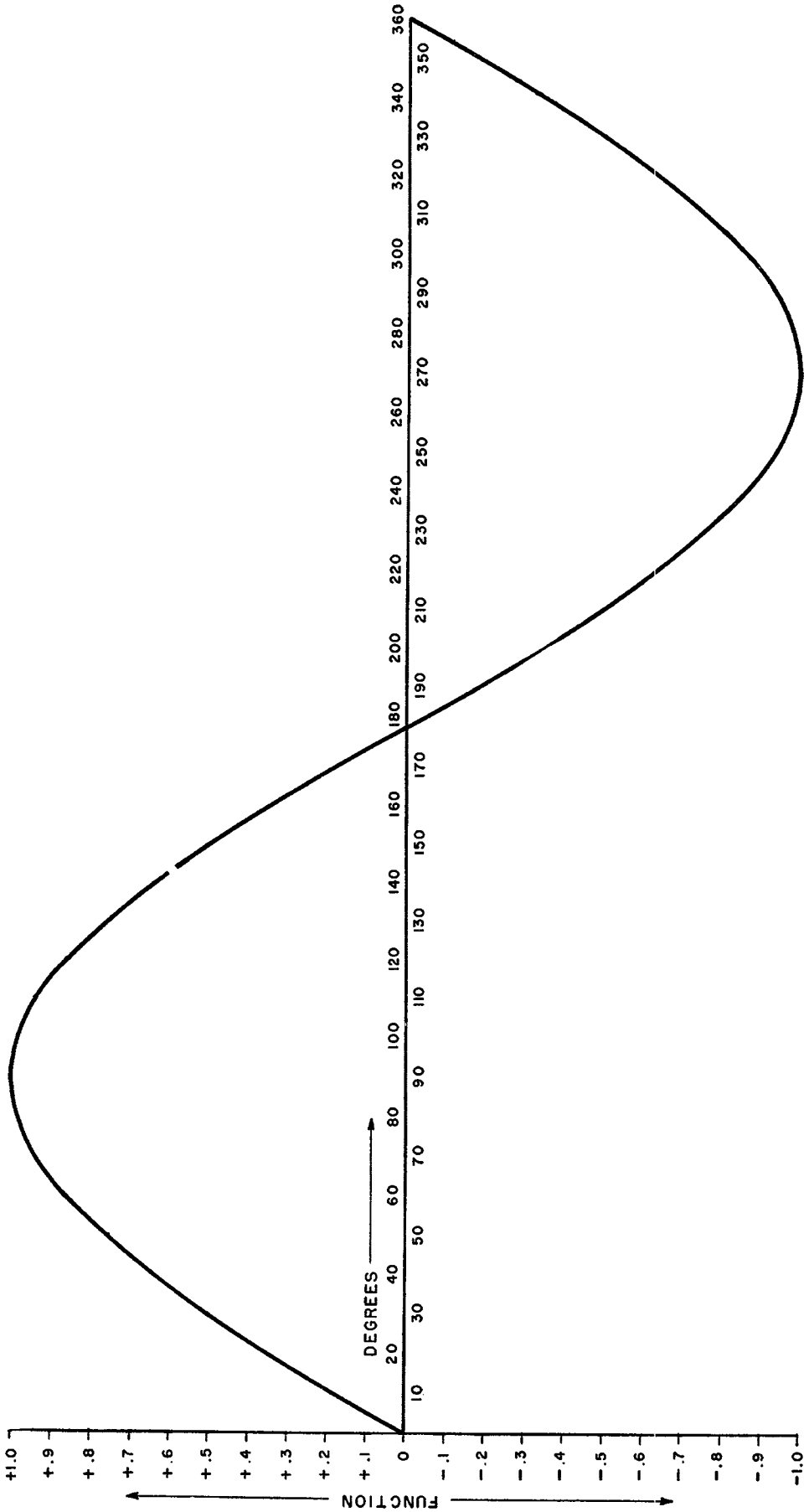


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Figure 246. Graph of sine curve.

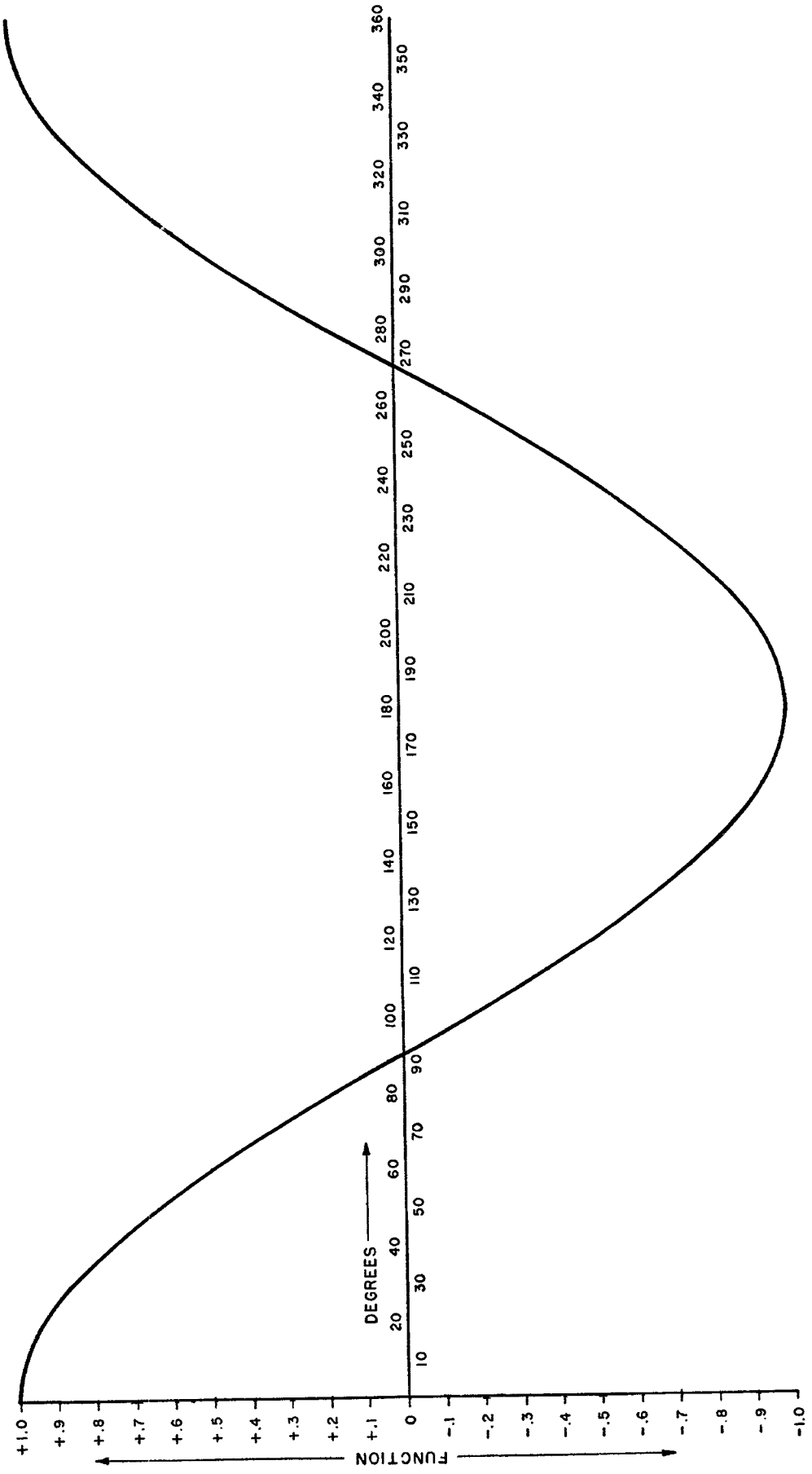

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Figure 247. Graph of cosine curve.

(1) Select the proper graph (sine or cosine graph of fig. 246 or 247).
(2) Locate the known function on the Y-axis of the graph.
(3) Locate the point on the sine or cosine curve which is directly to the right of the known function.
(4) Read the angle which is shown directly above (or below) the located point.

f. EXAMPLES.

(1) To find the angle whose sine function is .5, locate the function .5 on the Y-axis of figure 1. Then follow the horizontal line from .5 to the intersection on the sine curve. The angle can then be read on the X-axis directly below the intersection, or 30°. Notice that the horizontal line from the function, .5, also passes through the sine curve at the point which corresponds to 150°. Thus, the angles whose function is .5 are 30° and 150°. (The selection of the correct angle is usually possible by inspection of the trigonometric problem to be solved.)
(2) Using the procedures explained above, it can be seen that a function of —.5 corresponds to sine angles of 210° and 330°.
(3) To find the angle whose cosine function is .866, first locate the value .866 on the Y-axis of the graph in figure 247. Then locate the point directly to the right of this function at the intersection of the cosine curve. Finally, read the angle shown directly below the intersection, or 30°. Note that the angle of 330° also has the cosine function of .866.

2. How to Use Trigonometric Tables

When a high degree of accuracy is desired in finding functions of an angle, a trigonometric table can be used. Paragraph 3 in this appendix contains tables of natural trigonometric functions for decimal fractions of a degree. The table consists of sine, cosine, tangent, and cotangent functions. The accuracy obtainable by use of these tables is not less than four decimal places for sines and cosines.

a. DESCRIPTION OF TABLES. Each page of the tables is arranged in columns with the names of the functions (sin, cos, tan, and cot) located at the top and bottom of the columns. The angles in degrees and fractions of a degree are located in the degrees columns to the left and right of the function columns.

(1) For angles between 0° and 45°, the angles located to the left of the function columns and the names of the functions located at the top of the columns are used.
(2) For angles between 45° and 90°, the angles located to the right of the function columns and the names of the functions at the bottom of the columns are used.

b. PROCEDURE FOR DETERMINING FUNCTIONS OF ANGLES.

(1) To find the function of any angle between 0° and 45°, use the following procedure:
  (*a*) Find the desired angle in the degrees column located at the left side of the function columns.
  (*b*) Read the sine, cosine, tangent, and cotangent functions in the columns to the right of the desired angle. Use the function names shown at the top of the columns.
(2) To find the function of any angle between 45° and 90°, use the following procedure:
  (*a*) Find the desired angle in the degrees column located at the right side of the function columns.
  (*b*) Read the sine, cosine, tangent, and cotangent functions in the columns to the left of the desired angle. Use the function names shown at the bottom of the columns.

c. EXAMPLES.

(1) To find a function for an angle of 3.1°, locate 3.1 in the degrees column on the left of the function columns on the first page of the tables. To the right of the located angle (3.1), read the functions named at the top of the columns as follows:
  (*a*) Sin = .05408
  (*b*) Cos = .9985
  (*c*) Tan = .05416
  (*d*) Cot = 18.464
(2) To find a function for an angle of 55.8°, locate 55.8 in the degrees column on the right of the function columns on the third page of the tables. To the left of this located angle (55.8°), read the functions named at the bottom of the columns as follows:
  (*a*) Cos = .5621
  (*b*) Sin = .8271

(*c*) Cot = .6796
(*d*) Tan = 1.4715

d. PROCEDURE FOR DETERMINING ANGLES FROM FUNCTIONS. To find any desired angle when its function is known, the procedure described above is reversed. For example, to find the angle whose cosine is .5135, look through the cosine columns of the tables to locate .5135. Then read the angle of 59.1° in the degrees column to the right of the located function (.5135). The degrees column to the right of the function is read because the function appears in the column where the name cosine is located at the bottom of the page. If an angle between 0° and 45° were to be located, it would be found that the function would appear in a column named cosine at the top of the page. In this case, the angle would be read from the degrees column to the left of the function.

e. INTERPOLATION. Angles and functions which are between angles and functions tabulated here can be found by interpolation. In order to get a quick approximate result, we will assume that for small changes in a given angle the changes in the corresponding function are proportional. Then any angle or function can be found by setting up a proportion between the nearest two quantities.

(1) For example:
From the table:

$\sin 33.50° = .5519$
$\sin 33.45 = ?$
$\sin 33.40° = .5505$

From this it can be seen that an increase of .1 of one degree from 33.4° to 33.5° results in an increase of .5519 — .5505 = .0014. Then by proportionality an increase of .05 of a degree results in an increase in the function of .05 × (.5 — .4) × .0014 = .0007

Therefore: $\sin 33.45° = .5505 + .0007 = .5512$

(2) A similar process can be used to interpolate between two angles when the function is known. For example, find the angle whose tan is 1.5352.
First locate the entries which 1.5352 lies between. From the table:

$1.5399 = \tan 57°$
$1.5352 = \tan ?$
$1.5340 = \tan 56.9°$

Since 1.5352 is 12/59 or approximately 1/5 or .2 of the way between 1.5340 and 1.5399, we assume that the angle is .2 of the way from 56.9° to 57.0°.
Therefore:

$1.5352 = \tan (57 - 56.9) \times .2 + 56.9 = \tan 56.92$

3. Natural Trigonometric Functions For Decimal Fractions of a Degree

Deg.	Sin	Cos	Tan	Cot	Deg.	Deg.	Sin	Cos	Tan	Cot	Deg.
0.0	.00000	1.0000	.00000	∞	**90.0**	**6.0**	.10453	0.9945	.10510	9.514	**84.0**
.1	.00175	1.0000	.00175	573.0	.9	.1	.10626	.9943	.10687	9.357	.9
.2	.00349	1.0000	.00349	286.5	.8	.2	.10800	.9942	.10863	9.205	.8
.3	.00524	1.0000	.00524	191.0	.7	.3	.10973	.9940	.11040	9.058	.7
.4	.00698	1.0000	.00698	143.24	.6	.4	.11147	.9938	.11217	8.915	.6
.5	.00873	1.0000	.00873	114.59	.5	.5	.11320	.9936	.11394	8.777	.5
.6	.01047	0.9999	.01047	95.49	.4	.6	.11494	.9934	.11570	8.643	.4
.7	.01222	.9999	.01222	81.85	.3	.7	.11667	.9932	.11747	8.513	.3
.8	.01396	.9999	.01396	71.62	.2	.8	.11840	.9930	.11924	8.386	.2
.9	.01571	.9999	.01571	63.66	.1	.9	.12014	.9928	.12101	8.264	.1
1.0	.01745	0.9998	.01746	57.29	**89.0**	**7.0**	.12187	0.9925	.12278	8.144	**83.0**
.1	.01920	.9998	.01920	52.08	.9	.1	.12360	.9923	.12456	8.028	.9
.2	.02094	.9998	.02095	47.74	.8	.2	.12533	.9921	.12633	7.916	.8
.3	.02269	.9997	.02269	44.07	.7	.3	.12706	.9919	.12810	7.806	.7
.4	.02443	.9997	.02444	40.92	.6	.4	.12880	.9917	.12988	7.700	.6
.5	.02618	.9997	.02619	38.19	.5	.5	.13053	.9914	.13165	7.596	.5
.6	.02792	.9996	.02793	35.80	.4	.6	.13226	.9912	.13343	7.495	.4
.7	.02967	.9996	.02968	33.69	.3	.7	.13399	.9910	.13521	7.396	.3
.8	.03141	.9995	.03143	31.82	.2	.8	.13572	.9907	.13698	7.300	.2
.9	.03316	.9995	.03317	30.14	.1	.9	.13744	.9905	.13876	7.207	.1
2.0	.03490	0.9994	.03492	28.64	**88.0**	**8.0**	.13917	0.9903	.14054	7.115	**82.0**
.1	.03664	.9993	.03667	27.27	.9	.1	.14090	.9900	.14232	7.026	.9
.2	.03839	.9993	.03842	26.03	.8	.2	.14263	.9898	.14410	6.940	.8
.3	.04013	.9992	.04016	24.90	.7	.3	.14436	.9895	.14588	6.855	.7
.4	.04188	.9991	.04191	23.86	.6	.4	.14608	.9893	.14767	6.772	.6
.5	.04362	.9990	.04366	22.90	.5	.5	.14781	.9890	.14945	6.691	.5
.6	.04536	.9990	.04541	22.02	.4	.6	.14954	.9888	.15124	6.612	.4
.7	.04711	.9989	.04716	21.20	.3	.7	.15126	.9885	.15302	6.535	.3
.8	.04885	.9988	.04891	20.45	.2	.8	.15299	.9882	.15481	6.460	.2
.9	.05059	.9987	.05066	19.74	.1	.9	.15471	.9880	.15660	6.386	.1
3.0	.05234	0.9986	.05241	19.081	**87.0**	**9.0**	.15643	0.9877	.15838	6.314	**81.0**
.1	.05408	.9985	.05416	18.464	.9	.1	.15816	.9874	.16017	6.243	.9
.2	.05582	.9984	.05591	17.886	.8	.2	.15988	.9871	.16196	6.174	.8
.3	.05756	.9983	.05766	17.343	.7	.3	.16160	.9869	.16376	6.107	.7
.4	.05931	.9982	.05941	16.832	.6	.4	.16333	.9866	.16555	6.041	.6
.5	.06105	.9981	.06116	16.350	.5	.5	.16505	.9863	.16734	5.976	.5
.6	.06279	.9980	.06291	15.895	.4	.6	.16677	.9860	.16914	5.912	.4
.7	.06453	.9979	.06467	15.464	.3	.7	.16849	.9857	.17093	5.850	.3
.8	.06627	.9978	.06642	15.056	.2	.8	.17021	.9854	.17273	5.789	.2
.9	.06802	.9977	.06817	14.669	.1	.9	.17193	.9851	.17453	5.730	.1
4.0	.06976	0.9976	.06993	14.301	**86.0**	**10.0**	.1736	0.9848	.1763	5.671	**80.0**
.1	.07150	.9974	.07168	13.951	.9	.1	.1754	.9845	.1781	5.614	.9
.2	.07324	.9973	.07344	13.617	.8	.2	.1771	.9842	.1799	5.558	.8
.3	.07498	.9972	.07519	13.300	.7	.3	.1788	.9839	.1817	5.503	.7
.4	.07672	.9971	.07695	12.996	.6	.4	.1805	.9836	.1835	5.449	.6
.5	.07846	.9969	.07870	12.706	.5	.5	.1822	.9833	.1853	5.396	.5
.6	.08020	.9968	.08046	12.429	.4	.6	.1840	.9829	.1871	5.343	.4
.7	.08194	.9966	.08221	12.163	.3	.7	.1857	.9826	.1890	5.292	.3
.8	.08368	.9965	.08397	11.909	.2	.8	.1874	.9823	.1908	5.242	.2
.9	.08542	.9963	.08573	11.664	1	.9	.1891	.9820	.1926	5.193	.1
5.0	.08716	0.9962	.08749	11.430	**85.0**	**11.0**	.1908	0.9816	.1944	5.145	**79.0**
.1	.08889	.9960	.08925	11.205	.9	.1	.1925	.9813	.1962	5.097	.9
.2	.09063	.9959	.09101	10.988	.8	.2	.1942	.9810	.1980	5.050	.8
.3	.09237	.9957	.09277	10.780	.7	.3	.1959	.9806	.1998	5.005	.7
.4	.09411	.9956	.09453	10.579	.6	.4	.1977	.9803	.2016	4.959	.6
.5	.09585	.9954	.09629	10.385	.5	.5	.1994	.9799	.2035	4.915	.5
.6	.09758	.9952	.09805	10.199	.4	.6	.2011	.9796	.2053	4.872	.4
.7	.09932	.9951	.09981	10.019	.3	.7	.2028	.9792	.2071	4.829	.3
.8	.10106	.9949	.10158	9.845	.2	.8	.2045	.9789	.2089	4.787	.2
.9	.10279	.9947	.10334	9.677	.1	.9	.2062	.9785	.2107	4.745	.1
6.0	.10453	0.9945	.10510	9.514	**84.0**	**12.0**	.2079	0.9781	.2126	4.705	**78.0**
Deg.	Cos	Sin	Cot	Tan	Deg.	Deg.	Cos	Sin	Cot	Tan	Deg.

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Deg.	Sin	Cos	Tan	Cot	Deg.	Deg.	Sin	Cos	Tan	Cot	Deg.
12.0	0.2079	0.9781	0.2126	4.705	**78.0**	**18.0**	0.3090	0.9511	0.3249	3.078	**72.0**
.1	.2096	.9778	.2144	4.665	.9	.1	.3107	.9505	.3269	3.060	.9
.2	.2113	.9774	.2162	4.625	.8	.2	.3123	.9500	.3288	3.042	.8
.3	.2130	.9770	.2180	4.586	.7	.3	.3140	.9494	.3307	3.024	.7
.4	.2147	.9767	.2199	4.548	.6	.4	.3156	.9489	.3327	3.006	.6
.5	.2164	.9763	.2217	4.511	.5	.5	.3173	.9483	.3346	2.989	.5
.6	.2181	.9759	.2235	4.474	.4	.6	.3190	.9478	.3365	2.971	.4
.7	.2198	.9755	.2254	4.437	.3	.7	.3206	.9472	.3385	2.954	.3
.8	.2215	.9751	.2272	4.402	.2	.8	.3223	.9466	.3404	2.937	.2
.9	.2233	.9748	.2290	4.366	.1	.9	.3239	.9461	.3424	2.921	.1
13.0	0.2250	0.9744	0.2309	4.331	**77.0**	**19.0**	0.3256	0.9455	0.3443	2.904	**71.0**
.1	.2267	.9740	.2327	4.297	.9	.1	.3272	.9449	.3463	2.888	.9
.2	.2284	.9736	.2345	4.264	.8	.2	.3289	.9444	.3482	2.872	.8
.3	.2300	.9732	.2364	4.230	.7	.3	.3305	.9438	.3502	2.856	.7
.4	.2317	.9728	.2382	4.198	.6	.4	.3322	.9432	.3522	2.840	.6
.5	.2334	.9724	.2401	4.165	.5	.5	.3338	.9426	.3541	2.824	.5
.6	.2351	.9720	.2419	4.134	.4	.6	.3355	.9421	.3561	2.808	.4
.7	.2368	.9715	.2438	4.102	.3	.7	.3371	.9415	.3581	2.793	.3
.8	.2385	.9711	.2456	4.071	.2	.8	.3387	.9409	.3600	2.778	.2
.9	.2402	.9707	.2475	4.041	.1	.9	.3404	.9403	.3620	2.762	.1
14.0	0.2419	0.9703	0.2493	4.011	**76.0**	**20.0**	0.3420	0.9397	0.3640	2.747	**70.0**
.1	.2436	.9699	.2512	3.981	.9	.1	.3437	.9391	.3659	2.733	.9
.2	.2453	.9694	.2530	3.952	.8	.2	.3453	.9385	.3679	2.718	.8
.3	.2470	.9690	.2549	3.923	.7	.3	.3469	.9379	.3699	2.703	.7
.4	.2487	.9686	.2568	3.895	.6	.4	.3486	.9373	.3719	2.689	.6
.5	.2504	.9681	.2586	3.867	.5	.5	.3502	.9367	.3739	2.675	.5
.6	.2521	.9677	.2605	3.839	.4	.6	.3518	.9361	.3759	2.660	.4
.7	.2538	.9673	.2623	3.812	.3	.7	.3535	.9354	.3779	2.646	.3
.8	.2554	.9668	.2642	3.785	.2	.8	.3551	.9348	.3799	2.633	.2
.9	.2571	.9664	.2661	3.758	.1	.9	.3567	.9342	.3819	2.619	.1
15.0	0.2588	0.9659	0.2679	3.732	**75.0**	**21.0**	0.3584	0.9336	0.3839	2.605	**69.0**
.1	.2605	.9655	.2698	3.706	.9	.1	.3600	.9330	.3859	2.592	.9
.2	.2622	.9650	.2717	3.681	.8	.2	.3616	.9323	.3879	2.578	.8
.3	.2639	.9646	.2736	3.655	.7	.3	.3633	.9317	.3899	2.565	.7
.4	.2656	.9641	.2754	3.630	.6	.4	.3649	.9311	.3919	2.552	.6
.5	.2672	.9636	.2773	3.606	.5	.5	.3665	.9304	.3939	2.539	.5
.6	.2689	.9632	.2792	3.582	.4	.6	.3681	.9298	.3959	2.526	.4
.7	.2706	.9627	.2811	3.558	.3	.7	.3697	.9291	.3979	2.513	.3
.8	.2723	.9622	.2830	3.534	.2	.8	.3714	.9285	.4000	2.500	.2
.9	.2740	.9617	.2849	3.511	.1	.9	.3730	.9278	.4020	2.488	.1
16.0	0.2756	0.9613	0.2867	3.487	**74.0**	**22.0**	0.3746	0.9272	0.4040	2.475	**68.0**
.1	.2773	.9608	.2886	3.465	.9	.1	.3762	.9265	.4061	2.463	.9
.2	.2790	.9603	.2905	3.442	.8	.2	.3778	.9259	.4081	2.450	.8
.3	.2807	.9598	.2924	3.420	.7	.3	.3795	.9252	.4101	2.438	.7
.4	.2823	.9593	.2943	3.398	.6	.4	.3811	.9245	.4122	2.426	.6
.5	.2840	.9588	.2962	3.376	.5	.5	.3827	.9239	.4142	2.414	.5
.6	.2857	.9583	.2981	3.354	.4	.6	.3843	.9232	.4163	2.402	.4
.7	.2874	.9578	.3000	3.333	.3	.7	.3859	.9225	.4183	2.391	.3
.8	.2890	.9573	.3019	3.312	.2	.8	.3875	.9219	.4204	2.379	.2
.9	.2907	.9568	.3038	3.291	.1	.9	.3891	.9212	.4224	2.367	.1
17.0	0.2924	0.9563	0.3057	3.271	**73.0**	**23.0**	0.3907	0.9205	0.4245	2.356	**67.0**
.1	.2940	.9558	.3076	3.251	.9	.1	.3923	.9198	.4265	2.344	.9
.2	.2957	.9553	.3096	3.230	.8	.2	.3939	.9191	.4286	2.333	.8
.3	.2974	.9548	.3115	3.211	.7	.3	.3955	.9184	.4307	2.322	.7
.4	.2990	.9542	.3134	3.191	.6	.4	.3971	.9178	.4327	2.311	.6
.5	.3007	.9537	.3153	3.172	.5	.5	.3987	.9171	.4348	2.300	.5
.6	.3024	.9532	.3172	3.152	.4	.6	.4003	.9164	.4369	2.289	.4
.7	.3040	.9527	.3191	3.133	.3	.7	.4019	.9157	.4390	2.278	.3
.8	.3057	.9521	.3211	3.115	.2	.8	.4035	.9150	.4411	2.267	.2
.9	.3074	.9516	.3230	3.096	.1	.9	.4051	.9143	.4431	2.257	.1
18.0	0.3090	0.9511	0.3249	3.078	**72.0**	**24.0**	0.4067	0.9135	0.4452	2.246	**66.0**
Deg.	Cos	Sin	Cot	Tan	Deg.	Deg.	Cos	Sin	Cot	Tan	Deg.

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Deg.	Sin	Cos	Tan	Cot	Deg.	Deg.	Sin	Cos	Tan	Cot	Deg.
24.0	0.4067	0.9135	0.4452	2.246	**66.0**	**30.0**	0.5000	0.8660	0.5774	1.7321	**60.0**
.1	.4083	.9128	.4473	2.236	.9	.1	.5015	.8652	.5797	1.7251	.9
.2	.4099	.9121	.4494	2.225	.8	.2	.5030	.8643	.5820	1.7182	.8
.3	.4115	.9114	.4515	2.215	.7	.3	.5045	.8634	.5844	1.7113	.7
.4	.4131	.9107	.4536	2.204	.6	.4	.5060	.8625	.5867	1.7045	.6
.5	.4147	.9100	.4557	2.194	.5	.5	.5075	.8616	.5890	1.6977	.5
.6	.4163	.9092	.4578	2.184	.4	.6	.5090	.8607	.5914	1.6909	.4
.7	.4179	.9085	.4599	2.174	.3	.7	.5105	.8599	.5938	1.6842	.3
.8	.4195	.9078	.4621	2.164	.2	.8	.5120	.8590	.5961	1.6775	.2
.9	.4210	.9070	.4642	2.154	.1	.9	.5135	.8581	.5985	1.6709	.1
25.0	0.4226	0.9063	0.4663	2.145	**65.0**	**31.0**	0.5150	0.8572	0.6009	1.6643	**59.0**
.1	.4242	.9056	.4684	2.135	.9	.1	.5165	.8563	.6032	1.6577	.9
.2	.4258	.9048	.4706	2.125	.8	.2	.5180	.8554	.6056	1.6512	.8
.3	.4274	.9041	.4727	2.116	.7	.3	.5195	.8545	.6080	1.6447	.7
.4	.4289	.9033	.4748	2.106	.6	.4	.5210	.8536	.6104	1.6383	.6
.5	.4305	.9026	.4770	2.097	.5	.5	.5225	.8526	.6128	1.6319	.5
.6	.4321	.9018	.4791	2.087	.4	.6	.5240	.8517	.6152	1.6255	.4
.7	.4337	.9011	.4813	2.078	.3	.7	.5255	.8508	.6176	1.6191	.3
.8	.4352	.9003	.4834	2.069	.2	.8	.5270	.8499	.6200	1.6128	.2
.9	.4368	.8996	.4856	2.059	.1	.9	.5284	.8490	.6224	1.6066	.1
26.0	0.4384	0.8988	0.4877	2.050	**64.0**	**32.0**	0.5299	0.8480	0.6249	1.6003	**58.0**
.1	.4399	.8980	.4899	2.041	.9	.1	.5314	.8471	.6273	1.5941	.9
.2	.4415	.8973	.4921	2.032	.8	.2	.5329	.8462	.6297	1.5880	.8
.3	.4431	.8965	.4942	2.023	.7	.3	.5344	.8453	.6322	1.5818	.7
.4	.4446	.8957	.4964	2.014	.6	.4	.5358	.8443	.6346	1.5757	.6
.5	.4462	.8949	.4986	2.006	.5	.5	.5373	.8434	.6371	1.5697	.5
.6	.4478	.8942	.5008	1.997	.4	.6	.5388	.8425	.6395	1.5637	.4
.7	.4493	.8934	.5029	1.988	.3	.7	.5402	.8415	.6420	1.5577	.3
.8	.4509	.8926	.5051	1.980	.2	.8	.5417	.8406	.6445	1.5517	.2
.9	.4524	.8918	.5073	1.971	.1	.9	.5432	.8396	.6469	1.5458	.1
27.0	0.4540	0.8910	0.5095	1.963	**63.0**	**33.0**	0.5446	0.8387	0.6494	1.5399	**57.0**
.1	.4555	.8902	.5117	1.954	.9	.1	.5461	.8377	.6519	1.5340	.9
.2	.4571	.8894	.5139	1.946	.8	.2	.5476	.8368	.6544	1.5282	.8
.3	.4586	.8886	.5161	1.937	.7	.3	.5490	.8358	.6569	1.5224	.7
.4	.4602	.8878	.5184	1.929	.6	.4	.5505	.8348	.6594	1.5166	.6
.5	.4617	.8870	.5206	1.921	.5	.5	.5519	.8339	.6619	1.5108	.5
.6	.4633	.8862	.5228	1.913	.4	.6	.5534	.8329	.6644	1.5051	.4
.7	.4648	.8854	.5250	1.905	.3	.7	.5548	.8320	.6669	1.4994	.3
.8	.4664	.8846	.5272	1.897	.2	.8	.5563	.8310	.6694	1.4938	.2
.9	.4679	.8838	.5295	1.889	.1	.9	.5577	.8300	.6720	1.4882	.1
28.0	0.4695	0.8829	0.5317	1.881	**62.0**	**34.0**	0.5592	0.8290	0.6745	1.4826	**56.0**
.1	.4710	.8821	.5340	1.873	.9	.1	.5606	.8281	.6771	1.4770	.9
.2	.4726	.8813	.5362	1.865	.8	.2	.5621	.8271	.6796	1.4715	.8
.3	.4741	.8805	.5384	1.857	.7	.3	.5635	.8261	.6822	1.4659	.7
.4	.4756	.8796	.5407	1.849	.6	.4	.5650	.8251	.6847	1.4605	.6
.5	.4772	.8788	.5430	1.842	.5	.5	.5664	.8241	.6873	1.4550	.5
.6	.4787	.8780	.5452	1.834	.4	.6	.5678	.8231	.6899	1.4496	.4
.7	.4802	.8771	.5475	1.827	.3	.7	.5693	.8221	.6924	1.4442	.3
.8	.4818	.8763	.5498	1.819	.2	.8	.5707	.8211	.6950	1.4388	.2
.9	.4833	.8755	.5520	1.811	.1	.9	.5721	.8202	.6976	1.4335	.1
29.0	0.4848	0.8746	0.5543	1.804	**61.0**	**35.0**	0.5736	0.8192	0.7002	1.4281	**55.0**
.1	.4863	.8738	.5566	1.797	.9	.1	.5750	.8181	.7028	1.4229	.9
.2	.4879	.8729	.5589	1.789	.8	.2	.5764	.8171	.7054	1.4176	.8
.3	.4894	.8721	.5612	1.782	.7	.3	.5779	.8161	.7080	1.4124	.7
.4	.4909	.8712	.5635	1.775	.6	.4	.5793	.8151	.7107	1.4071	.6
.5	.4924	.8704	.5658	1.767	.5	.5	.5807	.8141	.7133	1.4019	.5
.6	.4939	.8695	.5681	1.760	.4	.6	.5821	.8131	.7159	1.3968	.4
.7	.4955	.8686	.5704	1.753	.3	.7	.5835	.8121	.7186	1.3916	.3
.8	.4970	.8678	.5727	1.746	.2	.8	.5850	.8111	.7212	1.3865	.2
.9	.4985	.8669	.5750	1.739	.1	.9	.5864	.8100	.7239	1.3814	.1
30.0	0.5000	0.8660	0.5774	1.732	**60.0**	**36.0**	0.5878	0.8090	0.7265	1.3764	**54.0**
Deg.	Cos	Sin	Cot	Tan	Deg.	Deg.	Cos	Sin	Cot	Tan	Deg.

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Deg.	Sin	Cos	Tan	Cot	Deg.
36.0	0.5878	0.8090	0.7265	1.3764	**54.0**
.1	.5892	.8080	.7292	1.3713	.9
.2	.5906	.8070	.7319	1.3663	.8
.3	.5920	.8059	.7346	1.3613	.7
.4	.5934	.8049	.7373	1.3564	.6
.5	.5948	.8039	.7400	1.3514	.5
.6	.5962	.8028	.7427	1.3465	.4
.7	.5976	.8018	.7454	1.3416	.3
.8	.5990	.8007	.7481	1.3367	.2
.9	.6004	.7997	.7508	1.3319	.1
37.0	0.6018	0.7986	0.7536	1.3270	**53.0**
.1	.6032	.7976	.7563	1.3222	.9
.2	.6046	.7965	.7590	1.3175	.8
.3	.6060	.7955	.7618	1.3127	.7
.4	.6074	.7944	.7646	1.3079	.6
.5	.6088	.7934	.7673	1.3032	.5
.6	.6101	.7923	.7701	1.2985	.4
.7	.6115	.7912	.7729	1.2938	.3
.8	.6129	.7902	.7757	1.2892	.2
.9	.6143	.7891	.7785	1.2846	.1
38.0	0.6157	0.7880	0.7813	1.2799	**52.0**
.1	.6170	.7869	.7841	1.2753	.9
.2	.6184	.7859	.7869	1.2708	.8
.3	.6198	.7848	.7898	1.2662	.7
.4	.6211	.7837	.7926	1.2617	.6
.5	.6225	.7826	.7954	1.2572	.5
.6	.6239	.7815	.7983	1.2527	.4
.7	.6252	.7804	.8012	1.2482	.3
.8	.6266	.7793	.8040	1.2437	.2
.9	.6280	.7782	.8069	1.2393	.1
39.0	0.6293	0.7771	0.8098	1.2349	**51.0**
.1	.6307	.7760	.8127	1.2305	.9
.2	.6320	.7749	.8156	1.2261	.8
.3	.6334	.7738	.8185	1.2218	.7
.4	.6347	.7727	.8214	1.2174	.6
.5	.6361	.7716	.8243	1.2131	.5
.6	.6374	.7705	.8273	1.2088	.4
.7	.6388	.7694	.8302	1.2045	.3
.8	.6401	.7683	.8332	1.2002	.2
.9	.6414	.7672	.8361	1.1960	.1
40.0	0.6428	0.7660	0.8391	1.1918	**50.0**
.1	.6441	.7649	.8421	1.1875	.9
.2	.6455	.7638	.8451	1.1833	.8
.3	.6468	.7627	.8481	1.1792	.7
.4	.6481	.7615	.8511	1.1750	.6
40.5	0.6494	0.7604	0.8541	1.1708	**49.5**
Deg.	Cos	Sin	Cot	Tan	Deg.

Deg.	Sin	Cos	Tan	Cot	Deg.
40.5	0.6494	0.7604	0.8541	1.1708	**49.5**
.6	.6508	.7593	.8571	1.1667	.4
.7	.6521	.7581	.8601	1.1626	.3
.8	.6534	.7570	.8632	1.1585	.2
.9	.6547	.7559	.8662	1.1544	.1
41.0	0.6561	0.7547	0.8693	1.1504	**49.0**
.1	.6574	.7536	.8724	1.1463	.9
.2	.6587	.7524	.8754	1.1423	.8
.3	.6600	.7513	.8785	1.1383	.7
.4	.6613	.7501	.8816	1.1343	.6
.5	.6626	.7490	.8847	1.1303	.5
.6	.6639	.7478	.8878	1.1263	.4
.7	.6652	.7466	.8910	1.1224	.3
.8	.6665	.7455	.8941	1.1184	.2
.9	.6678	.7443	.8972	1.1145	.1
42.0	0.6691	0.7431	0.9004	1.1106	**48.0**
.1	.6704	.7420	.9036	1.1067	.9
.2	.6717	.7408	.9067	1.1028	.8
.3	.6730	.7396	.9099	1.0990	.7
.4	.6743	.7385	.9131	1.0951	.6
.5	.6756	.7373	.9163	1.0913	.5
.6	.6769	.7361	.9195	1.0875	.4
.7	.6782	.7349	.9228	1.0837	.3
.8	.6794	.7337	.9260	1.0799	.2
.9	.6807	.7325	.9293	1.0761	.1
43.0	0.6820	0.7314	0.9325	1.0724	**47.0**
.1	.6833	.7302	.9358	1.0686	.9
.2	.6845	.7290	.9391	1.0649	.8
.3	.6858	.7278	.9424	1.0612	.7
.4	.6871	.7266	.9457	1.0575	.6
.5	.6884	.7254	.9490	1.0538	.5
.6	.6896	.7242	.9523	1.0501	.4
.7	.6909	.7230	.9556	1.0464	.3
.8	.6921	.7218	.9590	1.0428	.2
.9	.6934	.7206	.9623	1.0392	.1
44.0	0.6947	0.7193	0.9657	1.0355	**46.0**
.1	.6959	7181	.9691	1.0319	.9
.2	.6972	.7169	.9725	1.0283	.8
.3	.6984	.7157	.9759	1.0247	.7
.4	.6997	.7145	.9793	1.0212	.6
.5	.7009	.7133	.9827	1.0176	.5
.6	.7022	.7120	.9861	1.0141	.4
.7	.7034	.7108	.9896	1.0105	.3
.8	.7046	.7096	.9930	1.0070	.2
.9	.7059	.7083	.9965	1 0035	.1
45.0	0.7071	0.7071	1.0000	1 0000	**45.0**
Deg.	Cos	Sin	Cot	Tan	Deg.

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APPENDIX III

CAPACITOR COLOR AND MARKING CODES

1. Capacitor Codes

In the following analysis of capacitor codes, the most common types of paper, mica, and ceramic capacitors are described and illustrated. The two color codes analyzed are the JAN (Joint Army-Navy) and the RMA (Radio Manufacturers Association). These codes are used in the majority of capacitors manufactured. The JAN code supersedes all other codes in the Armed Forces, but if a particular capacitor is defective and a JAN-coded replacement capacitor is not available, an RMA-coded capacitor can be used, provided the proper precaution is taken with relation to capacity, voltage rating, and tolerance.

2. JAN Identification

a. The JAN code uses two methods of type identification:

(1) Coded colors both in bands and in dots.
(2) Letter markings (type-designation).

b. Under the JAN color-coding system, the first color is important in identifying the difference between paper molded and mica molded capacitors. Figure 248 shows a typical capacitor. The order of the colors is from left to right on the top row and from right to left on the bottom row, as shown by the direction of the arrows. *Silver* as the first color indicates a *paper* molded type. *Black* as the first color indicates a *mica* molded type. Ceramic capacitors are not indicated by a particular color in this manner, since they are cylindrical in form and easily recognized by physical shape.

c. Under the JAN letter and number identification system, the first two letters give the type of capacitor. Thus:

(1) CM = mica-molded.
(2) CN = paper-molded.
(3) CC = ceramic.

Examples of these markings are shown in paragraph 5 below.

3. JAN and RMA Codes

a. In addition to the type-identification shown above, information concerning capacity, voltage rating, tolerance, and characteristic is carried on the face of the capacitor in color codes and in letter and number markings. Because of the difficulty generally encountered in reading these codes, the following condensed tables have been prepared to make this information more readily available. Such condensed tables are made possible by the fact that *all* domestic manufacturers have agreed to the present *color* coding (capacitance in *micro-microfarads*).

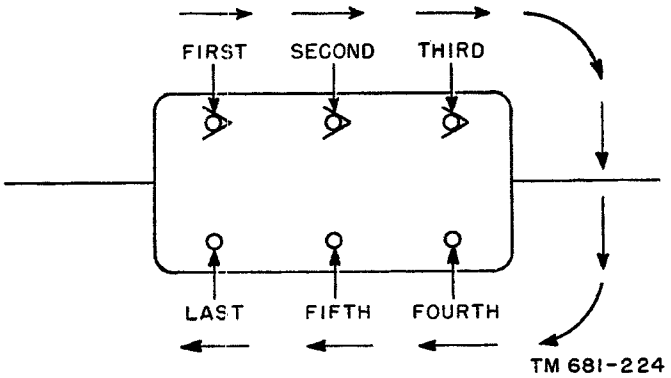


Figure 248. JAN molded capacitor sequence.

b. Table I includes both the common color-code analysis and the JAN type-designations, as seen under the JAN tolerance and characteristic columns. In analyzing an RMA-coded capacitor, the JAN tolerance and characteristic columns are disregarded. Table II refers only to ceramic capacitors, both JAN and RMA, but, as noted above, the JAN tolerance and characteristic columns are disregarded except for JAN capacitors. For example, a JAN mica capacitor showing G as a type-designation has a tolerance of ±2 percent (table I); a JAN ceramic capacitor showing G as a type-designation has a tolerance of ±2 percent if the capacity is greater than 10 $\mu\mu$f, or a

tolerance of ±2 μμf if the capacity is less than 10 μμf (table II). It should be noted, however, that detailed analysis of the characteristics of a capacitor is not necessary unless demanded by the circuit requirements.

4. Examples of JAN Color Codes

a. JAN PAPER CAPACITOR.

(1) Figure 249 shows the face of a JAN molded paper dielectric capacitor with explanations of the color-dot positions.

(2) Figure 250 is the color code as it appears on a typical JAN paper capacitor (molded).

(3) Since the arrows indicate the sequence of colors, the code is read as follows (table I):

(*a*) Silver identifies this capacitor as a paper dielectric capacitor.

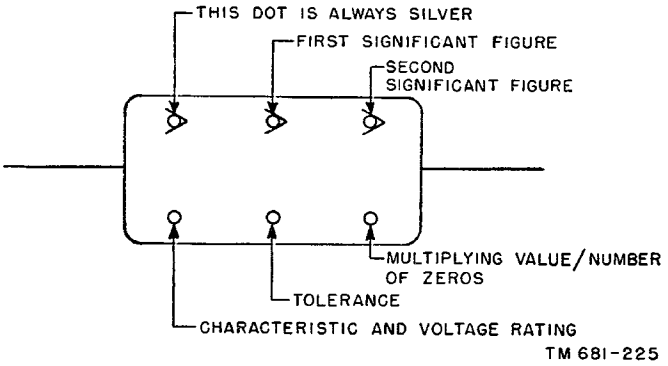


Figure 249. JAN identifications (paper capacitor).

(*b*) Red is the first significant figure (the number 2).

(*c*) Brown is the second significant figure (the number 1).

(*d*) Orange is the third significant figure, the decimal multiplier or number of zeros added (3).

(*e*) Silver is the tolerance (±10%).

(*f*) Red is the characteristic (200 volts).

(4) Therefore, this is a paper dielectric capacitor of 21,000-μμf capacity, rated at 200 working volts, and permitted a tolerance of ±2,100 μμf (10%).

b. JAN MICA CAPACITOR.

(1) Figure 251 shows the face of a JAN molded mica capacitor with explanations of the color-dot positions.

(2) Figure 252 is the color code as it appears on a typical JAN mica capacitor (molded).

(3) This capacitor is read in the same manner as the one above (table I):

(*a*) Black identifies this capacitor as a mica dielectric capacitor.

(*b*) Red is the first significant figure (the number 2).

(*c*) Green is the second significant figure (the number 5).

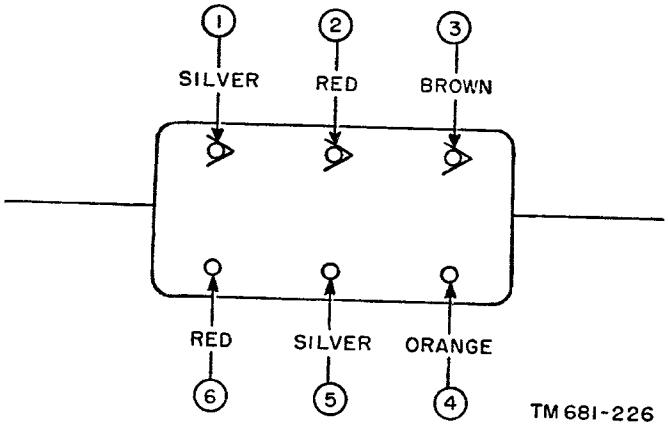


Figure 250. JAN paper dielectric capacitor.

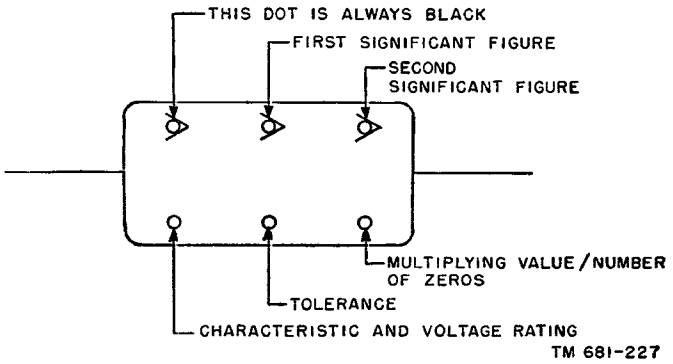


Figure 251. JAN identifications (paper capacitor).

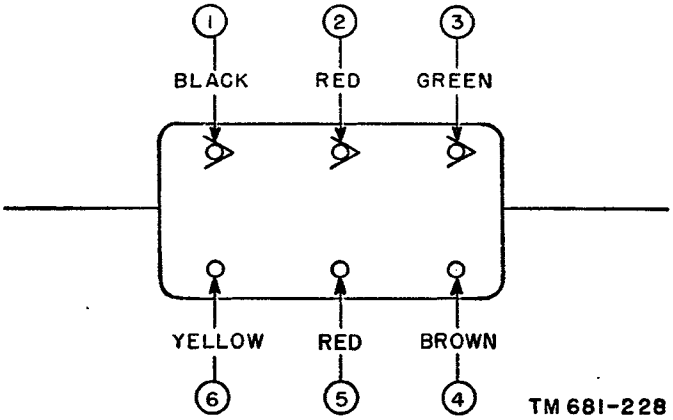


Figure 252. JAN mica dielectric capacitor.

(*d*) Brown is the third significant figure, the decimal multiplier or number of zeros added (1).

(*e*) Red is the tolerance (±2%).

(*f*) Yellow is the characteristic (400 volts).

(4) Consequently, this is a mica dielectric capacitor of 250-$\mu\mu$f capacity, rated at 400 working volts, and permitted a tolerance of ±5 $\mu\mu$f (2%).

c. JAN CERAMIC CAPACITOR.

(1) JAN ceramic capacitors generally are cylindrical in shape and resemble resistors of ¼- ½-, and 1-watt sizes. They may be easily distinguished, however, since resistors carry *four* colors or less; whereas JAN ceramic capacitors carry

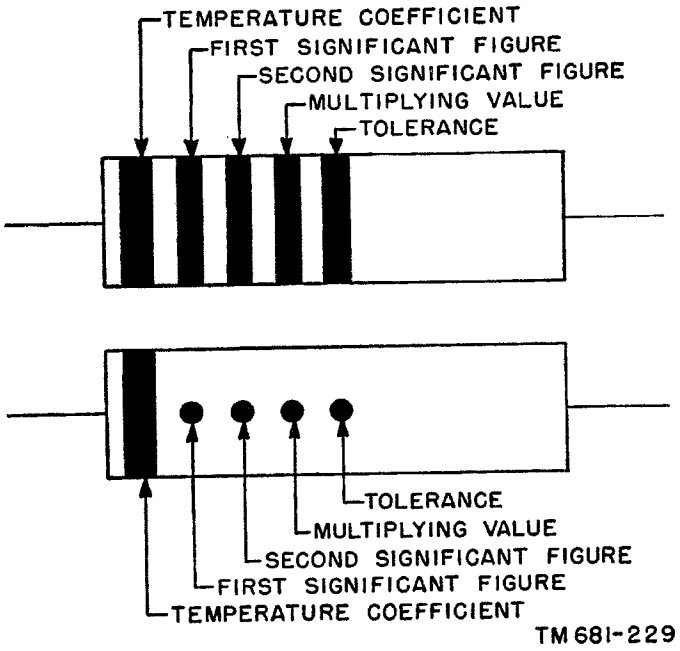


Figure 253. JAN identifications (ceramic capacitor).

five color bands or dots, as shown in figure 253. These capacitors are all rated at 500 working volts unless otherwise specified.

(2) The key to identification of this type is, first, the five bands or dots and, second, the wider first band or bigger first dot, both the temperature coefficient. It should be noted, however, that some RMA ceramic capacitors carry five bands or dots, but the larger band or dot is the last in order and also indicates the temperature coefficient.

(3) Figure 254 is the color code as it appears on a typical JAN ceramic capacitor.

(4) This capacitor is read as follows (table II):

(*a*) Violet is the temperature coefficient and characteristic.

(*b*) Gray is the first significant figure (the number 8).

(*c*) Red is the second significant figure (the number 2).

(*d*) Black is the third significant figure, the decimal multiplier (1) or number of zeros added (none).

(*e*) Green is the tolerance (±5%).

(5) Consequently, this is a ceramic dielectric capacitor of 82 $\mu\mu$f, rated at 500 working volts, and permitted a tolerance of ±4 $\mu\mu$f (5%).

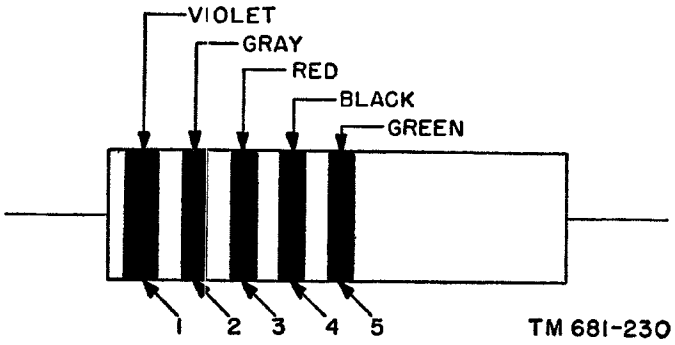


Figure 254. JAN ceramic dielectric capacitor.

5. Examples of JAN Letter and Number Type-designation

a. JAN PAPER CAPACITOR. The letter and number method of marking appears in the following manner on the face of a capacitor:

CN — 20 — H —

↓ component; ↓ case; ↓ characteristic

↓ 2 1 3 ↓ — K

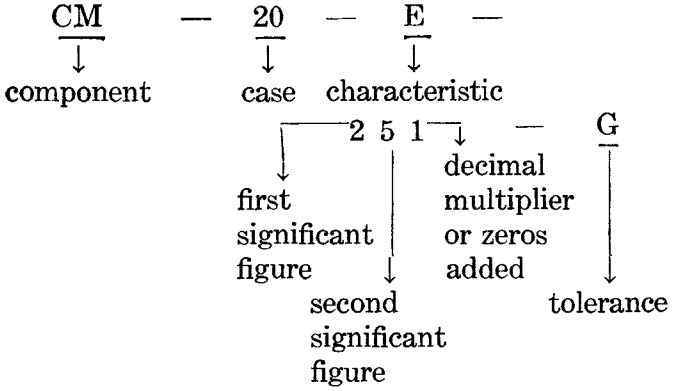
2 1: first significant figure; 3: decimal multiplier or zeros added; K: tolerance

↓ second significant figure

As previously noted, *CN* identifies this component as paper dielectric capacitor; 20 is the style of case; *H* is 200 working volts; 21,000 μf is the capacity; and *K* is the tolerance (±10%). With the exception of case style, which may be ignored, this information is found in table I. Since this capacitor is identical with the color-coded one of

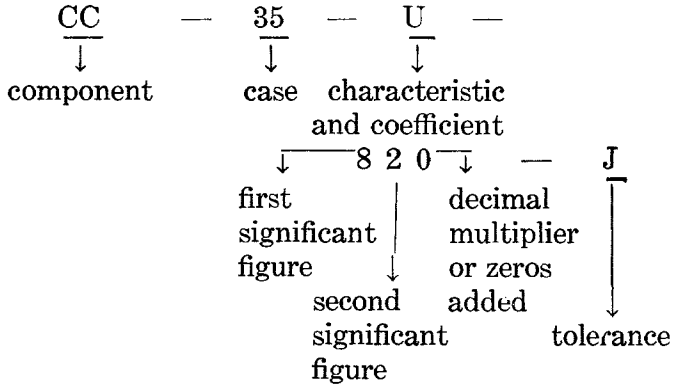
figure 250, the two methods of coding may be compared.

b. JAN MICA CAPACITOR. The JAN mica capacitor is marked as follows:



Then, *CM* identifies this component as a mica dielectric capacitor. From table I, *E* is 400 working volts, 250 $\mu\mu$f is the capacity, and *G* is the tolerance ($\pm 2\%$). This capacitor is identical with the color-coded unit of figure 252.

c. JAN CERAMIC CAPACITOR. The JAN ceramic capacitor is marked as follows:



Then, *CC* identifies this component as a ceramic dielectric capacitor. From table II, *U* is the characteristic, which may be disregarded (as previously noted, ceramic capacitors are rated at 500 working volts, unless otherwise indicated), 82 $\mu\mu$f is the capacity, and *J* is the tolerance ($\pm 5\%$). This capacitor is identical with the color-coded unit of figure 253.

6. Examples of RMA Color Codes

The RMA system of capacitor color coding is basically like the JAN system of color coding; therefore, the color listings of tables I and II may be used and the JAN characteristics columns disregarded. The RMA color coding makes no provision for indicating whether a molded capacitor uses paper or mica as a dielectric; however, the color of the molded plastic case gives this information. Thus, a black case is a paper dielectric capacitor; dark brown or tan indicates a mica dielectric; and red is a silver-mica dielectric (a better-grade mica). These molded case distinctions are equally true of JAN molded types, but the information is carried also in the color system, as detailed above. RMA color-coded capacitors differ from JAN capacitors in that three-dot, four-dot, five-dot, and six-dot (or band) combinations are used, whereas *all* JAN color-coded capacitors show six dots or bands.

a. THREE-DOT CAPACITOR. In A of figure 255, the outline of a three-dot capacitor is shown. Reading toward the arrowhead, the first dot is the first significant figure, the second dot is the second significant figure, and the third dot is the decimal multiplier or number of zeros added. Since there are no other markings, the tolerance is understood to be ± 20 percent and the working voltage 500 volts. Hence, a capacitor showing blue, gray, and brown is a 680-$\mu\mu$f capacitor of ± 136-$\mu\mu$f tolerance and rated at 500 volts.

b. FOUR-DOT CAPACITOR. In figure B of 255, the outline of a four-dot capacitor is shown. The order of the significant figures is the same as that of the three-dot capacitor (toward the arrowhead), and the extra dot indicates the tolerance. Since there is no marking for working voltage, the capacitor is rated at 500 working volts. For example, a capacitor showing red, violet, red, and yellow is a 2,700-$\mu\mu$f capacitor of ± 4-percent tolerance and rated at 500 working volts.

c. FIVE-DOT CAPACITOR. In figure C of 255, three outlines of five-dot capacitors are shown. The two dots on the back of the first capacitor outlined indicate tolerance and working voltage. The tolerance dot is the one near the arrowhead end of the capacitor. The two other capacitors outlined show the usual order of reading of the dots. For example, a capacitor showing green, blue, red, green, and yellow is a 5,600-$\mu\mu$f capacitor of ± 5-percent tolerance and rated at 400 working volts. It should be noted that when six dot positions are available, sometimes the fifth dot position is blank as shown, or the first dot position is left blank and all other positions are shifted.

d. SIX-DOT CAPACITOR. In D of figure 255, the outline of a six-dot capacitor is shown. This type capacitor differs from all others in that *three* significant figures are indicated, followed by the decimal factor, tolerance, and working voltage. The use of three significant figures makes possible the indication of capacities otherwise impossible.

Thus, a 1,550-$\mu\mu$f capacitor is coded as brown, green, green, brown, plus tolerance and working voltage. Under three-, four-, or five-dot systems this capacitance could not be indicated.

generally more than twice the size of a 2-watt resistor and always carry *five color bands*. Most of these capacitors are incased in a black plastic, but some manufacturers use a neutral colored clay

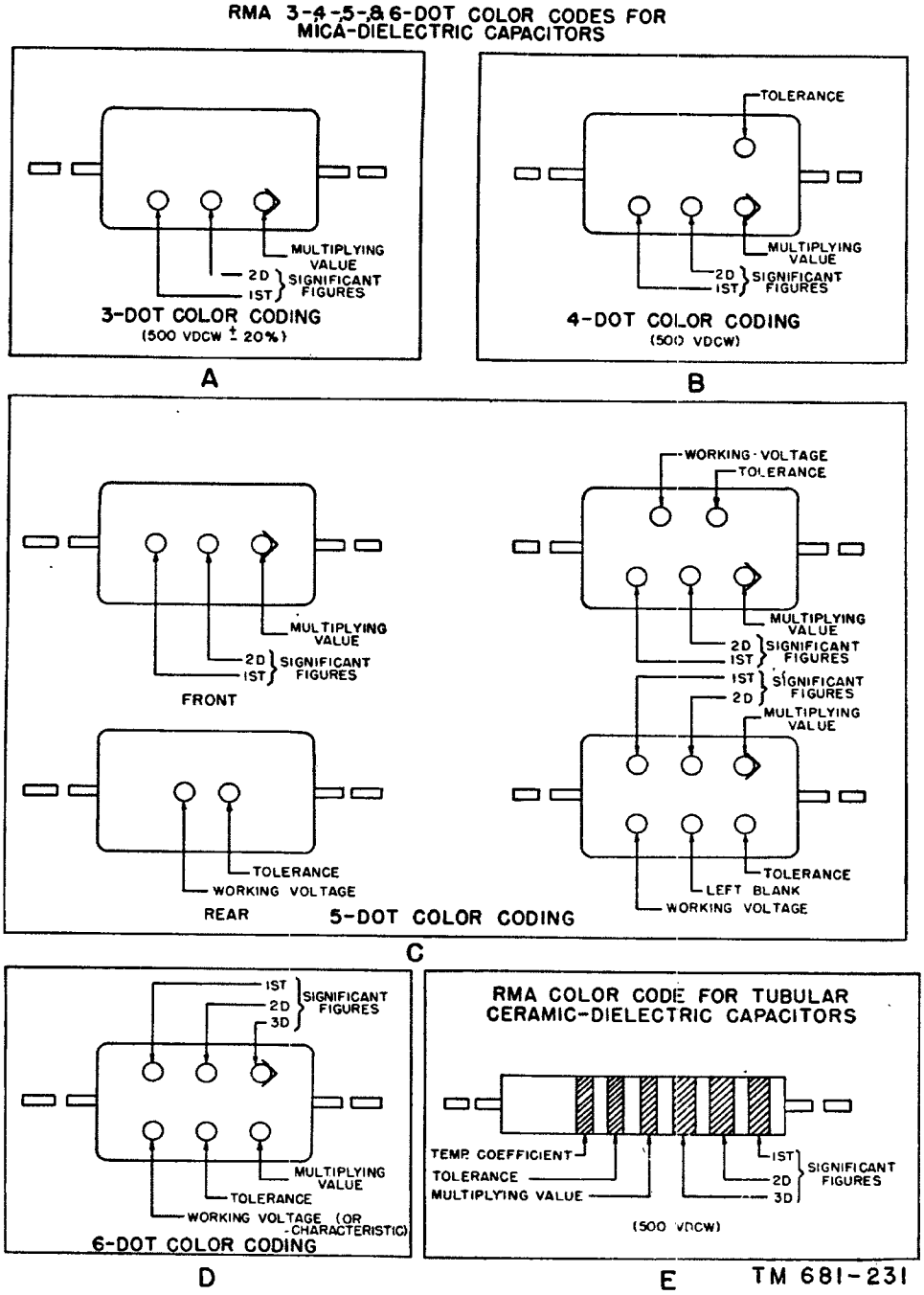


Figure 255. RMA color-coded capacitors.

e. RMA TUBULAR CAPACITOR. The RMA paper capacitors described above are also manufactured in tubular form. Mica capacitor are not made in tubular form. Like both JAN and RMA ceramic capacitors, these tubular paper capacitors may be mistaken for resistors, although they are case. The color bands are read in the same order as those described in the discussion of the conventional RMA five-dot molded capacitor (C of fig. 255).

f. RMA CERAMIC CAPACITOR. In E of figure 255 the outline of the RMA ceramic capacitor may

appear with either four, five, or six dots or bands. If four dots are shown, they are the two significant figures, the decimal multiplier and the tolerance. A five-dot ceramic capacitor includes the temperature coefficient; and a six-dot ceramic includes the *third* significant figure, as explained above. The working voltage in all cases is taken to be 500 volts. Since ceramic capacitors are often quite small, some difficulty may be experienced in discovering the proper order of the colors. One method of solving this predicament is analyses based on reading first in one direction and then in the other. One value will then appear to be highly appropriate and the other quite improbable. Should all attempts at analysis fail for a given capacitor, it is suggested that the component in question be tested on a calibrated capacitor analyser, or thrown away and replaced with one which lends itself more readily to the systems explained above.

Table I. Color-code Chart for JAN and RMA Paper- and Mica-molded Capacitors

Color scheme	Capacitance in Micromicrofarads			Tolerance	Voltage rating	JAN Mica-CM		JAN Paper-CN	
	First Sig. Fig.	Second and third Sig. Fig.	Decimal Multiplier or No. of zeros			JAN tolerance	Characteristic	JAN tolerance	JAN characteristic
Black	0	0	1	±20%	500	M	A	M	A
Brown	1	1	10	±1%	100	----------	B	----------	E
Red	2	2	100	±2%	200	G	C	----------	H
Orange	3	3	1,000	±3%	300	----------	D	N*	J
Yellow	4	4	10,000	±4%	400	----------	E	----------	P
Green	5	5	100,000	±5%	500	----------	F	----------	R
Blue	6	6	1,000,000	±6%	600	----------	G	----------	S
Violet	7	7	10,000,000	±7%	700	----------	----------	----------	T
Gray	8	8	100,000,000	±8%	800	----------	----------	----------	----------
White	9	9	1,000,000,000	±9%	900	----------	----------	----------	----------
Gold	----------	----------	0.1	±5%	1,000	J	----------	----------	----------
Silver	----------	----------	0.01	±10%	2,000	K	----------	K	----------
No Color	----------	----------	----------	±20%	500	----------	----------	----------	----------

* The tolerance of this capacitor is ±30%, not ±3%.

Table II. Color Coding for Ceramic Capacitors (CC), Capacitance in Micromicrofarads

Color	1, 2, or 3 Sig. Fig.	RMA Standards		JAN Standards					
					Tolerances for nominal capacitances greater than 10 μμf		Tolerance for nominal capacitances of 10 μμf or smaller		
		Multiplier	Tolerance	Multiplier	Percent	Designation	μμF	Designation	Characteristic
Black	0	1	----------	1	±20%	M	±2.0	G	C
Brown	1	10	±1%	10	±1%	F	----------	----------	H
Red	2	100	±2%	100	±2%	G	----------	----------	L
Orange	3	1,000	±3%	1,000	----------	----------	----------	----------	P
Yellow	4	10,000	±4%	----------	----------	----------	----------	----------	R
Green	5	----------	±5%	----------	±5%	J	±0.5	D	S
Blue	6	----------	±6%	----------	----------	----------	----------	----------	T
Violet	7	----------	±7%	----------	----------	----------	----------	----------	U
Gray	8	----------	±8%	0.01	----------	----------	±0.25	C	B
White	9	----------	±9%	0.1	±10%	K	±1.0	F	SL
Gold	----------	0.1	±5%	----------	----------	----------	----------	----------	A
Silver	----------	0.01	±10%	----------	----------	----------	----------	----------	----------
No color	----------	----------	±20%	----------	----------	----------	----------	----------	----------

APPENDIX IV

ABBREVIATIONS AND SYMBOLS

1. Abbreviations

a-c	alternating-current
a-f	audio-frequency
B	flux density
C	capacitance
cemf	instantaneous counter electromotive force
cm	centimeters
cps	cycles per second
C_T	total capacitance
d	distance between points
d-c	direct-current
de	change in voltage
di	change in current
dq	change in charge
dt	change in time
E	voltage
e	instantaneous voltage
E_c	capacitive voltage
e_c	instantaneous capacitive voltage
E_L	inductive voltage
e_L	instantaneous inductive voltage
E_m	maximum voltage
E_{max}	maximum voltage
emf	electromotive force
E_p	primary voltage
E_s	secondary voltage
f	frequency
f_r	frequency at resonance
H	magnetizing flux
h	henry
I	current
i	instantaneous current
I_c	capacitive current
i_c	instantaneous capacitive current
I_{eff}	effective current
i-f	intermediate-frequency
I_L	inductive current
i_L	instantaneous inductive current
I_m	maximum current
I_{max}	maximum current
I_p	plate current
I_R	current through resistance
i_R	instantaneous current through resistance
I_s	secondary current
I_T	total current
IØ	phase current
K	coefficient of coupling
kc	kilocycle
L	inductance
L-C	inductance-capacitance
L-C-R	inductance-capacitance-resistance
L_T	total inductance
mh	millihenry
N	revolutions per minute
N_p	primary turns
N_s	secondary turns
P	power
p	instantaneous power
P_{ap}	apparent power
P_{av}	average power
P_p	primary power
P_s	secondary power
Q	charge or quality
q	instantaneous charge
r-f	radio-frequency
R_G	grid resistance
R_o	load resistance
rpm	revolutions per minute
sq cm	square centimeters
t	time constant
t	time (seconds)
μf	microfarad
$\mu\mu$f	micromicrofarad
V, v	volt
X_c	capacitive reactance
X_L	inductive reactance
Z	impedance
Z_o	load impedance
Z_p	primary impedance
Z_s	secondary impedance
Z_T	total impedance

ϕ_M mutual flux
ϕ_P primary flux
ϕ_S secondary flux

2. Symbols

A complete set of symbols as used in this manual is shown in figure 256.

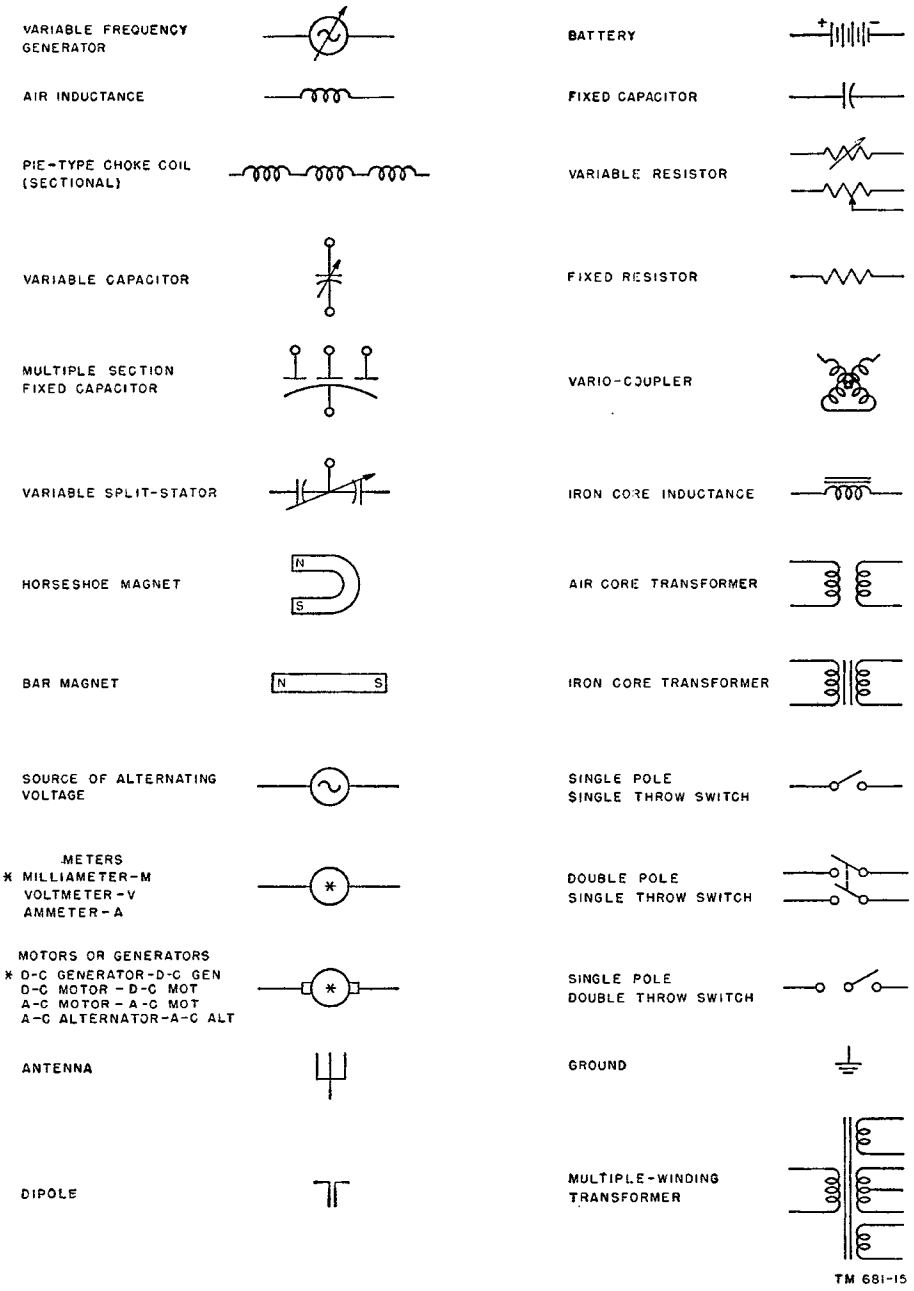


Figure 256. Symbols.

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